THE GROVND OF ARTS,

EACHING THE PERFECT worke and practise of Arithmeticke, both in whole Numb[illegible] and Fractions, after a more easie and exact forme then in former time hath beene set forth: Made by M. ROBERT RECORD D. *in Phisick.*

Afterward, augmented by Mr. Iohn Dee.

And since enlarged with a Third part of Rules of Practise, abridged into a briefer method then hitherto hath been published, with diuers necessary Rules incident to the Trade of Merchandise: with Tables of the valuation of all Coynes, as they are currant at this present time:

By IOHN MELLIS.

And now diligently perused, corrected, illustrated and enlarged; with an *Appendix* of figurate Numbers, and the Extraction of their Roots, according to the Method of *Christian Vrstitius*: with Tables of Board and Timber measure; and new Tables of *Interest* vpon *Interest*, after 10 and 8 *per* 100; with the true value of Annuities to be bought or sold present, Respited, or in Reuersion: the first calculated by R.C. but corrected, and the latter diligently calculated by *Robert Hartwell, Philomathemat.*

Scientia non habet inimicum, nisi ignorantem.
Fide ——— sed ——— vide.

LONDON:
Printed by THO. HARPER, for *Iohn Harison*, and are to be sold at his shop in Pater noster Row, at the signe of the Vnicorne 1636.

1888
1636
Year 1702

That which my friend hath well begun,
For very loue to Common weale,
Need not all whole to be new done,
But new increase I do reueale.

Something herein I once redrest,
And now againe for thy behoofe,
Of zeale I do, and at request,
Both mend and adde, fit for all proofe.

Of numbers vse, the endlesse might,
No wit nor language can expresse,
Apply and try both day and night,
And then this truth thou wilt confesse.

I. Dee.

The Books Verdict.

To please or displease sure I am,
But not of one sort to euery man:
To please the best sort would I faine,
The froward displease shall I certaine.
Yet wish I will, though not with hope,
All eares or mouthes to please or stop.

TO THE MOST mighty Prince *Edward* the sixth, by the grace of God King of *England, France*, and *Ireland*, *&c.*

THe excellency of mans nature being such, as it is by Gods diuine fauour (most mighty Prince) not only created in highnesse of degree farre aboue all other corporall things, but by perfection, reason and search of wit, much approaching toward the Image of God, as not onely the holy Scriptures do testifie, but also those naturall Philosophers, which exactly did consider the nature of man, and namely the far reach and infinite compasse of the works of the minde, were inforced to confesse, that man scarsely was able to know himselfe. And if he would duely ponder the nature of himselfe, he would find it so strange, that it might seeme vnto him a very miracle. And thereof sprang that saying: *Magnum miraculum est homo, maximum miraculum sapiens homo.* For vndoubtedly, as man is one of the greatest miracles that euer God wrought, so a wise man is plainly the greatest.

And therefore was it that some did account the head of a man the greatest miracle in the world, because not onely of the strange workmanship that is in it, but much more of the efficacie of reason, wit, memory, imagination, and such other powers, and works of the minde, which can more easily conceiue any thing in a manner, then vnderstand it selfe. Amongst all the creatures of God, it findeth none more difficult to be perceiued then these same powers of it selfe; whereby it doth conceiue and iudge: as it may be well coniectured by the diuersity of opinions, that the wisest Philosophers did vtter touching the spirit of man, & the substance of it: whereof, I now intend to make no rehearsall; but whoso listeth to

read thereof, may finde it largely set forth, not onely in *Aristotle* his books, *De anima*, but also in *Galene*, his book called *Historia Philosophica*: and againe in *Plutarch* his worke, *De Philosophorum placitis*, whose words are also repeated of *Eusebius* in the xv book, τῆς εὐαγγελικῆς προπαρασκευῆς, vnto whom I remit them that haue desired to vnderstand intricate difficulty of knowing our owne selues, as touching our best part, and that part where we deserue to beare the name of men.

This matter seemed so obscure and difficult in knowledge, that *Galen*, who for his excellent wisedome and iudgement in naturall works, is called of many men Miracle in nature, yet in searching the nature and substance of the spirit of man, he not onely confesseth himselfe ignorant, but counteth it plaine temerity to attempt to finde it. So farre aboue the hope of mans knowledge is that part, whereby man doth know and judge of things. And although the ignorant sort (which hate all things that they know not) do little esteeme the profoundnesse of mans spirit and of reason, the chiefe power and faculty of it: yet as there is a kinde of feare and obedience of vnreasonable Beasts vnto man, by the working power of God, so is there in those smal reasoned persons, a certaine kinde of reuerence toward wisedome and reason, which they do shew oftentimes, and by power of perswasion, are inforced to obey reason, will they nill they.

And hereby came it to passe, that the rudenesse of the first age of Man was brought vnto some more ciuil trade, as it is well declared by *Cicero*, in the beginning of his first book, *De Inventione Rhetorica*, where he saith thus: *Nam fuit quoddam tempus quum in agris homines passim bestiarum more vagabantur, & sibi victu ferino vitam propagabant, nec ratione animi quicquam, sed pleraq; viribus corporis administrabant. Nondum diuinæ religionis, non humani ratio colebatur. Nemo legitimas viderat nuptias, non certos quisquam inspexerat liberos; non ius æquabile quid vtilitatis haberet, acceperat: ita propter errorem atq; inscitiam cæca ac temeraria dominatrix animi cupiditas, ad se explendam viribus corporis abutebatur, perniciocissimis satellitibus. Quo tem-*

pore

pore quidam, magnus videlicet vir & sapiens, cognouit quæ materia esset, & quanta ad maximas res opportunitas in animis inesset hominum, si quis eam possit elicere, & præcipiendo meliorem reddere. Qui dispersos homines in Agris, & in tectis syluestribus abditos, ratione quadam compulit in vnum locum, & congregavit: & eos in vnamquamq; rem inducens vtilem atq; honestam, primò propter insolentiam reclamantes, deinde propter rationem atq; orationem studiosiùs audientes, ex feris & immanibus, mites reddidit, & mansuetos.

This long repetition of *Tullyes* words will seeme tedious to them that loue but little, and care much lesse for the knowledge of reason, but vnto your Majesty (I dare say) it is a delectable remembrance, and vnto me it seemed so pleasant, that I could scarce stay my pen from writing all that mine eyes did so greedily reade.

This sentence of *Cicero* am I loath to translate into English, partly for that vnto your Majesty it needeth no translation, but especially knowing how farre the grace of *Tullyes* Eloquence doth excell any Englishmans tongue, and much more exceedeth the basenesse of my barbarous style: yet for the fruit of the sentence, I had rather vnto my meer English countrimen vtter the rudenesse of my translation, then to defraud them the benefit of so good a lesson; trusting they will so learne to loue reason, that they will also gladly and greedily embrace all good Sciences, that may help to the just furniture of the same, when they consider that informed reason was the onely instrument, or at least the chiefest meanes to bring men into ciuill regiment, from barbarous manners and beastly conditions.

" For the time was (saith *Tully*) that men wandred a-
" broad in the fields vp and downe like beasts, and vsed
" no better order in feeding then they; so that by reasons
" rule they wrought nothing, but most of their do-
" ings did they atchieue by force of strength. At this
" time there was no iust regard of religion toward God,
" nor of duty toward man. No man had seen right vse of
" marriage, neither did any man know their owne chil-
" dren from other; nor no man had felt the commodity
" of iust Laws: so that through errour and ignorance,
" wilfull lust, like a blinde and heady ruler, abused bodi-

" ly strength as a most mortall minister for the satisfy-
"ing of his desire. At that time was there one which
" not onely in power, but also in wisedome was great,
" and he considered how that in the mindes of men
" was both apt instruments, and great occasion to the
" due accomplishment of most weighty affaires, if a man
" could apply them to vse, & by teaching of rules frame
" them to better trade. This man with perswasion of
" reason gathered into one place the people that were
" wandring about the fields, and lay lurking in wilde
" cottages and woods, & bringing them into one com-
" mon society, did trade them to al such things, as either
" were profitable or honest, although not without re-
" pining at the first, by reason that they had not been so
" accustomed before: Yet at length through reason and
" perswasion of words they obeyed him more diligent-
" ly, and so of a wilde and cruell people, he made them
" curteous and gentle.

Thus hath *Tully* set forth the efficacie of reason and perswasion, how it was able to conuert wilde people to a mildnesse, and to change their furious cruelnesse into gentle curtesie: were it not now a great reproach in this our time (when knowledge raigneth so large) that men should shew themselues lesse obsequious to reason? Vnlesse it may be thought, that now euery man hauing sufficient knowledge of himselfe, needeth not to hearken to the perswasion of others.

Indeed he that thinketh himself wise, will not esteeme the reason of any other, be he neuer so wise; so that of such a one it may well be said; He that thinketh himselfe wiser then he is, may justly be counted a double foole. Wherefore such men are not to be permitted in open audience to talke, but must be put to silence, and be made to giue eare to reason, which reason consisteth not in a multitude of words heaped rashly together, and applied for one purpose, but reason is the expressing of a just matter with witty perswasions furnished with learned knowledge: such knowledge had *Moses*, being expert in all learning of the Egyptians, as the Scriptures declare, and therefore was able to perswade the stubborn

people of the Iews, although not without paine. Such knowledge and such reasons did *Druys* shew, which was the first Law maker of all the West part of Europe. Like reason and wisedome did *Xamolxis* vse amongst the Goths: *Lycurgus* vnto the Lacedemonians, *Zeleucus* to the Iberians, *Solon* to the Athenienses, & *Dunwallo Molmutius* two thousand yeares past amongst the old Brittains of this Realme. And hereby came it to passe, that their Laws continued long, till more perfect reason altred many of them, and wilfull power oppressed most of them.

Druis was sonne to King Sarron, and succeeded him in his kingdome.

At the beginning when these wise men perceiued how hard it was to bring the rude people to vnderstand reason, they iudged the best meanes to attaine this honest purpose to depend of learning in euery kinde: for by learning (as *Ouid* saith,) *Pectora mollescunt asperitasque fugit*, Stout stomacks do waxe milde, and sharpe fiercenes is exilde. Therefore as *Berosius* doth testifie, *Sarron* that was the third King ouer all this West part of Europe, for to bring the people from beastly rage to manly reason, did erect Schools of liberall Arts, which took so good successe, that his name continued in that sort famous aboue two thousand yeares after, for *Diodorus Siculus* which was in the time of *Iulius Cæsar*, maketh mention of the learned men or Gothes of Celtes, and nameth them *Sarronides*, that is to say, *Sarron his Scholars and followers.*

Among these Arts that then were taught, some did informe the tongue, and make them able both to vtter aptly their minde, and also to perswade, as Grammar, Logicke, and Rhetoricke, although not so curiously as in this time; some other did appertaine to the iust order of partition of lands, the true vsing of weights, measures & reckonings in all sorts of bargains, and for order of building and sundry other vses; those were Arithmetick and Geometry. Againe, to incourage men to the honour of God, they taught Astronomy, whereby the wonderfull works of God were so manifestly set forth, that no mans tongue nor penne can in like sort expresse his infinite power, his vnspeakable wisedome, and his exceeding goodnesse toward man, whereby he doth bountifully

prouide for man all necessaries, not only to liue, but also to liue pleasantly. And so was their confidence in Gods prouidence strongly stayed, knowing his goodnesse to be such, that he would help man as he could, and his power to be so great, that he could do what he would: and thirdly his wisedome to be so pure, that he would do nothing, but that that was best. Beside these Sciences they taught also Musick, which most commonly they did apply partly to religious Seruices, to draw men to delight therein, & partly to songs made of the manners of men, in praise of Vertue, and discommendation of Vice, wherby it came to passe, that no man would displease them, nor do any thing euill that might come to their hearing: for their songs did make euill men more abhorred in that time than any excommunication doth in this time. The posterity of these Musitians continue yet both in Wales and Ireland, called *Bardes* vnto this day, by the ancient name of *Bardus*, their first founder.

This Bardus Druidius the 5. king of the Celtes, reigned 65 yeares, and died 1832 yeares before Christ.

And as these Sciences did encrease, so did Vertue encreale thereby. Againe, as those Sciences did decay, so vertue lost her estimation, and consequently was little in vse: whereof to make a full declaration were a thing meete for a Prince to heare, but it would require a peculiar Treatise. Wherefore at this present I count it sufficient lightly to haue touched this matter in generall words, and to say no more of the particularity thereof, but onely touching one of those Sciences, that is, *Arithmetick*, by which not onely just partition of lands was made, but also touching buying and selling, all Assises, Weights, and Measures were deuised, and all reckonings and accounts driuen, yea by proportion of it were the true orders of justice limited, *as Aristotle* in his Ethicks doth declare, and the degrees of estates in the commonwealth established: Although that proportion be called Geometricall, and not Arithmeticall, yet doth that proportion appertaine to the Art of Arithmeticke, and in Arithmeticke is taught the progression of such proportions, & all things thereto belonging. Wherefore I may well say, that seeing Arithmeticke is so many wayes needfull vnto the first planting of a Common-

wealth,

wealth, it must needs be as much required to the preseruation of it also, for by the same meanes is any Common-wealth continued, by which it was erected and established. And if I shal in small matters in appearance, but indeed very weighty, put one example or two: What shal we say for the Statutes of this Realme, which be the only stay of good order in manner now? As touching the measuring of ground by length and breadth, there is a good and an ancient Statute made by Art of Arithmetick, & now it shall be to little vse, if by the same Art it be not practised and tried. For the assise of Bread and Drinke, the two most common and most necessary things for sustentation of man, there was a goodly ordinance in the Law made, which by ignorance hath so growne out of knowledge and vse, that few men do vnderstand it, & therefore the Statute books wonderfully corrupted, and the Commons cruelly oppressed: notwithstanding some men haue written, that it is too doubtfull a matter to execute those assises by those Statutes, by reason they depend of the standard of the coyne, which is much changed from the state of that time, when those Statutes, were made. Thus shall euery man read (that listeth) in the Abridgement of the Statutes, in the title of Weights and Measures, in the seuenth number of the English Book, where he should haue translated a good ordinance which is set forth in the French book: but no maruell if the Abridgement doth omit it, seeing the great Book of Statutes doth omit the same Statute as it hath done diuers other very good Laws. And this is the fruit of ignorance, to reject and condemne all that it vnderstandeth not, although they vse some cloaks for it: but such cloaks as being allowed, might serue to repell all good Laws, which God forbid.

Againe, there is an ancient order for Assise of fire, Wood and Coals, which was renued not many yeares past; and now how Auarice and Ignorance doth canuase that Statute, it is too pittifull to talke of, and more miserable to feele.

Furthermore, for the Statute of Coynage, and the Standard thereof, if the people vnderstood rightly the

Statute,

Statute, they should not, nor would not (as they often do) gather an excuse for their folly thereby: but as I said, these Statutes by wisedome and good knowledge of Arithmeticke were made, and by the same must they be continued. And let ignorance no more meddle with the vse of them, than it did with the making of them. Oh in how miserable case is that Realm, where the ministers & interpreters of the Law are destitute of all good Sciences, which be the keyes of the Laws. How can they either make good Laws, or maintaine them, that lack that true knowledge whereby to iudge them? And happie may that Realme be accounted, where the Prince himselfe is studious of learning, and desireth to vnderstand equity in all Laws. Therefore most happy are we the louing Subjects of your Majestie, which may see in your Highnesse not onely such towardnes, but also such knowledge of diuers Arts, as seldome hath been seene in any Prince of such yeares, whereby we are enforced to conceiue this hope certainly, that he which in those yeares seeketh knowledge, when knowledge is least esteemed, and of such an age, can discerne them to bee enemies both to his Royall person, and to his Realmes, which labour to withdraw him from knowledge, to excessiue pastime, and from reasonable study, to idle or noysome pleasures, he must needs, when he commeth to more mature yeares, be a most prudent Prince, a most just Governour, and a right Iudge, not onely of his Subjects commonly, but also of the ministers of his Laws, yea, and of the Laws themselues: and to be able to conceiue the true equity and exact vnderstanding of all his Laws and Statutes, to the comfort of his good Subjects, and the confusion and reproach of them which labour to obscure or peruert the equity of the same Laws and Statutes. How some of these Statutes may be applyed to vse, as well in this our time, as in any other time, I haue peculiarly declared in this Booke, and some other I haue omitted for just considerations, till I may offer them first vnto your Majestie, to weigh them, as to your Highnesse shall seeme good: for many things in them are not to be published without your Highnesse

knowledge

knowledge and approbation, namely, because in them is declared all the rates of alloyes for all standards from one ounce vpward, with other mysteries of mint matters, and also most part of the varieties of coynes that haue been curant in this your Maiesties Realme by the space almost of six hundred yeares last past, and many of them that were currant in the time that the Romans ruled here.

All which with the ancient description of England and *Ireland*, and my simple censure of the same, I haue almost completed to be exhibited to your Highnesse. In the meane season most humbly beseeching your Maiesty to accept this simple Treatise, not worthy to be presented to so high a Prince, but that my lowly request to your Maiesty is, that this amongst other of my Books may passe vnder the protection of your Highnesse, whom I beseech God most earnestly and daily, according to my duty, to aduance in all honour, and Princely Regality, and to increase in all knowledge, iustice, and godly policy. Amen.

Your Maiesties most obedient Subiect and seruant,

ROBERT RECORD.

TO

TO THE LOVING *Readers: the Preface of* Mr. Robert Record.

SOre oft times haue I lamented with my selfe the vnfortunate condition of England, seeing so many great Clerks to arise in sundry other parts of the world, and so few to appeare in this our Nation: whereas for pregnancy of naturall wit (I thinke) few Nations do excell Englishmen. But I cannot impute the cause to any other thing, than to the contempt or misregard of learning. For as Englishmen are inferiour to no men in mother wit, so they passe all men in vaine pleasures, to which they may attain with great paine and labour: and are as slacke to any neuer so great commodity, if there hang of it any painfull study or trauelsome labour.

Howbeit, yet all men are not of that sort, though the most part be, the more pity it is: but of them that are so glad, not onely with painfull study, and studious paine to attaine learning, but also with as great study and paine to communicate their learning to other, and make all England (if it might be) partakers of the same; the most part are such, that vnneth they can support their owne necessary charges, so that they are not able to beare any charges in doing of that good, that else they desire to do.

But a greater cause of lamentation is this, that when learned men haue taken pains to do things for the

the aid of the vnlearned, scarce they shall be allowed for their wel-doing, but derided and scorned, and so vtterly discouraged to take in hand any like enterprise againe. So that if any be found (as there are some) that do fauour learning, & learned wits, and can be content to further knowledge, yea onely with their word; such persons though they be rare, yet shall they encourage learned men to enterprise some thing at the least that England may reioyce of. And I haue good hope th t England will (after she hath taken some sure taste of learning) not onely bring forth more fauorers of it, but also such learned men that she shall be able to compare with any Realme in the world. But in the meane season, where so few regarders of learning are, how greatly they are to be esteemed that do fauour and further it, my pen will not suffice at full to declare.

Therfore gentle Reader, whereas I do vpon most iust occasion iudge, yea and know assuredly, that there be some men in this Realme, which both loue and also much desire to further good learning, and yet am not wel able to write their condigne praise for the same, I thinke it better with silence to ouerpasse it, then either say too little of it, or to prouoke against them the malice of such other, which do nothing themselues that is praise-worthy, and therefore cannot abide to heare the praise of any other mans good deed.

And considering their great fauour vnto learning, though I my selfe be not worthy to be reckoned in the number of great learned men, yet am I bold to put my selfe in Presse, with such ability as God

hath

hath lent me, though not with so great cunning as many men, yet with as great affection as any man, to help my Countrey-men, and will not cease daily, (as much as my small ability will suffer me) to endite some such thing, that shall be to the instruction, though not of learned men, yet at the least of the vulgar sort, whose argument alwayes shall be such as it shall delight all learned wits; though they do not learne any great things out of it.

But to speak of this present book of Arithmetick, I dare not, nor will not set it forth with any words, but remit it to the iudgement of all gentle Readers, and namely, such as loue good learning, beseeching them so to esteeme it, as it doth seeme worthy. And so either to accept the thing for it selfe, either at the least to allow my good endeauour. But I perceiue I need not vse any perswasions vnto them; whose gentle nature and fauourable mind is ready to receiue thankfully, and interpret to the best all such enterprises attempted for so good an end, though the thing do not alwaies satisfie mens expectation. This considered, did bolden me to publish abroad this little book of the Art of numbring, which if you shall receiue fauourably, you shall encourage me to gratifie you hereafter with some greater thing.

And as I iudge some men of so louing a minde to their natiue Countrey, that they would much reioyce to see it prosper in good learning, and witty Arts; so I hope well of all the rest of Englishmen, that they will not be vnmindfull of his due praise, by whose meanes they are helped and furthered in any thing. Neither ought they to esteeme this thing

of

of so little value, as many men of little discretion oftentimes do. For who so setteth small price by the witty deuice and knowledge of numbring, he little considereth it to be the chiefe point, (in manner) whereby men differ from all bruit beasts: for as in all other things (almost) beasts are partakers with vs, so in numbring we differ cleane from them, and in manner peculiarly, sith that in many things they excell vs againe.

The Fox in crafty wit exceedeth most men,
A Dogge in smelling hath no man his peere:
To foresight of weather if you look then,
Many beasts excell men; this is cleere.
The wittinesse of Elephants doth letters attain,
But what cunning doth there in the Bee remain?
The Emmet foreseeing the hardnesse of Winter,
Prouideth victuals in time of Summer.
The Nightingale, the Linet, the Thrush, the Larke,
In Musicall harmony passe many a Clarke.
The Hedghogge of Astronomy seemeth to know,
And stoppeth his (ane where the winde will blow.
The pider in weauing such Art doth show,
No man can him mend, nor follow I trow.
When a house will fall, the Mice right quicke,
Flee thence before; can man do the like?

Many things else of the wittinesse of Beasts and Birds might I here say, saue that another time of them I intend to write wherin they excell in manner all men, as it is daily seene: but in number was there neuer beast found so cunning, that could know or discern one thing from many, as by daily experience you may well consider, when a Bitch hath many whelps, or a Hen many chickens: & likewise of other whatsoeuer they be, take from them all their young

sauing

sauing onely one, and you shall perceiue plainly, that they misse none, though they will resist you in taking them away, and will seeke them againe if they may know where they be, but else they will neuer misse them truly; but take away that one that is left, and then will they cry and complaine, & restore to them that one, then are they pleased againe. So that of number, this may I iustly say, it is the onely thing almost that separateth man from beasts. He therefore that shall contemne number, declareth himselfe as brutish as a beast, & vnworthy to be counted in the fellowship of men. But I trust there is no man so foule ouer-seene, though many right smally do it regard.

Why the Author wrot in Dialogue wise.

Therefore will I now stay to write against such, and returne againe to this my Booke, which I haue written in the forme of a Dialogue, because I iudge that to be the easiest way of instruction, when the Scholar may aske euery doubt orderly, and the Master may answer to his question plainly.

Howbeit I thinke not the contrary, but as it is easier to blame another mans worke, then to make the like, so there will be some that will find fault, because I write in a Dialogue: but as I coniecture, those shall be such as do not, cannot, or will not perceiue the reason of right teaching, and therefore are vnmeet to be answered vnto, for such men with no reason will be satisfied.

And if any man obiect, that other Books haue bin written of Arithmetick *already so sufficiently, that I needed not now to put Pen to the Booke, except I will condemne other mens writings: to them*

I answer. That as I condemne no mans diligence, so I know that no one man can satisfie euery man: and therefore like as many do esteeme greatly other Books, so I doubt not but some will like this my book aboue any other English Arithmeticke hitherto written; and namely, such as shall lack instructers, for whose sake I haue so plainly set forth the Examples, as no Book that I haue seene hath done hitherto: which thing shall be great ease to the rude Readers.

Therefore (gentle Reader) though this Booke can be but small aid to the learned sort, yet vnto the simple ignorant (which needeth most help) it may be a good furtherance and meane vnto knowledge.

And though vnto the King his Maiesty priuately I do it dedicate, yet I doubt not (such is his clemency) but that he can be content, yea, and much desirous, that all his louing Subiects shall take the vse of it, and imploy the same to their most profit: Which thing if I perceiue that they thankefully do, and receiue with as good will as it was written, then will I shortly with no lesse kindnesse set forth such introductions into Geometry and Cosmography, as I haue at times promised, and as hitherto in English hath not beene enterprised, wherewith I dare say all honest hearts will be pleased, and all studious wits greatly delighted.

I will say no more, but let euery man iudge as he shall see cause. And thus for this time I will stay my Pen, committing you all to that true fountaine of perfect number, which wrought the whole world by number and measure: he is Trinity in Unity, and Unity in Trinity: to whom be all praise, honour, and glory. Amen.

Here followeth a Table of the whole Contents of this Book.

¶ The Contents of the first Dialogue.

The

The Contents.

The Contents of the third part.

The Contents of this Booke

A Collection of ſuch Tables as are contained in this Treatiſe.

Before

Before the Introduction of Arithmeticke, it were very good to haue ſome vnderſtanding and knowledge of theſe Figures and Notes.

i	1	*one.*	xx	20	*twenty*
ii	2	*two.*	xl	40	*forty*
iii	3	*three.*	l.	50	*fifty*
iiii	4	*foure.*	lx.	60	*ſixty*
v	5	*fiue.*	lxx.	70	*ſeuenty*
vi	6	*ſix.*	xc.	90	*ninety*
vii	7	*ſeuen.*	C.	100	*a hundred*
viii	8	*eight.*	CC.	200	*2 hundred*
ix	9	*nine.*	D.	500	*5 hundred*
x	10	*ten.*	DC	600	*6 hundred*
xi	11	*eleuen.*	M.	1000	*a thouſand.*
xii	12	*twelue.*	MD	1500	*a thou. 5. hund.*
&c.	&c.	&c.	&c.	&c.	&c.

A Dia-

The wine gallon containes 231 inches

the beere gallon containes 282 inches

The Wine Gallon contains

231 Solid Inches

The Beer Gallon contains

282 Solid Inches

A Dialogue betweene the Master and the Scholar: teaching the ART and vse of Arithmetick with Pen.

The Scholar speaketh.

SIR, such is your authoritie in mine estimation, that I am content to consent to your saying, and to receiue it as truth, though I see none other reason that doth leade me thereunto: whereas else in mine owne conceit appeareth but vaine, to bestow any time priuately in learning of that thing, that euery childe may, and doth learne at all times and houres, when he doth any thing himselfe alone, and much more when he talketh or reasoneth with others.

Master. Lo, this is the fashion and chance of all them that seeke to defend their blinde ignorance, that when they thinke they haue made strong reason for themselues, then haue they proued quite contrary. For if numbring be so common (as you grant it to be) that no man can do any thing alone, and much lesse talke or bargaine with other, but

he

he shall still haue to do with number: this proueth not number to be contemptible and vile, but rather right excellent, and of high reputation, sith it is the ground of all mens affaires, so that without it no tale can be told, no communication without it can be continued, no bargaining without it can duely be ended, or no businesse that man hath, iustly completed. These commodities, if there were none other, are sufficient to approue the worthinesse of number. But there are other innumerable, farre passing all these, which declare number to excéed all praise. Wherefore in all great works are Clerks so much desired? Wherefore are Auditors so richly fed? What causeth Geometricians so highly to be enhaunsed? why are Astronomers so greatly aduanced? because that by number such things they finde, which else would farre excell mans minde.

Schollar. Verily Sir, if it be so, that these men by numbring, their cunning do attaine, at whose great works most men do wonder, then I sée well I was much deceiued, and numbring is a more cunning thing then I took it to be.

Master. If number were so vile a thing as you did estéeme it, then néed it not to be vsed so much in mens communication. Exclude Number, and answer to this question: how many yeares old are you?

Scholar.

Scholar. Mum.

Master. How many dayes in a weeke? How many weekes in a yeare? What lands hath your Father? How many men doth hée kéep? How long is it since you came from him to me?

Scholar. Mum.

Master. So that if number want, you answer all by Mummes: How many miles to London?

Scholar. A poake full of plummes.

Master. Why, thus you may sée, what rule number beareth, and that if number be lacking, it maketh men dumbe, so that to most questions they must answer Mum.

Scholar. This is the cause (sir) that I iudged it so vile, because it is so common in talking every while: For plenty is not daintie, as the common saying is.

Master. No, nor store is no sore perceiue you this? The more common that the thing is, being néedfully required, the better is the thing, and the more to be desired. But in numbring, as some of it is light and plaine, so the most part is difficult, and not easie to attaine. The easier part serueth all men in common, and the other part requireth some learning. Wherefore as without numbring a man can do almost nothing, so with the helpe of it you may attaine to all things.

Scholar. Yea sir, why then it were best to

learne

learne the Art of numbring, first of all other learning, and then a man need learne no more, if all other come with it.

Master. Nay, not so: but if it be first learned, then shall a man be able (I meane) to learne, perceiue, and attaine to other Sciences; which without it he could neuer get.

Scholar. I perceiue by your former words that Astronomy and Geometry depend much on the helpe of numbring: but that other Sciences, as Musicke, Physicke, Law, Grammer, and such like, haue any helpe of Arithmeticke, I perceiue not.

Master. I may perceiue your great Clerklinesse by the ordering of your Sciences: but I will let that passe now, because it toucheth not the matter that I intend, and I will shew you how Arithmeticke doth profit in all these somewhat grosly, according to your small vnderstanding, omitting other reasons more substantiall.

First (as you reckon them) Musicke hath not onely great helpe of Arithmeticke, but is made, and hath his perfectnesse of it: for all Musicke standeth by number and proportion: And in Physicke, beside the calculation of criticall dayes, with other things, which I omit, how can any man iudge the pulse rightly, that is ignorant of the proportion of numbers?

Musick.

Phisick.

And as for the Law, it is plaine, that the man

Law.

man

man that is ignorant of Arithmeticke, is neither meet to be a Iudge, neither an Aduocate, nor yet a Proctor. For how can he well understand another mans cause, appertaining to distribution of goods, or other debts, or of summes of money, if he be ignorant of Arithmeticke? This oftentimes causeth right to be hindered, when the Judge either delighteth not to heare of a matter that he perceiueth not, or cannot iudge for lacke of understanding: this commeth by ignorance of Arithmeticke.

Now, as for Grammer, me thinketh you should not doubt in what it néedeth number, sith you haue learned that Nounes of all sorts, Pronounes, Verbes, and Participles are distinct diuersly by numbers: besides the variety of Nounes of Number, and Aduerbes. And if you take away number from Grammer, then is all the quantity of Syllables lost. And many other wayes doth number helpe Grammer. Whereby were all kinds of Méeters found and made? was it not by Number? Grammer.

But how néedfull Arithmeticke is to all parts of Philosophy, they may soone sée, that do reade either Aristotle, Plato, or any other Philosophers writings. For all their examples almost, and their probations, depend of Arithmeticke. It is the saying of Aristotle, that he that is ignorant of Arithmeticke, is méete for no Science. And Plato his Master Philosophy.

wrote

wrote a little sentence ouer his Schoolhouse dore, Let none enter in hither (quoth he) that is ignorant of Geometry. Seeing he would haue al his Scholars expert in Geometry, much rather he would the same in Arithmeticke, without which Geometry cannot stand.

Diuinity. And how needfull Arithmeticke is to Diuinity, it appeareth, seeing so many Doctors gather so great mysteries out of number, and so much do write of it. And if I should go about to write all the commodities of Arithmeticke in ciuill acts, as in gouernance of commonweales in time of peace, and in due prouision, and order of armies, in time of warre, for numbring of the host, summing of their wages; prouisions of victuals, viewing of Artillery, with other Armour; beside the cunningest point of all, for casting of ground, for encamping of men, with such other like: And how many waies also Arithmeticke is conducible for all priuate weales, of Lords and all possessioners, of Merchants and all other occupiers, and generally for al estates of men, besides Auditors, Treasurers, Receiuers, Stewards, Bailiffes, and such like, whose offices without Arithmetick, are nothing: if I should (I say) particularly repeat all such commodities of the noble Science of Arithmetick, it were enough to make a very great book.

Armie.

Scholar. No, no sir, you shall not need: For I doubt not, but this, that you haue said, were enough

enough to perswade any man to thinke this Art to be right excellent and good, and so necessary for man, that (as I thinke now) so much as a man lacketh of it, so much he lacketh of his sense and wit.

Master. What, are you so farre changed since, by hearing these few commodities in generall: by likelyhood you would be farre changed if you knew all the particular commodities.

Schollar. I beseech you Sir, reserue those commodities that rest yet behinde unto their place more conuenient: and if ye will be so good as to utter at this time this excellent treasure, so that I may be somewhat inriched thereby, if euer I shall be able, I will requite your paine.

Master. I am very glad of your request, and will do it spedily, sith that to learne it you be so ready.

Schollar And I to your authority my wit do subdue, whatsoeuer you say, I take it for true. The duty of a Schollar.

Master. That is too much, and meete for no man to be beleeued in all things, without shewing of reason. Though I might of my Schollar some credence require, yet except I shew reason, I do it not desire. But now sith you are so earnestly set this Art to attaine, best it is to omit no time, lest some other passion coole this great heate, and then Perseuerance in study.

you leaue off before you sée the end.

Scholar. Though many there be so vnconstant of minde, that flitter and turne with euerie winde, which often begin, and neuer come to the end, I am none of this sort, as I trust you partly know. For by my good will what I once begin, till I haue it fully ended, I would neuer blin.

Master. So haue I found you hitherto indéed, and I trust you will increase rather then go backe. For better it were neuer to assay, then to shrinke and flie in the mid-way: But I trust you will not so do, therefore tell mée briefly: What call you the Science that you desire so greatly?

Scholar. Why Sir, you know.

Master. That maketh no matter, I would heare whether you know, and therefore I aske you. For great rebuke it were to haue studied a Science, and yet cannot tell how it is named.

Schollar. Some call it Arsmetricke, and some Augrime.

Master. And what do these names betoken?

Schol. That, if it please you, of you would I learne.

Master. Both names are corruptly written: Arsmetricke for Arithmeticke, as the Greekes call it, and Augrim for Algorisme, as the Arabians sound it: which doth betoken

Αριθμη-τική.

ken the Science of Numbring: for Arithmos in Greeke is called Number; and of it commeth Arithmeticke, the Art of Numbring, So that Arithmeticke is a Science or Art teaching the manner and vse of Numbring: This Art may be wrought diuersly, with Pen or with Counters. But I will first shew you the working with the Pen, and then the other in order. Αριθμος

Scholar. This I will remember. But how many things are to be learned to attaine this Art fully?

Master. There are reckoned commonly seuen parts or workes of it.

Numeration, Addition, Subtraction, Multiplication, Diuision, Progression, and Extraction of roots: to these some men adde Duplication, Triplation, and Mediation. But as for these three last they are contained vnder the other seuen. For Duplication and Triplation are contained vnder Multiplication, as it shall appeare in their place: And Mediation is contained vnder Diuision, as I will declare in his place also.

Scholar. Yet then there remaine the first seuen kinds of Numbring.

Master. So there doth: Howbeit if I shall speake exactly of the parts of Numbring, I must make but fiue of them: for Progression is a compound operation of Addition, Multiplication, and Diuision. And so is the Ex-

you leaue off before you see the end.

Scholar. Though many there be so uncon-stant of minde, that flitter and torne with euerie winde, which often begin, and neuer come to the end, I am none of this sort, as I trust you partly know. For by my good will what I once begin, till I haue it fully ended, I would neuer blin.

Master. So haue I found you hitherto indeed, and I trust you will increase rather then go backe. For better it were neuer to assay, then to shrinke and flie in the mid-way: But I trust you will not so do, therefore tell me briefly: What call you the Science that you desire so greatly?

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Scholar. Yet then there remaine the first seuen kinds of Numbring.

Master. So there doth: Howbeit if I shall speake exactly of the parts of Numbring, I must make but fiue of them: for Progression is a compound operation of Addition, Multiplication, and Diuision. And so is the Ex-

tractions of rots. But it is no harme to name them as kinds ſeuerall, ſeeing they appeare to haue ſome ſeuerall working. For it forceth not ſo much to contend for the number of them, as for the due knowledge and practiſing of them.

Schollar Then you will that I ſhall name them as ſeuen kindes diſtinct. But now I deſire you to inſtruct me in the vſe of each of them.

Maſter. So I will, but it muſt be done in order: for you may not learne the laſt as ſoone as the firſt, but you muſt learne them in that order, as I did rehearſe them, if you will learn them ſpeedily, and well.

Scholar. Euen as you pleaſe. Then to begin, Numeration is the firſt in order: what ſhall I do with it?

Maſter. Firſt, you muſt know what the thing is, and then after learne the vſe of the ſame.

Numeration.

N*Vmeration is that Arithmeticall* skill, whereby wee may duely value, expreſſe, and reade any *number* or *ſumme* propounded: or elſe in apt *figures* and *places* ſet downe any *number* knowne or named.

Schollar.

Schollar. Why? then me thinketh you put a difference betweene the value and the figures.

Master. Yea so do I. For the value is one thing, and the Figures are another thing: and that commeth partly by the diuersity of Figures, but chiefly in the places wherein they be set.

Schollar. Then I must know here three things: the value, the figure, and the place.

Master. Euen so: but yet adde Order to them as the fourth. And first marke, that there are but ten figures that are vsed in Arithmeticke; and of those ten, one doth signifie nothing, which is make like an o, and is priuately called a Cypher, though all the other sometime be likewise named. The other nine are called signifying Figures, and be thus figured. A Cypher. Figuras.

1. 2. 3. 4 5. 6. 7. 8. 9.

And this is their value:

i. ii. iii. iiii. v. vi. vii. viii. ix.

But here you must marke, that euery Figure hath two values: One alwayes certaine, that it signifieth properly, which it hath of his forme; and the other vncertaine, which he taketh of his place.

A place is called the seate or roome that a Figure standeth in. And looke how many A place.

figures are written in one summe, so many places hath that whole number. And that must be called the first place that is next to the right hand, and so reckoning by order towards the left hand, so that that place is last that is next to the left hand. As for example: If there stood before you six men in a row, side by side, and you should tell them as they stand in order, beginning with the man that were next to your right hand: then he that were next him should be called the second, and so forth to the furthest from your right hand, which is the sixt and the last.

Scholar. Sir, I perceive you well: so might I reckon Letters or any other thing. As if I should write eight letters after this order, a, b, c, d, e, f, g, h, then must I say, h, is the first, g, the second, f, the third, e, the fourth, d, the fifth, c, the sixth, b, the seventh, a, the eight.

Master. That is well done. And after the same sort use hereafter, that what I declare by one example, do you expresse by another: and so shall I perceive whether you understand it or no. And so passe over nothing, till you perceive it well, and be expert therein.

Scholar. Sir, I pray you how many of these places be there in all?

Master. There is no certaine number of them, but they are sometimes more and sometimes fewer, according to the summe that is expres-

expressed. For so many as the figures are, so many are the places: and the last place is so called, not because it is last of all other, but it is the last of that present summe, and it may be the middle place in another summe.

Scholar. Me seemeth I perceive this very well, as touching the order of reckoning of the places; but as for the number of them, you say there is no certainty. Now there resteth to declare the value of the figures by the diversity of places, which you called the value vncertaine.

Value vncertaine.

Master. But first let me heare whether you know perfectly the certaine value.

Value certaine.

Scholar. Yes Sir, as you wrote them, so I marked them.

Master. How write you then five?

Scholar. By this figure 5.

Master. And how six?

Scholar. Thus 6.

Master. Write these thrée numbers, each by it selfe, as I spake them, vii. iiii. iii.

Scholar. 7 4 3.

Master. How write you these foure other, ii. i. ix. viii.?

Scholar. Thus (I trow) 2. 1 6. 8.

Master. Nay, there you misse; look on mine example againe.

Scholar. Sir, true it is, I was to blame, I take 6 for 9, but I will beware hereafter.

Master. Now then take héed, those certaine

taine values every Figure representeth when it is alone written without other Figures ioyned to him. And also when it is in the first place, though many other do follow: as for example, This figure 9, is ix, standing now alone.

Scholar. How is he alone, and standeth in the middle of so many letters?

Master. The letters are none of his fellows. For if you were in France in the middle of a thousand Frenchmen, if there were no Englishman with you, you would reckon your selfe to be alone.

Scholar. So it is. Then 9. without moe figures of Arithmeticke betokeneth ix. whatsoever other letters be about it.

Master. Euen so, and so doth it, if it be in the first place ioyned with other, how many soeuer do follow, as in this example, 3679. You see 9. in the first place, and doth betoken nine, as it were alone.

Scholar. I perceiue that, and doth not 7. that standeth in the second place, betoken vii, and 6. in the third place betoken vi. and so 3. in the fourth place betoken three?

Master. Their figures be as you haue said, but their values are not so. For as in the first place euery figure betokeneth his owne value certaine onely, so in the second place euery figure betokeneth his owne value certaine, ten times: as in the example 7. in the second

place

place is ſeuen times ten, and is, lxx. And in the third place euery figure betokeneth his owne value an hundred times so the 6. in that place betokeneth vi. C. and in the fourth place euery figure betokeneth his owne value a M. times, as in the aforesaid number, 3. in the fourth place standeth for 3. M. and in the fifth place, euery figure ſtandeth for his owne value x M times, and in the ſixth place a C M times, and in the ſeuenth place a M M times, and in the eight place, x M M, ſo that euery place exceedeth the former ten times.

Scholar. As thus: if I make this number at all aduentures, 91359684, here are eight places. In the firſt place is 4. and betokeneth but foure. in the ſecond place is 8, and betokeneth ten times 8. that is 80. in the third place is 6, and betokeneth ſix hundred: in the fourth place 9, is nine thouſand, and 5 in the fifth place is x M times 5. that is fifty M. So 3. in the ſixth place is a C M times 3. that is, CCCM. Then 1. in the ſeuenth place, one MM. and 9. in the eight place, ten thouſand thouſand times 9. that is, xc M M. But now I cannot eaſily nor quickly reade it in order. A generall rule.

Maſter. That ſhall you practice by this meanes. Firſt put a pricke ouer the fourth figure, and ſo ouer the ſeuenth. And (if you haue ſo many) ouer the tenth, thirteenth, ſixteenth, and ſo forth, ſtill leauing two figures betweene each two prickes. And thoſe two

roomes

Ternaries. roomes betwéene the prickes are called Ternaries.

Then begin at the last pricke, and sée how many figures are betwéene him and the end, which cannot passe thrée, reckoning himselfe for one: then pronounce them as if they were written alone from the rest, and at the end of their value, so many times thousands, as your numbers haue pricks.

After that, come to the next thrée figures, and sound them as if they were apart from the rest, and adde to their value so many times thousands, as there are prickes betwéene them and the first place of your whole number. And so do by euery other thrée Figures following, if vou haue moe. As in example, 9135968 4: this was your number.

Put a pricke ouer 9 in the fourth place, and ouer 1 in the seuenth place, and then no more (for your places come not to ten) as thus: 9 1 3 5 9 6 8 4.

Now go to the last prick ouer 1, and take it and the figure 9 that followeth it, and value them alone.

Scholar. 91, that is xci.

Master. So it is, then adde for the number of your pricks twice M.

Scholar. That is, xci. thousand thousand.

Master. So it is. Then take the thrée other figures from one to the next pricke, and value them.

Scholar.

Scholar. 3 5 9. that is, CCC.lix.

Maſter. Now adde for the one pricke, that is betwéene them and the firſt place, M.

Scholar. CCC.lix. thouſand.

Maſter. Then come to the other thrée Figures that remaine.

Scholar. 6 8 4 that is, viC lxxxiiii.

Maſter. Now haue you valued all. And at the end of the laſt number you ſhall adde nothing, becauſe there remaineth no pricke nor number after it; yet proue it in another number, as thus, 2 3 0 8 6 4 0 8 9 1 0 5 3 4 0.

Schol. 2 3 0 8 6 4 0 8 9 1 0 5 3 4 0, I haue pricked them as you taught me; but I am in doubt whether I haue done well or no, becauſe of the Ciphers; for I remember you told me that they do ſignifie nothing, and therefore I doubt whether I ſhould reckon them for a figure in ſetting of the pricks, and againe, I know not wherefore they ſerue.

Maſter. That will I tell you now Indéed they are of no value themſelues, but they ſerue to make vp the number of places, and ſo make the figure following them to be in a further place, and therefore to ſignifie the more value, as in this example 90, the Cypher is of no value, but yet he occupieth the firſt place and cauſeth 9 to be in the ſecond place, and ſo to ſignifie ten times 9, that is xc. ſo that two ciphers thruſt the figure following them into the third place, and ſo forth. The uſe of Cyphers.

Schollar

Scholar. Then I perceiue in the example aboue, I haue pricked well enough: for though that Cypher that is pricked signifie nothing, yet must he haue the pricke, because he came in the thirteenth place. Then will I proue to number that summe. First, there is 230, M.M.M, M, & then followeth 864, M, M, M. And what shall I now do? There is a Cypher in the third place, and no figure after him, but they that I haue reckoned.

Master. He did serue for them that you haue already reckoned, to make them in a place farther then they should be, if he were away; and therefore now ye shall let him go. And so do alwayes when he occupyeth that place next before any pricke, which is the last of that Ternary, and a Cipher in the last place doth nothing.

Scholar. Then shall I say but 89. M. M.

Master. So, but go forth.

Scholar. 105. thousand. Now are all my prickes spent, and yet remaine 340. so that I must value them, CCC.xl. onely.

Master. Now can you reckon after this sort: and remember, that euery such roome so parted, is called a Ternary or Trinity: for you haue numbred or valued the summe most truly, and by the aide of the pricks each denomination is distinct most plainly.

Trinity.

Scholar. What call you Denomination?

Denominations.

Master.

Master. It is the last value or name added to any summe. As when I say, an hundred two and twenty pounds. Pounds is the Denomination. And likewise in saying 25 men: Men is the Denomination, and so of other: But in this place (that I spake of before) the last number of every Ternary is the Denomination of it. As for the first ternary, the Denomination is vnites, and of the second Ternary, the Denomination is thousands: And of the third Ternary, thousand thousands: or Millions: of the fourth, thousand thousand thousands, or thousand Millions: and so forth.

Schollar. And what shall I call the value of the three Figures that may be pronounced before the Denomination? as in saying: 203000000, that is, two hundred three millions. I perceive by your words, that Millions is the Denomination: but what shall I call CCiii. ioyned before the Millions?

Master. That is called the Numerator or valuer, and the whole summe that resulteth of them both, is called the summe, value, or number. Numerator. Summe or value.

Schol. Now is there any thing else to be learned in Numeration? or else haue I learned it fully?

Master. I might shew you here who were the first Inuentors of this art, & the reason of all

all these things that I haue taught you, but that I will reserue till ye haue learned ouer all the practise of this Art, lest I should trouble you with ouer many things at the first.

Three kinds of numbers. But yet this must you marke, that there are three kindes of numbers, one called Digits, another Articles, and the third Mixt numbers.

Articles.

Digits. A Digit is any number vnder ten, as these: 1.2.3.4.5.6.7.8.9.

And 10 with all other that may be diuided into ten parts iust, and nothing remaine, are called Articles, such as are 10. 20. 30. 40. 50. &c. 100. 200. &c. 1000, &c.

Mixt. And that number is called Mixt, that containeth Articles, or at the least one Article, and a Digit, as 12 16. 19. 21. 38. 107. 1005. and so forth: and for the more ease of vnderstanding and remembrance, marke this; The Digit number is neuer written with more than one figure, but the Article and the Mixt number are euer written with more then one figure. And thus they differ; that the Article hath euermore this Cypher o in the first place; and the Mixt number hath euer there some Digit.

Scholar. By these last words I perceiue it much better then I did before, and now (I thinke) I will neuer misse to know those three asunder.

Master. If you remember now all that I haue

haue said, you haue learned sufficiently this first kinde of Arithmeticke called Numeration. Howbeit I will exhort you now to remember both this that I haue said, and all that I shall say, and to exercise your selfe in the practise of it, for rules without practise, are but a light knowledge, and practise it is, that maketh men perfect and prompt in all things. Vse makath mastery.

And as you haue learned to gather and expresse the value of a summe propounded, and set downe before you, so must you practise to marke, note, or write downe with apt Figures and in due places, any number onely named or recited to you, or if your selfe imagined; as for a proofe. How note you or write downe this summe, fiue thousand two hundred fifty and seuen?

Schollar. This troubleth me now, whether I should begin at the first or at the last. For reason (me thinketh) should cause me to begin at the first; and yet if I write it as you speake it, I must begin at the last.

Master. When you know your places perfectly, you may begin where you list; but the more ease for your hand is to begin with the last, that is to say, as I did speake them. yet for the more surety, a while you may begin with the first, repeating my words backward thus; Seuen, fifty, two hundred, fiue thousand: or else sounding them all by their digit or

value

valewer, as thus: ſeuen, fiue, two, fiue: for that way is eaſieſt: But then muſt you looke well whether there be any Cypher in your ſumme, that he may be ſet in his place. As if the laſt valewer of your ſumme (as you ſpeake it) be aboue 9, then is there a Cipher in the firſt place. And if it be an hundred, or aboue, then is there two Cyphers, one in the firſt place, and another in the ſecond, and ſo forth.

But becauſe this thing is ſuch that cannot be ſet forth without many words, I thinke beſt here now at the end of Numeration to adde a Table eaſie and ready for the firſt exerciſe of it.

Loe this is the Table.

The right side or hand.
The names of Digits, values certaine, or values.

The denominators of the place or value vncertaine	Nine.	Eight.	Seuen.	Six.	Fiue.	Foure.	Three.	Two.	One.	Cipher	The order of places.
Vnites.	9	8	7	6	5	4	3	2	1	0	First.
Tennes.	9	8	7	6	5	4	3	2	1	0	Second.
Hundreds.	9	8	7	6	5	4	3	2	1	0	Third.
Thousands.	9	8	7	6	5	4	3	2	1	0	Fourth.
X Thousands.	9	8	7	6	5	4	3	2	1	0	Fift.
C. of Thousands.	9	8	7	6	5	4	3	2	1	0	Sixt.
Millions.	9	8	7	6	5	4	3	2	1	0	Seuenth.
X. of Millions	9	8	7	6	5	4	3	2	1	0	Eight.
C. of Millions.	9	8	7	6	5	4	3	2	1	0	Ninth.
M. of Millions.	9	8	7	6	5	4	3	2	1	0	Tenth.
X. M. of Millions	9	8	7	6	5	4	3	2	1	0	Eleuenth

The left side or hand.

This Table (as yée may sée) hath eleuen places, and in each of them are set all the Digits, whose certaine value is written on the right hand of the Table, and the value vncertaine on the left hand, so that by this Table you may learne both how to expresse any number that you list, if that it excéed not

D eleuen

eleuen places) that is to say, XC. thousand millions, and so may you by helpe of it, value all summes proposed vnder the said number.

For example: take the summe that I proposed before, which was fiue thousand, two hundred fifty and seuen. And if you will expresse it, take the first number (as I speak it) which is fiue M. whose valuer or certaine value is v, and his vncertaine value or denomination is M. First, you shall seeke at the right hand of the valuer 5. Then seeke along vnder the title of denomination toward the left hand till you finde thousands, and vnder it, right at the foot of the Table is the number of the place that is in the fourth, wherein you must write your digit or valuer 5.

Afterward come to the second part of the number two hundred, whose valewer is 2, and his denomination C. Seeke two at the right hand of the Table, and go along vnder the denomination toward the left hand, til you come vnder C: then looke to the foot of the Table, and there you shal see the number of the place, that is to say, the third, wherein you must set your digit 2.

Then do so by your other two numbers that remaine, and you shall finde 5 in the second place for your fifty, and 7 in the first place for your seuen. And thus may you do with other numbers.

Scholar. Master, I thanke you heartily. I perceiue you seeke to instruct me most plainly and

and briefly, and not to hide your knowledge with subtile words, as many do. For this rule is so plaine, that I can desire it no plainer. And though it séeme somewhat long, yet I perceiue it to be a sure way.

Master. So it is, and though it be long, yet it is neither too long, neither too plaine for yong learners that lacke practise: for this Table is in stead of a teacher to them that lacke one. But now I trust I haue said enough of Numeration: which after you haue well practised, then may you learne forth.

Scholar. Yet I pray you in one thing to tel me your iudgement. Why do men reckon the order of the places backward, from the right hand to the left?

Master. In that thing all men do agrée, that the Chaldees, which first inuented this Art, did set these figures as they set all their Letters, for they write backward, as you tearme it, and so do they reade. And that may appeare in all Hebrew, Chaldee, and Arabicke books; for they be not onely written from the right hand to the left, and so must be read, but also the right end of the booke is the beginning of it, whereas the Greekes, Latines, and all Nations of Europe, do write and reade from the left hand toward the right: and all their bookes begin at the left side. Why numbers are writ- ten back- ward.

Scholar. That reason hath satisfied me.

Master. It neither satisfieth me, neither liketh me wel, because I sée that the Chaldees

and Hebrews do not so use their owne numbers, as at another time I will declare. But this plaine reason may best satisfie you presently: that séeing in pronouncing of numbers we kéepe the order of our owne reading, from the left hand to the right: and againe, we do ever name the greater numbers before the smaller: it was reason that the lesser places, containing the lesser numbers, should be set on the right hand, and the greater places containing the greater numbers, to procéed toward the left hand.

Scholar. This reason is to me so plaine, that it séemeth now against reason to make a doubt of that order. So that now for Numeration I am satisfied; hoping that practise shall make me fully ready and expert in it. And in the meane season, I desire to learne the other kinds of Arithmetick.

Master. That is well said: but what should you next learne? can you tell?

Scholar. I remember you said that Addition was next.

Master. Euen so, and what that is, must you first know.

Addition.

Addition.

A*Ddition is the gathering together and bringing of two numbers or more into one summe.* As if I haue 160 Books in the Latine tongue, and 136 in the Greeke tongue, and would know how many they be in all, I must write these two numbers one ouer another, writing the greatest number highest, so that the first figure of the one being vnder the first figure of the other: and the second vnder the second, and so forth in order.

When you haue so done, draw vnder them a right line, then will they stand thus:

Now begin at the first places toward the right hand alwayes, and put together the two first figures of these two numbers, and looke what commeth of them; write vnder them, right vnder the line. As in saying 6, and 0 is 6, write 6 vnder 6 as thus:

$$\begin{array}{r} 160 \\ \underline{136} \end{array}$$

$$\begin{array}{r} 160 \\ \underline{136} \\ 6 \end{array}$$

And then go to the second figures, and do likewise: as saying 3 and 6 is 9 write 9 vnder 6 and 3, as here you see.

$$\begin{array}{r} 160 \\ \underline{136} \\ 96 \end{array}$$

And likewise do you with the figures that be in the third place, saying, 1 and 1 be 2, write 2 vnder them, and then will your whole summe appeare thus:

$$\begin{array}{r} 160 \\ \underline{136} \\ 296 \end{array}$$

So that now you see, that 160, and 136, do make in all 296.

Scholar. What? this is very easie to do, me thinketh I can do it euen since.

There came through Cheapeside two droues of cattell: in the first was 848 sheepe, and in the second was 186 other beasts.

Those two summes I must write as you taught me, thus: Then if I put the two first figures together, saying, 6, & 8. they make 14. That must I write under 6, and 8, thus:

$$\begin{array}{r} 848 \\ 186 \\ \hline 1034 \end{array}$$

☞ Master. Not so, and here are you twice deceiued. First, in going about to adde together two summes of sundry things, which you ought not to do, except you seeke onely the number of them, and care not for the things. For the summe that should result of that addition, should be a summe neither of sheepe, nor of other beasts, but a confused summe of both. Howbeit sometimes ye shall haue summes of diuers denominations to be added, of which I will tell you anon: but first I will shew you where you were deceiued in another point, and that was in writing 14. which came of 6, and 8, under 6.8. which is unpossible; for how can two Figures of two places be written under one Figure and one place.

Scholar. Truth it is, but yet I did so under-

stand

stand you.

Master. I said indéed, that you should write that vnder them that did resulte of them both together: which saying is alwayes true, if that summe do not excéed a digit. But if it be a mixt number, then must you write the digit of it vnder your Figures as you haue said before: and if it be an article, then write o vnder them, and in both sorts you shall kéep the article in your mind; and therefore when you haue added your second Figures, which occupie the place of tens, you shall put that one thereto, which you kept in your minde; for though it were ten indéed, yet in that place it is but as one, because that euery one of that place is ten, for that it is the place of tens. And in like manner, if you haue in the second place so great a number that it amounteth aboue 9, then write the digit, and reserue the article in your minde, euer adding it to the next place following, and so of all other places, how many soeuer you haue. And if you haue a mixt number when you haue added your last figures, then write the digit vnder the last figures, and the article in the next place beyond them: so shall your number resulting of Addition, haue one place more then the numbers which you shall adde together. A place:

Schollar. Now do I perceiue you, and the reason of this, is, (as I vnderstand) because that no one place can containe aboue 9, which is the greatest figure that is, and then all tens

or Articles must be put to the next place following: for every place (as I may see) exceedeth the other place next before him by 10.

Now (if it please you) I will returne to my example of cattell. But I remember you said I might not adde summes of sundry things together, and that I may see by reason.

Master. Truth it is, if you seeke the true summe of any thing, but if you onely seeke a bare summe, and have no respect to the thing, then were it better to name the summe onely without any thing: as in saying 848 without naming sheepe or any thing else. And likewise 186 naming nothing.

Now let mee see how can you adde those two summes?

Schollar. I must first set them so that the two first figures stand one over another, and the other each one over his fellow of the same place: then shall I draw a line under them both. And so likewise of other figures, setting alwayes the greatest number highest, thus as followeth.

Then must I adde 6 to 8, which make 14, that is a mixt number, therefore must I take the Digit which is 4, and write it under 6 and 8, keeping the Article 1 in my minde thus:	848 186 ——— 4

Next that, I doe come to the second figures, adding them together, saying 8 and 4 make 12, to the which I put the one reserved in my minde, and that maketh 13. of which

which number I write the digit 3 under 8 and 4, and keepe the Article in my minde, thus:

848
186
34

Then come I to the third figures, saying, 1 and 8 make 9. and 1 in my minde maketh 10. Sir, shall I write the Cypher under 1 and 8?

Master. Yea.

Scholar. Then of 10 I write the Cypher under 1 and 8, and keepe the article in my minde.

Master. What needeth that, seeing there follow no more figures?

Scholar. Sir, I had forgotten, but I will remember better hereafter. Then seeing I am come to the last figures, I must write the Cypher under them, and the Article in a further place after the Cypher, thus:

848
186
1034

Master. So now you see, that of 848 and 186 added together, there amounteth 1034.

Scholar. Now I thinke I am perfect in Addition.

Master. That will I proue by this example. There are two armies of Souldiers: in the one are 106800, and in the other, 9400. How many are there in both armies say you?

Scholar. First, I set them one ouer another, beginning with the first numbers of the right hand thus:

106800
9402

But the nether number will not match the ouer number.

Master.

Master. That forceth not.

Scholar. Then do I adde 0. to 0. and there amounteth 0. that must I write under the first place thus:

106800
9400
0

Master. Well said.

Scholar. Then likewise in the second place I adde 0 to 0, and there ariseth 0, which I write under the second place thus;

106800
9400
00

Then I come to the third place, saying 4 & 8 make 12, of which I write the digit 2, and keep the article 1 in my mind, thus;

106800
9400
200

Then I adde 9 to 6, which makes 15, to that I adde the article 1, that was in my mind, and it is 16, I write 6 under 6, & 9, and keepe one in minde thus.

106800
9400
6200

Master. Why do you not write both Figures, seeing you are come to the last couple of numbers?

Schollar. Nay, reason sheweth me, that I must adde that article that is in my minde unto the next figure of the over summe, though there be no more in the neather summe.

Master. That is well considered: then do so.

Scholar. Then say I, 0 in the over summe and 1 in my minde, maketh 1: that write I

under

vnder 0. Then followeth there yet one more in the ouer summe, which hath none to be added to it, for there is none in the neather summe, nor yet in my mind: therefore I think I must write that euen as it is.

Master. Yea.

Schollar. Then doth my whole summe appeare thus.

106800
9400
116200

Master. If you mark this you haue learned perfectly the common Addition of all summes which are of one denomination: so that ye obserue this also, that in Addition you must haue two numbers at the least: or else how can you say, that you do adde? And euer let the greatest number be written highest, for that is the best way, though it be not necessary.

And forget not this, that (if you haue many numbers to adde together) you shall haue oftentimes an article of a greater value then 10. sometimes 20. sometimes 30. sometimes more, yea (peraduenture) 100. Therefore as you did with the article 10. so do with them, reseruing them in your minde, and adding to the number next following so many, as their valuer or value certaine is: that is to say 2. for 20. 3 for 30. 5 for 50. 10 for 100. 12 for 120. and so forth of other like. So that, if the article be 100, then must you set downe the 0. and keepe 10 in minde, to be carried to the next row of figures or place, if any such

happen to come. For your better vnderstanding take this example for all.

I would adde these thirteen summes into one, which I set after this manner: then do I begin and gather the summe of the first row of Figures, which come to 107, (for I take 9 there ten times, and that is 90) then 9 and 8 is 17, that is in all 107, of which summe I write the 7 vnder the first row of Figures, and then for that 100 is ten tens, I keepe ten in minde, which ten I must adde vnto the next row of Figures, which are in the second place:	4889 4599 2290 3699 2299 4099 1099 3298 299 699 499 899 389
	29057

which second row of figures (when they are added together with that ten that I had in my minde) make in all 125. of which summe write the digit 5 vnder the second row, and then (for that 120 containeth twelue tens) I keepe twelue in minde to be added to the third place or row of figures: which being added together, make in all 60, the cypher 0, I set downe vnder the row of figures in the third place.

And the figure 6 I keepe in minde to be added to the row of figures in the fourth place, which (when they are added together) make 29. The figure or digit 9 I set downe vnder the fourth place. And because it is my last worke, I set downe the 2 also that I haue in

my

my mind to the 9 in the fift place; ſo thoſe ſummes do make in all 29057.

¶ But (for your more eaſe in worke) when you haue an addition of ſo many ſummes to bée added together, you were beſt part that ſumme into two or thrée parts, and worke them ſeuerall, and ſo put their additions together, and this were the beſt things you could doe when euer many ſummes fall to bée added.

4889
4599
2290
3699
2299
4099
1099
3298
299
699
499
899
389
29057

Scholar. This ſéemeth ſomewhat hard by the reaſon of ſo many numbers together.

Howbeit, I thinke (if I do often proue, euen with the ſame example, either by working of it alone, or elſe by parting it as you ſaid euen now) that I ſhall be able to do ſo ſhortly with any other ſumme.

Maſter. So ſhall you. For it is often practiſe that maketh a man quicke and ripe in all things; but becauſe, as well in great ſummes as in ſmall, there may chance to be ſome errour, I will teach you how you ſhall proue whether you haue done well or no.

Scholar. That were a great help and eaſe.

The proofe of Addition.

Maſter. Begin firſt with the higheſt number, and then to all the other orderly, and adde them together, not hauing regard to their places, but as though they were all vnites: and

and

and still (as your number increaseth aboue 9) cast away 9. Then go forth, euer casting away 9 as often as it amounteth thereto: and so do till you haue gone ouer all the numbers that you intended first to adde; and whatsoeuer remaineth after such addition and castings away of 9, write it in some void place by the end of a line, for the better remembrance: and thus is the first part of your worke proued. Then secondly, put together the figures that resulte of the addition vnder the line, still casting away 9 also. And then that that remaineth write at the other end of that line; and if those two figures be like, then haue you well done; but if they be vnlike, then haue you missed. As for example, in this present summe: The first figure of the ouer line is 9, let him go, then 8 and 8 is 16, take away 9, there resteth 7, and adde that 7 to 4 that followeth, and it maketh 11, from which if you take 9, there resteth 2. Then come to the next row, whose first and second numbers are 9. therefore ouerpasse them both, and take the 5, to the 2, which did remaine in the first row, that maketh 7, put thereto the 4 following, and that maketh 11. thence take 9, and there remaineth 2. Next vnto that, go to the third line, whose two first numbers you may let passe because they are nines: then take the two figures of 2, which (with the other two that remained, in the second row) make 6. Then go to the fourth row, whose two first numbers

numbers let go, and take the 6 to the 6, that remained, and that maketh 12: take away 9, and there resteth 3, which with the 3 that is next, maketh 6. And so go through all the other numbers, and you shall finde that there remaineth 5, after you haue cast away 9, as often as you can finde it: therefore write 5 at the end of the line in a void place, thus:

5 ———————

Then gather all the figures of the totall summe, which is vnder the lowest line, and cast away 9, as often as you can finde it: as thus: 7, and 5 make 12, take away 9, there resteth 3, to that if you adde the 2 that is last, (for you may omit the 9) then doth it make 5, which 5 you must write at the other end of the line that you made in the void place, thus:

5 ——————— 5

And then you see that those two figures be like, whereby you may know that you haue done well, and so you may proue in any other.

Scholar. (If it please you) I will proue in another summe.

Master. With a good will.

Scholar. Then will I take one of your former examples, which was this.

First in the highest line 8 and 6 make 14, then 9 taken away, there remaine 5, to which I adde the 1 that followeth, & that maketh 6. Then come I to the second line, where I finde first 4,

106800
9400
116200

which

which with 6 maketh 10, from that I take 9, and there resteth 1. the next figure is 9, and therefore I let him alone, so finde I 1 remaining which I set at the end of a line, thus:

1 ———————

Then I come to the totall summe, and there I find that all the figures put together, make ten, from which I take nine, and there resteth 1 also, which I put at the other end of the line thus. 1 ———— 1

And because they be like, I know that I haue well added.

Master. So you know now both how to adde two summes or more together, and also how to proue whether you haue done well or no; and now I will teach you how to adde summes of diuers denominations together: which thing can neuer be but when the one Denomination is such that it containeth the other certaine times. And yet you shall adde them to the other, not after this sort (as you did them that were of one denomination) but after such a sort as I will now shew you, that is to say:

Addition of numbers of diuers denominations

If you haue a summe of diuers denominations, then looke that you set euery denomination by himselfe, with some note or figure of his denomination, as they are wont to be written. Then write your other summes so vnder that first, that euery one be set vnder the other of the same denominations: As for example, if your denominations bée

pounds,

pounds, ſhillings, and pence, write pounds vnder pounds, ſhillings vnder ſhillings, and pense vnder pence: and not ſhillings vnder pence, nor pence vnder pounds.

Schollar. Now that you haue ſpoken it, me thinketh it néedeth not to warne me of it, for it were againſt reaſon ſo to confound ſums: but yet if you had not ſpoken of it, peraduenture I ſhould haue béene deceiued in it.

Maſter. If you do ſay it is plaine, I will ſpeake no more of it, but with an example make the matter to appeare euidently.

Firſt one man oweth me 22 li 6. s 8 d. another oweth me 5 li 16. s 6. d. and another oweth me 4 li 3 s. I would know what this is altogether: Therefore muſt I firſt ſet downe my great ſumme, and then the other, euery one vnder his Denomination agréeing to the greateſt ſumme, as here you ſée with a line vnder them.

li.	ſ	d
22	6	8
5	16	6
4	3	0

Then muſt I begin at the ſmalleſt numbers (which muſt alwayes be ſet next to the right) and adde them together: and (if the ſum will make 1 or 2 or 3 of the next denomination) then muſt I kéepe it in my minde till I come to that place, and vnder that firſt place muſt I note the reſidue (if there remaine any of the ſame denomination:) but, if there remaine none, then néed I to write vnder it nothing. And this is all that you muſt marke in this Addition: for all other things are like to the

manner of Addition before mentioned: *Therefore, the chiefest point of this Addition is, to know the values of Common Coines and rated summs.* As how many shillings be in a pound? how many pence in a shilling? of which (and of other like things) I will instruct you hereafter in the thing of Reduction. But now I may not disturbe your wit from the thing that we are about.

Therefore let us returne to that former example which I proposed of the Debtors: which summes when I had set orderly, they stood thus with a line under them.

Then to adde them into one summe I must begin at the right hand where the smallest denomination is, and adde them together, first saying, 6 and 8 make 14. Now, seeing these 14 are pence, which containe one shilling and 2 pence: the 2 pence I set downe under the like of pence: and the one shilling I keepe in my minde to carry to the next row being the place of shillings.

li.	ſ	d.
22	6	8
5	16	6
4	3	0
		2

Then do I adde the shillings together, saying, 1 in my minde and 3 make 4 and 6 make 10 and 6 make 16, and 1 in the second place, which standeth for 10 make 26, which is 1 pound 6 s. The 6 s. I set down under the place of shillings, as appeareth in the example. And the 1 pound I keepe to carry to the pounds.

li.	ſ	d
22	6	8
5	16	6
4	3	0
	6	2

Then

Then come I to the pounds, adding them all together ſaying, 1 that I keepe and 4 make 5, and 5 make 10, and 2 make 12. The figure or digit 2 I ſet downe right vnder that place or row of pounds where I gather them, and the article 1 I keep to carry to the next place, ſaying. 1. in minde and 2 is 3, which 3 I ſet downe directly vnder the 2. And then appeareth my whole ſumme thus.

li.	ſ	d
22	6	8
5	16	6
4	3	0
32	6	2

And thus muſt you do with any ſuch like ſummes whatſoeuer, whether they be money, weight, or meaſure, which (if you practice diuers ſummes) you ſhall be well acquainted with the feat of Addition.

But now, can you tell how to proue this Addition, or ſuch other like of diuers Denominations, and to try whether you haue well done or no?

*S*chollar. I would I could.

Maſter. That ſhall you do by this meanes: You muſt make a Croſſe which ſhall haue ſo many lines as you haue ſundry Denominations in your Addition: As if you haue but two Denominations then you may make it thus: that the ouer part and nether part may ſerue for one Denomination. And if you haue three Denominations (as pounds, ſhillings, and pence) then muſt you make

Proofe of Addition of divers denominations.

thrée lines, thus: The upright line may serue for pounds, and the highest thwart line for shillings, & the lowest for pence, as for example the summe which we last wrought.

li.	s	d
22	6	8
5	16	6
4	3	0

	5	
6		6
2		2
	5	

For the proofe of the which, because it containeth thrée denominations, I must make a crosse of 3 lines as in the example before. Then I reckon first at the right hand, the pence: 6 and 8 make 14, from which I take 12 for the next Denomination, that is to say, a shilling, and there resteth 2, which I must write at one end of the neather thwart line.

After that, I gather the summe of the shillings, 3. 16. 6. which maketh 25, to whom I put 1, that I tooke of the pence, & that maketh 26, from those I take 20. the quantity of the next greater Denomination, that is to say, a pound, and there resteth 6, which I write at the end of the highest thwart line.

Thirdly, I adde together the pounds 4. 5. and 2 which make 11. to them I adde the one that came of the shillings, and they make 12, from whence I cast 9, and there resteth 3. That 3 I ioyne to the 2 in the next place, and they make 5, which 5 I set at the Crosse also

also. And thus is my first part of my worke proued.

That done, I come to the totall summe vnder the line, and examine it, beginning at the pence, where I finde but 2. and cannot take 9. from him: therefore, I set him at the other end of the neather thwart line: then I come to the shillings, where I finde onely 6 which (because it is lesse then nine) I set it at the other end of the line of shillings, that is, the overmost thwart line.

Last of all, of the 32 li. I take three times 9. which is 27, and there remaineth 5, which I write vnder the vpright line: either else I may reckon them simply without any respect of their valuation or place: saying, 2 and 3 make 5, which (because it is lesse then nine) I set vnder the vpright line as before. Then I consider every number, comparing it to the number, that is against it: and because I finde them to be every one like his match, I know that I haue well done.

Schollar. This crosse I perceiued doth serue for these 3 denominations, pounds, shillings, pence: but what if I had l. s. d. ob. & q?

Master. These lines (as I haue said) do serue for three denominations, such as they be, as here three do serue for pounds, shillings, and pence: but if you haue no pounds in your sum, then may they serue for shillings, pence, and halfe penies: yea, for d. ob. and q or in weight for C. q. & l. or, in measure, for Elles,

 Quarters,

Quarters, and Nailes, if you haue no greater Denomination: ſo that you remember that the vpright line ſerueth for the greateſt Denomination, and the higheſt thwart line for the next, and the loweſt for the leaſt.

And ſo, if you haue foure Denominations, you muſt make your croſſe with ſo many lines. And (if your ſumme be of more denominations) make ſo many lines in your Croſſe. And thus will I make an end of Addition, ſauing that here (for the better vnderſtanding of this Rule) I haue ſet you downe certaine examples both of money, weigh and meaſures, with their workes and proofes.

Examples of Addition.

li.	ſ	d	li.	ſ	d
23	10	4	130	17	10
45	6	8	28	6	8
37	2	9	13	13	4
25	13	6	120	0	0
121	13	3	291	17	10

The proofes

	4			4	
4		4	8		8
3		3	1		1
	4			4	

C.	q.	li
24	1	3
12	2	2
7	3	4
13	0	13
57	2	22

3
2 — — 2
4 — — 4
3

yards	q.	nayles.
17	3	3
35	2	1
26	1	3
54	2	0
134	1	3

8
1 — — 1
3 — — 3
8

The Subt

Subtraction

The Subt
ction

Quarters, and Nailes, if you haue no greater Denomination: ſo that you remember that the vpright line ſerueth for the greateſt Denomination, and the higheſt thwart line for the next, and the loweſt for the leaſt.

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Examples of Addition.

li.	ſ	d
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The proofes

4
4 — — 4
3 — — 3
4

8
8 — — 8
1 — — 1
4

C.	q.	li
24	1	3
12	2	2
7	3	4
13	0	13
57	2	22

3

2 — 2

4 — 4

3

yards	q.	nayles.
17	3	3
35	2	1
26	1	3
54	2	0
134	1	3

8

1 — 1

3 — 3

8

Subtraction

Subtraction.

Schollar.

THen haue I learned the two first kinds of Arithmeticke: now (as I remember) doth follow Subtraction, whose name (me thinketh) doth sound contrary to Addition.

Master. So it is indeed: for, as Addition increaseth one grosse summe, by bringing many into one: so contrariwise, Subtraction diminisheth a grosse summe by withdrawing of other from it. *So that Subtraction or Rebating is nothing else but an Art to withdraw and abate one summe from another, that the remainer may appeare.*

Subtraction.

Schollar. What do you call the remainer?

Master. That you may perceiue by the name.

Schollar. So me thinketh: but yet it is good to aske the truth of all such things, lest in trusting to mine owne coniecture, I be deceiued.

Master. So it is the surest way. And (as I see cause) I will still declare things vnto you so plainly that you shall not need to doubt. Howbeit, if I do ouer passe it sometimes (as the manner of men is to forget the small knowledge of them to whom they speake) then do you put mee in remembrance your selfe,

selfe, and that way is surest.

And, as for this word that you last asked mée, take you this description: The Remainer is a summe left after one Subtraction made, which declareth the excesse or difference of the other two numbers: as if I would abate or subtract 14 out of 18, there should remaine 4, which is called the Remainer, and is the difference betwéene those two numbers 14 and 18. Remainer.

Schollar. I perceiue then what Subtraction is: Now resteth to know the order to worke it.

Master. That shall you do by this meanes. First, you must consider, that if you should go about to rebate, you must haue two sundrie summes proposed: the first, which is your grosse summe (or summe totall) and it must be set highest: and then the rebatement (or summe to be withdrawne) which must be set vnder the first, (whether it be in one parcell or in many) and that in such sort, that the first figures be one iust ouer another, and so the second, and third, and all other following as you did in Addition: then shall you draw vnder them a line, and so are your summes duely set to begin your working. ☞

Then begin you at the right hand (as you did in Addition) and withdraw the neather number out of the higher, and if there remaine any thing, write that right vnder them beneath the line: and if there remaine nothing

(by

(by reason that the two figures were equall) then write under them a Cypher of nought. And so do you with all the other Figures, evermore abating the lower out of the higher, and write under them the Remainer still, till you come to the end. And so will there appeare under the line what remaineth of your grosse summe, after you have deducted the other sum from it, as in this example.

I received of your Father 48 s. of which I have layd out for you 36 s. now would I know what doth remaine? And therefore I set my number thus in order. First, I write the greatest summe, and under him the lesser, so that the Figures at the right side be even one under another, and so the other, thus.

s
48
36
12

Then do I rebate 6 out of 8, and there resteth two, which I write under them right beneath the line thus.

48
36
12

Then I go to the second figures, and do rebate 3 out of 4 where there remaineth 1, which I write under them right, and then the whole summe and operation appeareth thus:

48
36
12

Whereby it appeareth, that if I withdraw 36 out of 48, there remaineth 12.

Schollar

Schollar. Now will I prove in a greater summe, and I will subtract 2367924 out of 3468946 those sums I set in order thus

3468946
2367924
——— ———

Then do I begin at the right side, and deduct 4 out of 6, and there resteth 2 which I write under them. Then go I to the second figures, and withdraw 2 out of 4, & there remaineth 2. which I set under them also, then I take 9 out of 9, and there resteth 0. which I write under them (for you say that if the Figures be equall; so that nothing do remaine, I must write this Cypher 0 under them.)

Master. It was well remembred: now go forth.

Schollar. Then I come to the fourth place, and draw 7 out of 8, and there remaineth 1. which I write under them also: Then in the fifth place I take 6 out of 6, and there resteth 0. (for if I write under them the Cypher 0) Then in the sixt place 3 rebated from 4, there remaineth 1. which I write under them, and likewise in the seventh and the last place 2 taken from 3. there is left 1.

Which I write under them: so have I done my whole working, and my summes do appeare thus. Whereby

3468946
2367924
——— ———
1101022

I see, that (if I do rebate 2367924 out of 3468946) there remaineth 1101022.

Master. This is well done. And that you may be sure to perceive fully the Art of Sub, traction,

traction, let me sée how you can subtract 529847 32. out of 8250003456.

Schollar. First, I set downe the greatest summe, and after that, I write under it the lesser number, beginning at the right side, and then my figures will stand thus.

8250003456
529847 32

Note. Then take I 2 from 6, and the rest is 4, which I write under them. Then do I withdraw 3 from 5, and there remaines 2. which I write under them. Then take I 7 out of 4, but that I cannot, what shall I now do?

Master. Marke well what I shall tell you now, how you shall do in this case, and in all other the like: if any figure of the neather summe be greater then the figure of the summe that is ouer him (so that it cannot bée taken out of the figure ouer him) then must you put 10 to the ouer figure, and then consider how much it is, and out of that whole summe withdraw the neather figure, and write the rest under them. Can you remember this?

Schollar. Yes, that I trust I shall. Now then in mine example where I should haue taken 7 out of 4. and could not, I put 10 to that 4 which maketh 14. from it I take away 7. and there resteth 7 also, which I write under them.

Master. So haue you done well, but now must you marke another thing also: that (when-

(whensoeuer you do so put ten to any figure of the ouer number) you must adde one still to the figure or place that followeth next in the neather line: as in this example there followeth 4, to which you must put 1, and make him 5, and then go on as I haue taught you.

8	2	5	0	0	0	3	4	5 6
5	2	9	8	4	7	3	2	
	0	1	8	7	3	2		

Schollar. Then shall I say, 4 and 1 (which I must put to him for the 10 that I added to 4 before) make 5, which I should take out of 3, but that cannot be: therefore I must put to it also 10, and then it will be 13, from which I take 5 and there resteth 8 to be written vnder them: and because of that 10 added to the 3. I must adde 1 to 8 that followeth in the neather line, and that maketh 9. which I should take out of 0 and cannot: therfore I put thereto 10, & that maketh 10, from 10 I take 9, and there remaines 1, which I write vnder them.

Thus do I adde 1 likewise to the next figure beneath, which is 9, and that maketh 10, that 10 should I take out of the figure aboue, but I cannot: for it is 0, therefore I put 10 to it, and so take I 10 out of 10, and there resteth 0 to be written vnder them.

Then come I to the next figure, which is 2, and to him I do adde 1 which maketh 3: that 3 I cannot take out of naught: therefore of that naught I make 10, and thence do take 3, so there remaineth 7 to be written vnder them: likewise do I put 1 to 5, that

that 5, and make it 15. from which I rebate 6 and there remaineth 9, which I write under them. Now have I spent all the neather figures, and what shall I do more?

	8250003456
	5298473 2
	8197018724

Master. You should have added one to the next figure following (if there had béene any) because you added 10 to the last figure before of the over line: but séeing there is no figure following, you must adde that one to the place following, and then deduct that one from the number above.

Schollar. Then shall I say, because I borrowed 10 to the over 5, I must put 1 in the next place beneath, that is under 2, then must I subtract that 1 from 2, and there resteth 1, to be written under that in the ninth place. Now I have no more to subtract, for there is not any figure remaining beneath, neither yet any Vnite to be added, because I borrowed not 10 to the figure last before: and yet is there 8 remaining in the over line, which I thinke (by reason) should be set at the end of the figures in the lowest row, which is under the line, for because there was nothing taken from it.

Master. That is well considered, and reason teacheth so indéed.

Schollar. But Sir, I beséech you, shall I alwayes when any number so remaineth alone, as this 8 did, write him under the line

straight

straight against his owne place?

Master. Yea, what else? Whether they be one or many: and this well remembred, you haue sufficiently learned Subtraction: Howbeit, because of certaine things that might deceiue you, if you did not take good heed to your working, I will propose to you another example of many numbers to be subtracted, as thus. I receiued of a friend of mine to keepe 2869 crownes, of which at one time I deliuered him againe 500. at another time 368. at another time 440. at another time 80 and another time 64. now would I know how many do rest behinde? Therefore first I set downe my grosse summe and vnderneath it I set all the parcels thus, and vnder them a double line.

2869 *Crowns receiued*

500
368
440 } *deliuered.*
80
64

Then first I begin at the first place, and gather together the summe of all those lines (saue the ouermost) in their first figures: and so I do with all the figures of the second place, and so forth, as I did in Addition, saue that I leaue out the highest row of numbers (as the line warneth mee) and that summe so gathered betweene the double line, is the summe deliuered in all: which summe I do afterwards subtract out of the highest row of numbers; and the remainer do I set vnder the neathermost line: Note.

as for example.

I set the summes as before: then do I gather the first figures of all the places deliuered, together: where I find but 4, and 8, that maketh 12. (for three Cyphers increase no summe in Addition, as you learned before:) of the 12 therefore, do I write the Digit 2. betweene the double line, and keepe the Article in my minde, till I come to the second place, where I finde 6. 8. 4. 6. that maketh 24. to them I put the Article in my minde, and it is 25. of which I write 5. vnder the second place, and keepe the Digit 2 in my minde for the third place, where I finde 4. 3. 5. that makes 12. to the which I adde the 2 in my minde, and it maketh 14, thereof I write the 4 vnder the third place, and because there remaines no moe figures to be added, I write the Digit in the fourth place, as you see in the example: and so it appeareth, I haue deliuered in all, a thousand foure hundred fifty two crownes.

2869 *Crowns receiued.*

$\left.\begin{array}{r} 500 \\ 368 \\ 440 \\ 80 \\ 64 \end{array}\right\}$ *Deliuered.*

1452 *Deliuered in all.*

1417 *Rest behinde.*

Then come I to the subtracting of this summe betweene the lines, for by Addition it is equall to the fiue parcels ouer it. Therefore I proceed to subtract it from the ouermost summe, saying, 2 from 9 remaine 7, to be written vnder them beneath the lowest line.

line. Then in the second place I take 5 from 6 and there resteth 1 to be written vnder them. Then in the third place, 4 from 8, resteth 4. Last of all, in the fourth place, 1 from 2, remaineth 1. And thus I see that after those fiue summes are subtracted from 2869, the Remainer is 1417.

Scholar. This I perceiue: but is there no shorter way and more speedy?

Master. Yea, when you are a while exercised in it: for you may (as fast as you can gather the numbers together) withdraw them out of the highest summe. But if in quantity those numbers added together, exceed the highest summe or vpper number, then shall you (as before hath beene taught you) imagin to borrow 10, 20, or 30 moe, as need shall require, and put them to the vpper number, to helpe to further the abatement, reseruing or restoring the Articles that you borrowed to the next place againe: & so still go forward till you haue ended your worke: as for example. In the last summe proposed. I gather first in the first place 4 and 8 that maketh 12, which 12 I should deduct or take out of 9 in the vpper number aboue the line, but I cannot: that therefore I ad vnto 9 an article of 10, & maketh the vpper number 19, from whence I take 12, then there resteth 7; then for the Article 10 I adde to the next place of money deliuered; saying, 1 that I bring and 6 make 7, & 8 make 15, & 4 make 19 and 6 make 25.

An abridgment of the former manner of Subtractiō

F which

which 25 I should take out of 6 in the vpper number, but I cannot. Therefore I adde 2 tens or 20 vnto 6 in the vpper number, and that maketh 26, then 25 out of 26, resteth 1; then the tens which I borrowed, or haue in minde, I adde to the next row or sum deliuered; saying, 2 that I bring, and 4 make 6, and 3 make 9, and 5 make 14; then 14 out of 8 I cannot take, but 14 out of 18 resteth 4. Now because there are no more places to bee added, the one that I borrowed, or haue in minde, I rebate from 2 in the vpper line, and there remaineth 1, which I set downe in the remainer line; and so my summe appeareth (as before) to be 14 17 Crownes.

Loe thus haue you now a shorter way.

Scholar. I like both waies well, and I perceiue both well: yet, as in the working seemeth somewhat long, so in the other it leaueth very much (me seemeth) to remembrance, and therefore may cause errour quickly, except a man haue a quicke and an exercised remembrance. But yet for the sharpning of my wit by your patience (if you will giue me leaue) I will try what I can do in a like summe, to worke it the shortest way: whereupon I would subtract out of 4030 1964, these three parcels.

There-

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Therefore I set them first in due order: then I gather the parcels of the first place which are 8. 2. 1. that is a

40301964	Charge.
20003428	}
10002432	} Disch.
10101461	}
43	

11, which I should take or deduct out of 4. which is ouer him, but I cannot: therefore I adde an article or one ten to 4, which maketh 14, then 11 out of 14, there resteth 3 to be written vnder the first place betwéene the two lines.

Then come I to the second place, saying, 1 that I borrowed to haue in my mind, & 6 make 7, and 3 make 10, and 2 make 12, which I cannot take from 6, therefore I adde 10 to 6, which maketh 16, and then 12 from 16 resteth 4, which I write vnder the second place betwéene the two lines.

Then come I to the third place, saying, 1 that I borrowed or haue in mind and 4 make 5 and 4 is 9, and 4 make 13, which I should take out of 9 that is ouer them, but I cannot: therefore I adde 10 to 9 which make 19, then 13 out of 19, rest 6.

Then come I to the fourth place, saying, 1 in minde and 1 is 2, and 2 is 4 and 3 make 7, which because it cannot be taken from 1, I take it from 11, and there resteth 4.

After that, I come to the fifth place, where are onely thrée Ciphers, which make nothing,

Hugh Roberts His Book

unto which I adde 1 in minde, then should I take that (that is to say) 1 from the figure over them, which is also a Cipher: therefore I say thus, I cannot take 1 from 0, but 1 from 10 remaineth 9: so must I write 9 under them. Then in the sixt place I finde but 1, and 1 in minde make 2, which I take out of 3 over him, and the remainer is 1: that must be written betwéene the two lines in the sixth place. So I go to the seventh place, where I find onely Ciphers, & in the grosse summe over them a Cipher also: therefore must I write the remainer (which is nothing) with a Cypher also. Then in the eight and last place, I gather 1, 1, 2, that maketh 4, which if I take out of that 4 that is over them, there will nothing remaine. And that must be noted with a Cipher betwéene the two lines (as I have often said) and so have I ended my worke, and the figures stand as followeth.

But Sir, I remember you taught mée that Ciphers should not come in the last place, for because they serve onely to increase the value of other figures which follow them and serve not those figures that go before them: and now in my Example I have set two Ciphers in the two last places.

Master. I commend you for your remembrance. And truth it is, you should not have set them here, but onely because that I would make you plainly to perceive the art of Subtraction,

traction: Therefore seeing that you do now perceiue it, whensoeuer you would write downe a Cypher, looke whether any other figures be yet behinde: and if not, then let go the o also, for it needeth not to write him in the latter places, where no other figure doth follow, except it be (as I did now suffer you) to teach the vse of Subtraction the plainer.

Therefore your figures must stand thus when the worke is ended.

40301964	*Charge.*	
20003428		
10002433	*Disch.*	
10101461		
194643	*Rest.*	

Scholar. Sir, I do thinke with that that you taught me before, and by these two summes that you taught me last also, that now I could subtract any summe.

Master. So may you, if you haue marked what I haue taught you. But, because this thing (as all other) must be learned surely by often practise, I will propound here two examples to you: wherein if you often exercise your selfe, you shall be ripe and perfect to subtract any other summe lightly, for in them is contained all the obseruances of whole numbers. And because you shall perceiue somewhat both how to do it, and also whether it be well done when you haue proued to do it; therefore haue I written vnder them both, the Remainers.

30606.	*Lent.*	308964.	*Debt.*
10354	*Paid.*	103145	*Paide.*
10249		102597	
163		101024	
20766	*Paid in all.*	02198	*Rest.*
9840	*Rest to pay.*		

Scholar. Sir, I thanke you: but I thinke I might the better do it, if you did ſhew mee the working of it.

Maſter. Yea, but you muſt proue your ſelfe to do ſome things without my aid, or elſe you ſhall not be able to do any more then you are taught: And that were rather to learne by wrote (as they call it) then by reaſon. And againe, there is nothing in theſe examples, or any other of whole numbers, but I haue taught you the rules of them already.

Scholar. Then I truſt by practiſe to attaine the vſe of it. And is this all that I ſhall learne of Subtraction?

Maſter. Yea ſauing that (as you haue ſeene in Addition) there are numbers of diuers Denominations, in which the working is not much vnlike: yet (without ſome inſtructions be giuen of it) it might ſeeme to a learner more difficult then indeed it is. Therefore I will briefly ſhew you the vſe of it onely by an example or two.

A certaine man owed to me:	14 l, 12 s, 8 d,
of which he paid me at one time	4 l, 6 s, 8 d,
at another time 3 l, *at another*	2 l, 3 s, 4 d,
and last of all 6 s, 8 d.	

Now would I know what remaineth unpaid yet? therefore I set my summes thus, every one in their due place: As pounds under pounds, shillings under shillings, pence under pence

li	s	d
14	12	8
4	6	8
3	0	0
2	3	4
	6	8

Scholar. Sir, I pray you why do you write 2 l. for the common speech useth rather to say, 40 s.

Master. Wee must here use the Denomination that is greatest in any summe, so that wee may not write according as wee use to speake, saying, 16 d, 18 d, or likewise 7 groats, 8 groats, 24 s, 40 s, 48 s, and such other: but we must write every Denomination that is in any summe by it selfe. Note how the pence differeth from the common order of counters.

Namely, shillings and pounds. So must we write for the last summes now named, 1 s, 4 d; 1 s; 6 d; 2 d, 4 d; 2 s, 8 d; 1 l, 4 s; 2 l, 8 s, and so forth of other like.

Scholar. So that we may not write in Arithmeticke, pence, when the summe amounteth to shillings, nor shillings, when the summe maketh pounds. Now, (if it please you) end your example.

Master. When my summes are so set as I shewed, then (according to the rules of Addition) I gather all the particular summes which bée payd mée into one totall summe, directly to bée set vnder them betwéene the two lines, not medling with the 14 l, 12 s, 8 d, as the line warneth mée: therefore must I beginne with the smallest Denomination, saying, 8, 4, 8, is 20, pence, which maketh one shilling and 8 pence, the 8 d I set downe vnder the place of pence, and the one shilling I kéepe in mind to carry to the next denomination of shillings. Then come I to the shillings, and say, one that I bring or haue in minde, and 6 is 7, and 3 is 10 and 6 makes 16, which, because it containeth not one pound, I set directly vnder the place of shillings. Then come I to the pounds, whose parcels are 2, 3, 4, that is in all 9, that 9 do I set downe directly vnder the pounds: And so the totall or whole Addition of all the particulars paid, amounteth to 9 l 16 s. 8 d.

li	s	d	
14	12	8	
4	6	8	
3	0	0	
2	3	4	
	6	8	
9	16	8	
4	16	0	*Rest.*

Now for the worke of Subtraction, I must rebate that totall summe of Addition out of the highest number, that is to say, from the

the

the 14 li 12 s 8 d.

Therefore to performe the worke, I say, 8 d. out of 8 d. remaineth or resteth nothing, therefore in the place of the rest or remaine, right under the denomination, I set downe 0. Then comming to the shillings, where I find 16. which should be taken out of 12, but I cannot: therefore I imagine to borrow 1 of the next Denomination, that is, of the 14 li, and put that one pound so borrowed unto 12 s, that maketh 32 s.

Now 16 s out of 32 s, resteth 16 s, which 16 s I set downe directly under the place of the rest.

Lastly, comming to the pounds, saying, one pound in minde that I borrowed, and 9 make 10, then 10 out of 14, there resteth 4.

So doth my whole rest or remaine, appeare to be 4 li 16 s. 0 d.

This I account the easiest way for a young beginner to practise, though it be something long.

Schollar. Is there any shorter way for this worke also?

Master. Yes, as in this last example I will also shew you, for you may adde together the particular summes as

they

li	s	d
14	12	8
4	6	8
3	0	0
2	3	4
0	6	8
4	16	0

they are set in order, beginning with the pence, saying, 8, 4 8, make 20 d. which 20 d. you should take out of the 8 d. above the line, but you cannot, therefore shall you borrow 1 of the next denomination, that is to say 1 of the shillings, & put it to the 8 d. that maketh 20 d. now 20, out of 20 d resteth 0, which Cypher I set down directly under them.

Then one shilling that I borrowed or had in mind, and 6 make 7, and 3 make 10, and 6 make 16, the 16 out of 12 I cannot take, therefore of the next Denomination I do borrow one li, and put it to 12 s, which maketh 32 s, then 16 s. out of 32 s. resteth 16 s.

Lastly, I came to the pounds, saying 1 li in minde, or that I borrowed, and 2 make 3, and 3 is 6, and 4 is 10, then 10 out of 14, there resteth 4.

So doth my remainer or rest appeare as before to be 4 li, 16 s, 0 d.

Scholar. Then do I perceive very well, and if there be no other things to be learned in Substraction, then may I come to Multiplication, for that you reckoned to be next in order.

Mast. We have done indeed with the Art of Subtraction, as touching the working.

But yet before we go to Multiplication, I

will

will instruct you how to examine your worke, whether it be well done or not. For the performance whereof, if you marke what I said in Addition, you may easily perceiue what is to be done for the proofe of Subtraction, which is best made by the aide of Addition thus. Proofe of Subtractiō

Draw vnder the lowest number (which is your Remainer) a line, and then adde this Remainer and all the other that you did substract before, together, and write that that amounteth vnder the lower line; and if the summe that commeth thereof, be equall to the highest of the Substraction, then is the Subtraction well wrought, or else not. As you may see for example in the summes set downe before, and first in summes of one Denomination, whereof one was this.

Where the number 529847 32 is substracted from 8250003456. & the Remainer is 81970 18724	8250003456
	52984732
	8197018724

Now to proue whether it be truly wrought or not. I adde the Remainer and the number substracted, together, beginning at the right hand; and first I say 4 and 2 is 6 which is set vnder the line: Example in a sum of one denomination.

The number giuen	8250003456
The number to Subtract	52984732
The remainer	8197018724
The proofe.	1250003456

Then againe in the second place I say 2 and 3 is 5, which I write vnder, next that in the third

third place, 7 and 7 are 14, of which I write the Digit 4, and kepe the Article 1 in my minde. Then in the fourth place 8 and 4 is 12 and 1 in my minde maketh 13, whereof I write downe the Digit 3, and kepe the Article 1 in my mind. Againe in the fift place, 1 and 8 is 9, and 1 in my minde is 10. Whereof I set downe 0 and kepe the 1 in my minde. And so going on to the rest (as it is taught in Addition) when I have made an end, I see that the lowest line of numbers & the highest be alike: wherefore I know that I have well done.

So likewise the profe is to be made in numbers of diverse Denominations: as for example in our summe of that kinde which in the first forme of working, stood thus; (all the particular numbers to be subtracted, being drawne into one.)

Example in a sum of divers denominations.

Where, in the title of pence, I find 8 and 0; the 8 I set downe directly under in that of pence.

Then in the place of shillings I find 16 and 16 which make 32 shillings, wherein is contained 1 li and 12 s. the 12 s. I set downe directly under them in the due place

	li	s	d
	14	12	8
	4	6	8
	3	0	4
	2	3	0
		6	8
Paid in all	9	16	8
Rest.	4	16	0
Proofe	14	[illegible]	8

of

of ſhillings, and one pound I kéepe.

Then comming to the pounds, I ſay 1 that I kéepe, and 4 is 5, and 9 is 14, which 14 in due order I ſet downe directly vnder them as this figure ſheweth. And the whole ſumme is 14 li, 12 s. 8 d. agréeing with the vpper number aboue. So I finde the worke is good, and the Subtraction well wrought.

The ſame thing is to be done for the latter forme of Subtraction (where the particular ſummes are not gathered together into one groſſe.) For the Remainer and all the particular ſummes ſubtracted, being added together, if the ſumme that commeth thereof bée equall to the higheſt number aboue, then is the Subtraction well wrought, or elſe not.

Example of a proofe in the latter forme of Subtraction.

As for example alſo in the laſt ſums which ſtood thus.

li	ſ	d
14	12	8
4	6	8
3	0	0
2	3	4
0	6	8
4	16	0
14	12	8

Firſt in the title of pence, I adde 8, 4, 8, that maketh 20 d, which containeth one ſhilling and 8 pence.

The 8 I ſet downe vnder the loweſt line in the row or title of pence, and that one ſhilling I kéepe to carry the next Denomination or place of ſhillings.

Then returning to the ſhillings, ſaying: one

one in minde, or that I keepe, and 16 make 17, and 6 make 23, and 3 make 26, and 6 make 32 shillings, which amounteth to one pound, 12 s. the 12 s I set downe under the title of shillings, and 1 pound I keepe or haue in minde to carry to the next Denomination or place of pounds. Then come I to the pounds, saying, 1 that I bring and 4 make 5, and 2 make 7, and 3 is 10, and 4 make 14, then do I write 14 under the pounds, and so haue I ended the Addition: and I see that the lowest line is like unto the uppermost line in number, wherefore I know that I haue well done.

And thus haue I taught you the Art of Subtraction, and the meanes to proue whether it be well wrought or not. Therefore now will I make an end thereof, and will instruct you in Multiplication.

Multipli-

Multiplication.

M*Vltipllcation is an operation wherby two summes produce the third: which third summe so many times shall containe the first, as there are Vnites in the second. And it serueth instead of many Additions.* As for example: When I would know how many are 30 times 48. if I should adde 48 thirty times, it would be a long worke. Therefore was this worke of Multiplication deuised, which shall do that at once that Addition should do at many times. Multiplication what it is.

Schollar. I perceiue the commodity of it partly, but I shall not see the full profit of it till I know the whole vse of it. Therefore Sir, I beseech you, teach me the working of it.

Master. So I iudge it best, but because that great summes cannot be multiplied, but by the Multiplication of Digits, therefore I thinke it best to shew you the way of multiplying them. As when I say, 9 times 8 or 8 times 9, &c. And as for the small Digits, vnder 5 it were but folly to teach any rule, seeing they are so easie that euery childe can do it: but for the Multiplication of the greater Digits, thus shall you do. Multiplication of Digits.

First, set your Digits one right ouer the

other,

other then from the uppermost downwards, and from the nethermost upward; draw straight lines, so that they make a crosse, commonly called Saint Andrews crosse, as you sée here. Then looke how many each of them lacketh of 10, and write that against each of them at the end of the lines, and that is called the difference: as if I would know how many are 7 times 8, I must write those Digits thus.

The difference.

Digit difference

8

7

Then doe I looke how much 8 doth differ from 10, and I finde it to be 2: that 2 doe I write at the right hand of 8, at the end of the line thus.

Digit difference

8 2

7

After that I take the difference of 7 likewise from 10, that is 3, and I write that at the right side of 7, as you sée in this example.

Digit difference

8 2

7 3

Then do I draw a line under them, as in Addition thus

Last of all, I multiply the two difference, saying 2 times 3 make 6, that must I ever set under the differences, beneath the line: then must I take one of the differences (which I will, for all is like) from the other digit (not from his owne) as the lines of the Crosse

warne

warne me, and that that is left, must I write vnder the digits. As in this example, if I take 2 from 7, or 3 from 8, there remaineth 5: that 5 must I write vnder the digits, and then there appeareth the multiplication of 7 times 8 to be 56. And so likewise of any other digits, if they be aboue 5, for if they be vnder 5, then will their difference be greater then themselues, so that they cannot be taken out of them. And againe, such little summes euery childe can multiply, as to say, 2 times 3, or 4 times 5, and such like.

Digit difference.

Scholar. Truth it is. And seeing me seemeth that I vnderstand the multiplying of the greater digits, I will proue by an example how I can do it. I would know how many are 9 times 6.

Master. It is all one in value to say 9 times 6, or 6 times 9: but yet the order is best to put the lesse summe first, saying, 6 times 9, and so of all other summes.

Scholar. Then would I know how many are 6 times 9: therefore I set the digits thus, and make the crosse, thus.

Then do I set their differences from 10 at the right side, the difference of 9, which is 1, against it, and the difference of 6 which is 4, against it also, as in this example.

9	1
6	4

And under them draw a line, Then do I multiply the differences together, saying: 1 time 4 maketh 4, that 4 doe I write under them thus.

9	1
6	4
	4

Then take I one of the differences from the other digit, as, 1 from 6, or else 4 from 9, and each wayes there resteth 5, which I doe write under the digits thus. And so appeareth the multiplication of 6 times 9 to be 54. Thus I see the feat of this manner of multiplication of digits

9	1
6	4
5	4

Master Now might you go straight to the multiplication of great numbers, save that both for your ease and surety in working I will draw you here a Table, whereby shall appeare the multiplication of all the Digits, and this is it that followeth.

1	1	2	3	4	5	6	7	8	9
	2	4	6	8	10	12	14	16	18
		3	9	12	15	18	21	24	27
			4	16	20	24	28	32	36
				5	25	30	35	40	45
					6	36	42	48	54
						7	49	56	63
							8	64	72
								9	81

In which Table when you would know the product in any multiplication of Digits, seeke your first or last Digit in the greater figures, and from it go right forth towards the right hand, till you come vnder the number of your second Digit, which is in the highest row, and then the number that is in the meeting of the rows of little squares (which come directly from both your propounded Digits) is the Multiplication that amounteth of them. As if I would know by this table the multiplication of 7 times 9, seeke first 7 in the greater figures, and then go right forth toward the right hand, till you come vnder 9 of the highest row, in which place where you so come vnder the other digit (as here for example you come vnder 9) is alwayes contained the off come or product, which you seek,

and that place we terme to be in the common angle, in respect of the two numbers so taken on the outsides: as here in that common angle, where the rows of little squares directly proceeding from 7 and 9 do meet, you have 63, which 63 is the summe of the multiplication of 9 by 7.

To multiply greater summes.

Scholar. This is very good and ready. And so may I find the multiplication of any digits: but now how shall I do in greater summes?

Master. *When you would multiply any summe by another, you shall marke that it is the meetest order to set the greatest number highest, which is the place of the number that must be multiplied: and likewise the lesser number under it, for that is the place of the Multiplier or Multiplicator, that is to say, the number by which the Multiplication is made, and is in English alwayes put before this word, Times: in such speaking when I say, 20 times 70. And the number that followeth this word Times, is that which must be multiplied.*

Multiplier

Times,

Therefore when I would multiply one number by another, I must write the greatest highest and the lesser under it, as in Addition. And under them must I draw a line. As for example. If I would multiply 264 by 29, I must set them thus.

264
29

¶. Of which numbers thus set downe to be multiplied, may be formed a question, as thus. There are 29 men, and each man hath

264 Lambes. The queſtion is, how many Lambes they haue in all.

To the performance whereof, I muſt multiply euery figure of the higher row, by euery figure of the neather row: and that that amounteth I muſt ſet vnder the line, as thus.

Firſt I do multiply 4 by 9, ſaying: 9 times 4 (or 4 times 9 which is al one) and that maketh 36, as the Table before of digits doth declare, of that 36 I muſt write the 6, that is the Digit, vnder the 9, and the Article 3 I keepe in minde to carry to the next place.

264
29
———
6

Then come I to the ſecond figure of the higher row, which is 6, and ſay: 9 times 6 make 54, and with the 3 in my minde make 57, the 7 I ſet downe vnder the 2, and 5 I keepe in minde.

264
29
———
76

After that I come to the next figure, which is 2, and multiply it by 9, and that maketh 18 and with 5 that I haue in minde, maketh 23: wherefore becauſe it is the laſt worke of the Multiplier, I ſet it downe in order as you ſee;

264
29
———
2376

And ſo haue I ended the firſt figure of the Multiplier. Wherefore I giue it now a fine daſh with my pen.

Then begin I with the next figure, and multiply it into all the higher figures, as thus:

	2 6 4
	2 9
2	3 7 6

First, 2 times 4 make 8, that 8 do I write under the second place: for evermore the Digit or first figure of the Multiplication that amounteth of the figure of the higher number, must be set under the Multiplier of it, the other in their order toward the left hand.

Scholar. I understand you thus, that the Digit of the summe amounting of the Multiplication of the first figure of the higher row, by the first figure of the lower row, or Multiplier, must be set under the first place: and that that amounteth of the same first figure by the second Multiplier; must be set under the second place and so of the other, if there be more Multipliers.

Master. So meane I indeed: and if there amount but a Digit, then must it be set under the Multiplier.

And now to go forth: I multiply by the same 2, the second figure of the higher row, which is 6, saying, two times 6, make 12, whereof I write the digit 2 under the third place, and the Article 1 I keepe in minde.

	2 6 4
	2 9
2	3 7 6
	2 8

Then do I multiply the last figure of the higher summe by that same 2, saying, two times 2 is 4, and with the 1 that I haue in minde maketh 5, which 5 I write vnder the fourth place. And so

so haue I ended the whole Multiplication: wherefore I also giue the 2 a dash with my pen, thus: and so I doe euer as soone as I haue dispatched any Digit by which I multiply: and the summes stand thus.

264
29
2376
528

Then must I draw a line vnder all those summes that mount of the multiplication, and must adde all them into one summe, as in the example you may see.

264
29
2376
528
7656

Where in the first place I finde but 6, and therefore write 6 it vnder the line. Then in the second place 8 and 7 make 15, whereof I write 5, and keepe one in my minde, and so forth as you learned in Addition. And so appeareth the whole summe to be 7656, which amounteth of the Multiplication of 264, by 29 and that is the iust number of the Lambes that 29 men had.

Scholar. If there be no more to be obserued in it, then can I do it, I suppose, as by this example I shall proue.

¶ There is a peece of ground which containeth 1365 yards in length, and 236 yards in breadth: I would know how many yards square there is in all this peece of ground: which numbers I set downe with the greater aboue, and the lesser vnder, as you see.

1365
236

Then doe I multiply 5 by 6, saying, 6 times 5 make 30, of which I write the Cipher in the first place and the Article 3 I doe kéepe in minde to carry to the next place.

```
1365
 236
----
   0
```

Then do I by the same 6 multiply the second figure of the higher summe, which is 6, saying, 6 times 6 make 36, and 3 in my minde make 39, of which I write the 9 under the second place and the Article 3 I kéepe in minde.

```
1365
 236
----
  90
```

Then do I multiply the third figure, which is 3 by the same 6, and that maketh 18, and 3 in my minde make 21. The 1 I set downe, and kéepe 2 in minde.

```
1365
 236
----
 190
```

Then come I to the last figure of the higher summe, and multiply it by 6, saying, 6 times 1 make 6, and 2 in my minde make 8, that 8 doe I write under the fourth place. And so haue I ended the first Multiplier, and dash him sleightly with my Pen.

```
1365
 236
----
8190
```

Then beginne I with the second Multiplier, and say, first 3 times 5 that maketh 15, of which I set the 5 under the second place, because that the Multiplier is there, and the Article 1 I kéepe in minde.

Then

Then come I to the second Figure that is 6, and multiply it by 3, which maketh 18. and with one in minde maketh 19, the 9 I set downe vnder the third place, and 1 I kéepe in minde.

```
1365
 236
----
8190
  95
```

Then come I to the third Figure, which is 3, and multiply it by 3, saying, 3 times 3 make 9, and with 1 in minde make 10, the Cypher I set vnder the fourth place, and the Article 1 I kéepe in minde.

```
1365
 236
----
8190
095
```

And then comming to the last figure 1, I multiply it by 3: and it maketh 3, and with the one in mind, it maketh 4, which 4 I set in the first place, and then haue I ended two of the Multipliers, and the summes stand as you may sée in the latter end of the page going before, and then I giue 3 his dash.

```
 1365
  236
 8190
4095
```

Then come I to the third Multiplier, and multiply it into euery figure of the higher sum, and first I say, 2 times 5 make 10, of which I set the Cypher vnder the Multiplier in the third place, and the Article 1 I kéepe in minde.

```
 1365
  236
 ----
 8190
4095
  0
```

And so multiplying the second figure 6 by

that ſame 2, there amounteth 12, and 1 in my minde maketh 13, whereof I write the Digit 3 vnder the fourth place, and the Article 1 I kéepe in minde:

```
 1365
  236
 ----
 8190
4095
  30
```

Then do I multiply the ſaid 2 by the third figure of the higher ſumme, which is 3, and that maketh 6, and the one in minde make 7, which 7 I ſet downe vnder the fifth place, as appeareth by the example.

```
 1365
  236
 ----
 8190
4095
 730
```

Then come I to the laſt place, and multiply that 1 by 2, and there amounteth 2, which I ſet in the ſixth place, and then doth the ſumme ſtand thus.

```
  1365
   236
  ----
  8190
 4095
2730
```

And ſo haue I ended the whole Multiplication.

But now (as you taught me) to know what this whole ſumme is, I muſt adde all thoſe parcels together, and then vnder the line will appeare, as you may ſée, the groſſe or totall ſum is, 322140. Whereby I know there is ſo many yards ſquare in that péece of ground.

```
   1365
    236
   ----
   8190
  4095
 2730
------
322140
```

Maſter. This is well done.

Scholar. Then me thinketh I could call it well done, when I know, whether I had well done or no.

Maſter.

Master. It is to be proued by 9 as Addition was, but the surest prooſe is by Diuision, and therefore I will reserue that prooſe by Diuision, till you haue learned the Art of Diuision. And anone I will shew you how it is commonly proued.

¶ *But first, for your further instruction in this exercise of Multiplication, I will with one example more try your cunning, and so make an end: And the question is this. I would know how many dayes it is since the Natiuity of our Lord and Sauiour Iesus Christ, unto this yeare 1630. Which to performe; you must multiply this present yeare 1630. by the dayes in one whole yeare, which are 365.*

Schollar. Now for that you haue giuen me so much light into the question, you shall see I will handsomely finish the worke, for according to your former instructions, I set them downe with a line under them, thus.

$$\begin{array}{r} 1630 \\ \underline{361} \end{array}$$

Then say I, 5 times 0 is 0, which I set downe under the first place, as here appeareth. Then say I, 5 times 3 make 15, the digit 5 I set downe in the second place under 3, and the Article 1 I keepe in minde to bee added to the next Multiplication. Then saying fiue times 6 make 30, and 1 in minde 31, the 1 I set downe in the third place; and 3 I keepe in minde. Then comming to the last figure, I say once 5 is 5, and 3 in minde make 8, that I do I set downe under

the

the fourth place: and thus haue I ended my first Multiplier, and therefore I giue it a dash with my Pen.

Then come I to the second Multiplier, which is 6, and do likewise multiply it into the vpper number, saying, 6 times 0 is 0, which I set down in the second place right vnder his Multiplier: then say I, 6 times 3 make 18, the 8 I set down vnder the third place, and 1 I kéepe in minde. Then say I, 6 times 6 make 36, and 1 I kéepe in minde make 37. the Digit 7 I set downe in the fourth place, & 3 I kéepe in mind: Then say I 6 times 1 is 6, or once 6 is 6, and 3 in mind make 9, which I set down next, & so haue I ended two Multipliers: wherefore I dash the 6 with my Pen.

1590
365
7950
9540

Then I begin to multiply the third Multiplier into the ouer number, saying, 3 times 0 is 0; the 0 I set down in the third place right vnder his Multiplier. Then say I 3 times 3 make 9, which I set down in order next; then say I, 3 times 6 is 18, the 8 I set down, and 1 I kéepe Lastly, I say, once 3 is 3 and 1 I kéepe is 4, which I set downe orderly next: And so haue I ended the Multiplication, and my figures stand thus.

	or thus.
1630	1630
365	365
8150	815
9780	978
4890	489
594950	594950

Master.

Master. I commend you for your diligence, the worke is very perfectly done, which parcels if you now adde together into one summe it will be 594950; which is the grosse or totall summe of that Multiplication, and declareth the number of daies since our Lord and Sauiour his incarnation, unto the end of 1630. *yeares, besides 407 dayes, and twelue houres for leape yeares.*

Scholar. This is meruellous, me thinke, that such great matters may so easily be atchieued by this Art, which heretofore I euer thought had béene impossible, as infinite sorts of people are of that minde.

Master. Truth it is, that knowledge hath no greater enemy then ignorance, for this is one of the least of ten thousand things that may be done by this Art, as hereafter you shall be able to iustifie.

Scholar. The manner of Multiplication I perceiue, if there be no more in it.

Master. *Yes, there are other formes and helpes for ease, and shorter labour of the worke of Multiplication but I will remit them till you haue a litle tasted Diuision, where also the like helpe into Diuision may be vsed: and so therefore vnder one example for both, will I shew you both ease in Multiplication, and also in Diuision.*

But sith the other formes and workings do nothing differ from these works in effect, but onely in setting of the numbers, I will ouer-passe them till a more méete place and time.

time. And now will I instruct you in Division, so that you thinke your selfe sufficiently to perceiue what I haue taught you.

Scholar. Yes Sir, I thanke you, but I do not perceiue how to examine my worke, to try whether I haue well done, or no: therefore as you promised me ere-while, I pray you first shew me how I shall proue it.

Master. *That is commonly vsed by the proofe of 9, as you learned before in Addition, sauing that it differeth from that forme in diuers respects: As for example.*

First, you must make a crosse after this manner.

Proofe of Multiplication.

Then must you examine your summe that should be multiplied, and looke what remaineth after casting away of 9, that set you at the one side of the crosse, then examine the Multiplier, and whatsoeuer remaineth in it after casting away 9 so often as you can, write that at the other side of the crosse: then must you multiply those two numbers together, and looke what amounteth thereof, if it be vnder 9, write it at the higher part of the crosse: but if it be aboue 9, then take thence 9 as often as ye can, and write the rest at the head of the crosse: As for example, we will proue the example you put forth of the peece of ground that contained 1365 yards in length, and 236 yards in breadth.

Therefore first I cast away all the nines from the summe to be multiplyed, saying, 5 and

and 6 make 11, cast away 9 rest 2: then 3 and 2 makes 5, and 1 is 6, that 6 I write at one side of the crosse thus.

Then do I examine the Multiplier, which is 2 3 6, wherein when the 9 is cast out, there remaineth 2, that 2 therefore I set at the other side of the Crosse.

Then do I multiply 6 by 2, and it maketh 12, from which 12 I withdraw 9, then resteth 3, which 3 doe I set at the head of the crosse. Then do I examine the grosse summe amounting of the Multiplication, which is 322140, where I finde 9 once, and 3 remaining; that 3 I set at the foot of the crosse, and then I see it to agree with the other 3 at the top of the crosse, and so know I that I have done well: for if they two did differ, then were my work vaine, and the Multiplication false.

This is the common proofe: but the most certaine proofe is by Diuision, of which I will anon instruct you.

A sure proofe of Multiplication.

Scholar. Sir, what is the chiefe use of Multiplication?

Master. The use of it is greater then you can yet understand: howbeit, these plaine commodities it hath, that if you would resolue any great and whole value into many small and lesse portions, as if you would change

pounds

pounds in shillings and pence, oz any other greater oz smaller parcels by Multiplication, ye shall do it speedily and easily. Also if you should need to adde one summe to it selfe, oz to any other often times, you shall do it by Multiplication much moze speedily, readily, easily, and surely, then by often and sundzie Additions. Take you these commodities grosly shewed foz an answer at this time, and hereafter I will moze abundantly make you to perceiue the vse of it.

Robert Combe[s]
his Book

Diuision.

Diuiſion.

Scholar.

WEll Sir, then in Diuiſion I pray you to inſtruct me. But me thinketh by the name of it, that it ſhould be all one with Multiplication: for I call that Diuiſion, when any thing is parted into diuers and many parts.

Maſter. You take it as it is taken commonly: howbeit, if you marke well, you ſhall perceiue that it is quite contrarie to Multiplication, and doth not part one thing, or few things into many, but contrarie wayes, it bringeth many parcels into few; but yet ſo, that theſe few taken together, are equall in value to the other many: for by Diuiſion, pence are turned into ſhillings, and ſhillings into pounds: As for example, of 120 ſhillings it maketh 6 pounds, ſo are 120 turned into 6, which is a ſmaller number: but then if you conſider the Denominators, you ſhall ſee that they are ſuch that one of the latter is equall to 20 of the firſt, and ſo in value the ſummes are one, though in number they doe differ, and the latter ſumme is the leſſer, and ſo it is alwayes in Diuiſion, howbeit, yet

in the working, the summe is parted by ano- ther, and thereof doth it take the name.

*S*cholar. I thinke I shall better vnder- stand the reason of the name when I know the vse of the work, therefore now would I glad- ly learne that.

D'uision what it is.

Master. *Diuision is a distributing of a grea- ter summe by the vnites of a lesser: Or, Diuision is an Arithmeticall producing of a third number, in respect of two propounded numbers; which third number shall so often containe an vnite, as the greater of the two propounded numbers can con- taine the lesser.* So that as Multiplication did seeme to serue instéed of many Additions, so Diuision may séeme to be in place of many Subtractions: Because that third number briefly expresseth how many times the lesser of your two propounded numbers may be subtracted from the greater: as in practise will more plainly appeare. Therefore (as you may perceiue) vnto Diuision are required three numbers: the first, which should be di- uided, and that must (generally) be the grea- ter: and the second, by which the other must be diuided, and that is (generally) the lesser, and is called, the Diuisor: And the third, which answereth to the question (How ma- ny times?) and therefore is called the quo- tient.

A generall rule for placing the figure.

The first must be first written, and the second so set vnder it, that the last figure of the lower number be right vnder the last of the

the higher, contrariwise to the work of other kinds of Arithmeticke: for in them the two first figures were set euer meet one vnder the other: but in Diuision, the last figures must be set meet, except it chance so that the last figure of the Diuisor be greater then the last of the higher number for then you shal set the last of the Diuisor vnder the last saue one of the higher number, as for example. An exception.

If you should diuide 365 (which are the summe of the daies of a yeare) by 28, which are the daies of a common moneth, then should you set them thus.

	365
	28

But if you should diuide those 365 dayes by 52, which is the number of weekes in one yeare, then should you set them thus.

	365
	52

Likewise, if I would diuide the same 365 by 4, which is the summe of the quarters of yeares, then must I set them thus.

	365
	4

Scholar. Sir, this do I vnderstand, but now how should I do to diuide the one by the other?

Master. You must begin with the last figure next the left hand, and see how many times the last figure of the Diuisor may bee taken out of the last figure of the other number, and that shall you note within a crooked line toward your right hand. As for example, I would diuide 365 by 28,

28, then set I those two summes thus.

365(
28

And I looke how many times I may finde 2 (which is the last figure of the Diuisor) in 3, (which is the last of the number to be diuided) and considering that I can take 2 out of 3 but once, I make a crooked line at the right hand of the numbers, and within it I set 1, and that is called the Quotient number, as I told you. Then because that when 2 is taken out of 3, there remaineth 1, I must write that 1 over 3, and deface or cancell the 3 and the 2, then will the figures stand thus.

Quotient number.

1
~~3~~65(1
~~2~~8

Then come I to the next figure of the Diuisor, and take it likewise so many times out of the figures that be over it, and looke what doth remaine, that I must write over them, and cancell them, as in this example.

Therefore now do I take once 8, out of 16, and there remaineth 8, which I must set over the 6, and cancell or crosse out the 16, and the 8 of the Diuisor: and then will the figures stand thus. And so haue I once wrought.

~~1~~8
~~36~~5(1
~~28~~

Scholar. So I perceiue that you take the neather figure, not onely out of the other that is right over him, but out of that with the other, also

also that remaineth before, and are written toward the left hand.

Master. So must you do: for you must so take the Diuisor out of the ouer number, that there remaine not ouer it so great a summe as it selfe is, for then were your worke in vaine.

But yet againe here must you marke, that when you seeke how many times the last figure of the Diuisor may be found in the number ouer him, that you looke also whether you may as often find all the figures following in those that are aboue them (considering all the remainers, if there be any) if not, take your Quotient lesse by one, and then proue againe, and so still till you finde a meete Quotient: and by that meet Quotient must you alwayes multiply your Diuisor, and set the product vnder your Diuisor, so that the first figure stand vnder the first figure of your Diuisor, and the second vnder the second, and so forth: and then subtract that product from the number to be diuided that standeth directly ouer it, as you haue seene me do.

When you haue thus wrought once, then must you begin againe, and write your Diuisor anew, nearer toward the right hand by one place, as in this example, you shall set 2 vnder 8, and 8 vnder 5, thus.

	~~1~~ 8
	~~3~~ ~~6~~ 5 (1
	~~2~~ 8 8
	2

Then (as before) seeke how many times you may take

your diuiſor out of the number ouer him now.

Scholar. That may I do here 4 times.

Maſter. Truth it is, that you may finde 2 foure times in 8: but then mark whether you can find the figure following ſo many times in the other that is ouer him. Can you finde 8 foure times in 5?

Scholar. No, neither yet once.

Maſter. Therefoꝛe take 2 out of 8 once leſſe.

Scholar. That is thꝛee times.

Maſter. Well then 3 times 2 make 6: if I take 6 out of 8, there remaineth 2: which 2 with the 5 following make 25, in which ſumme I find 8 thꝛee times alſo: and therefoꝛe I take 3 as a true quotient, and wꝛite it within the crooked line of the quotient befoꝛe the 1, thus.

Mark how to conſider this kinde of Remainer.

Then ſay I, 3 times 2 make 6, then 6 out of 8 reſteth 2, therefoꝛe I cancell the 8, and wꝛite ouer it the 2 that doth remaine, thus:

2
~~1~~8
~~3~~ ~~6~~ 5 (13
~~2~~ 8 8
~~2~~

Then do I take 8 as many times out of 25, ſaying: 3 times 8 make 24, and if I take 24 out of 25, there remaineth 1, ſo then I cancell 25 and 8, and ouer the 5 ſet 1, thus.

Or, you might (after you found 3 to bée a fit Quotient) straightway haue multiplied the whole Diuisor 28, by that at once: which giueth 84, which being set vnder 28, and duly subtracted from 85, of the number diuided, giueth 1, the remainer of the whole Diuision, as before you had. Worke which way you list, here you sée also the forme.

2
281
285 (13
288
2

And now haue I done with the diuiding, for I cannot finde my Diuisor 28 no more in the ouer summe.

Scholar. No, except you would part the 1, that remaineth into 28 parts.

Master. That is well said, and so must we do in such cases, when there remaineth any thing: but I will let that passe now, and will make you perfect in Diuision of whole numbers, and will hereafter teach you particularly of broken numbers, called Fraction. Now if you do perceiue the order of diuision, then do you diuide this summe 136280 by 452.

Scholar. First, I set downe the number that should be diuided, then do I set the Diuisor vnder the last figure of the ouer number. Then will it be thus.

136280
452

Maſter. Can you take the laſt of your Diuiſor (which is 4) out of 1, which is the laſt of the ouer number?

Scholar. I had forgotten, becauſe the laſt of the Diuiſor cannot be taken out of the laſt of the ouer number, in ſo much as it is the greater, therefore muſt I ſet the Diuiſor one place more forward toward the right hand thus.

1 3 6 2 8 0
4 5 2

And then muſt I looke how often I may find the laſt figure of the Diuiſor (that is 4) in 13, which I may do 3 times, therefore do I ſay, 3 times 4 is 12, which I take out of 13, and there remaineth 1. Then do I make at the right hand of my ſummes a crooked line, and write before it my Quotient 3, and I cancell 13 and 4, and ouer the 3 I ſet the 1 that remaineth, and then the figures ſtand thus:

1
$\cancel{1}\cancel{3}6280$ (3
$\cancel{4}52$

Then I multiply the ſame Quotient into euery figure of the Diuiſor, and withdraw the ſumme that amounteth out of the numbers ouer them, as firſt I ſay, 3 times 5 make 15, which I take from 16, and there reſteth 1, I cancell therefore 16 and 5, and write ouer the 6 that 1 that remaineth thus,

$\cancel{1}1$
$\cancel{1}\cancel{3}\cancel{6}280$) 3
$\cancel{4}\cancel{5}2$

Then do I ſay likewiſe, 3 times 2 make 6, which I take out of 12, and there reſteth 6, there-

therefore I cancell the 12 and the 2, and ouer the 2 I write the 6 that remaineth thus.

~~1~~~~1~~6
~~2~~~~3~~~~6~~~~2~~80 (3
~~4~~~~5~~~~2~~

Then should I set forward the Diuisor into the next place toward the right hand thus.

~~1~~~~1~~6
~~1~~~~3~~~~6~~~~2~~80 (3
~~4~~~~5~~~~2~~2
45

Master. But you may see that ouer the 4 is no figure, therefore I must set the Diuisor yet forwarder by another place.

And marke, whensoeuer it chanceth so, that you should set forward the diuisor, and that it cannot stand there, because there is no number ouer the last place, or if there be any, it is lesser then the last figure of the diuisor, then must you remoue the diuisor, yet once againe: and because that his first place of remouing serued not to subtract him so much as once, therefore you shall write in the Quotient a cipher, and if you should by chance need to do so oft times, for euery time write a Cipher in the Quotient. The reason of this will I shew you hereafter.

Scholar. Then must I set my summes thus.

~~1~~~~1~~6
~~2~~~~3~~~~6~~~~2~~80 (30
~~4~~~~5~~~~2~~~~2~~
~~4~~~~5~~

And because I remoued the diuisor, so that I ouerskipped one place, I must write a Cypher in the Quotient: and then must I seeke a new Quotient, as

in this example I must say; How many times 4 is there in 6? (and sith it can be but once) therefore doe I write 1 in the quotient: and then say I, 1 time 4 taken out of 6, remaineth 2, I cancell the 6 and the 4, and write 2 over them thus.

```
     2
   ~~2~~~~2~~63
 ~~2~~~~3~~~~6~~~~2~~80(301
  ~~4~~~~5~~~~2~~~~2~~2
    ~~4~~~~5~~~~5~~
     ~~4~~
```

Then say I againe, once 5 out of 28, remaineth 23: I let the 2 stand as it did, and over that 8 I set 3, cancelling the 8 and the 5 vnder it thus.

```
     2
   ~~2~~~~2~~~~6~~3
 ~~2~~~~3~~~~6~~~~2~~80 (301
  ~~4~~5~~2~~~~2~~2
   ~~4~~~~5~~~~5~~
     ~~4~~
```

Master. You might as well haue said, once 5 out of 8, and so remaineth 3, but now go forward:

Scholar. Then once 2 out of 0 cannot be: What shall I now do?

Master. Borrow of the next number that is behinde (for there is 230) and do as you learned in Subtraction in a like case.

Scholar. Then must I borrow 1 of the 3 comming behinde next, and make that 0

to

to be 10, and then take 3 2 out of 10, and there resteth 8. And because I borrowed one of the 3, I must cancell the 3, and write 2 ouer it: then doth the figure stand thus.

22
11628
136280 (301
45222
455
4

Master. Now haue you done, and yet remaineth 228, and your quotient sheweth you, that if you diuide 136280 by 452, you shall finde your Diuisor in your greater number 301, that is C C C times and once, and 228 remaining.

And in the other example (where I diuided 365 by 28, the quotient was 13, and 1 remained, whereby I knew that in a yeare (which containeth 365 dayes) there are 13 moneths, reckoning 28 dayes (or 4 weekes) iust to a moneth, and 1 day more.

Scholar. Why then do we call a yeare but twelue moneths?

Master. Of that at a more conuenient time will I fully instruct you: but now it is not conuenient to intangle your minde with other things then do directly pertaine to your matter. Therefore if you remember what you haue heard, you haue learned a short manner of diuision, which I would haue you often to practise, so that you may be perfect in it, and hereafter I will shew you certaine other proper points touching it.

Scholar. Then I pray you tell mee how

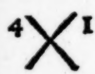
I

I will examine and try my worke, whether I haue done well or no, that though no man be by me to tell me, yet I may perceiue it my selfe.

Proofe of Diuision.

Master. Some men (yea and commonly most) do try it by the rule of 9, as in all the other kindes, saue that their order is: First, they cast away 9 as often as they can out of the Diuisor, and that remaineth they set at one side of a crosse, as in our first example the diuisor was 28, from which you may take 9 thrée times, and 1 remaineth: which they set by a Crosse thus.

When they likewise examine the Quotient (which in our example is 13) and from thence they cast away 9 as often as they can, and the remainder they set at the other side of the Crosse, and then they multiply together those two remainers; and to it that amounteth they adde the remainer of the Diuision, if there were any from that whole summe they withdraw 9 as often as they can, and the rest they set at the head of the crosse, as in our example the Quotient is 13, from which take 9, and there remaineth onely 4, and therefore must you set 4 at the other side of the Crosse thus.



Then multiply 4 by 1, and it yéeldeth but 4, thereto adde the remainer of the Diuision (which was 1) and it will be 5, which summe doth

doth

doth not amount to 9, and thereforе muſt be ſet wholly at the head of the croſſe as you ſee here.

And this number on the head of the Croſſe is the firſt proofe, to which if you find another like in the number that was diuided, then you haue done well.

Therefore now ſhall you likewiſe examine the whole ſumme that was diuided, and take away 9 as often as you can, and that that remaineth, ſet at the foot of the Croſſe: and if it be equall to that in the head of the Croſſe, then haue you done well, elſe not.

As in our example the whole ſumme was 365, which maketh 14, from that take 9, and there reſteth 5, which ſet at the foot of the Croſſe, thus:

And you ſhall ſee that they agree: therefore haue you well done.

Now will I likewiſe examine our ſecond example, where the diuiſor was 452, which maketh 11; from thence I take 9, and the 2 that remaineth I ſet at the right ſide of the croſſe thus:

Then examine I the quotient, which was 301, where I finde but onely 4: that I ſet at the other ſide of the croſſe, thus:

Then I multiply 4 by

2, and it maketh 8: to that I adde the remainer of the diuision (which was 228, and it maketh 12) and they two make 20, wherein I find twice 9, and 2 remaining: that 2 must I set at the head of the Crosse thus:

Then I examine the whole number to be diuided, which was 136280, where I finde twice 9 and 2 remaining which I set at the foot of the Crosse thus:

And because it doth agree with the figure at the head of the Crosse, I know that the diuision was well wrought.

Master. This is the common proofe. Howbeit, the more certaine working is by the contrary kind: as, to proue diuision by Multiplication thus:

The proof of Diuision more certaine by Multiplication.

Multiply the quotient by the diuisor, and if the summe that amounteth, be equall to the summe that should be diuided, then haue you well diuided: else not.

Howbeit, this must you marke, that if there remained any thing after the diuision, that must you adde to the summe that amounteth of the Multiplication. As in our first example our Quotient was 13, and the diuisor was 28: Now multiply the one by the other, and the sum will be 364: to that if you adde the 1 that remained after the diuision,

then

then will it be 365, which was the summe that should be diuided: and therefore I know that I haue well done.

Schollar. Now will I proue the same in the second example, whose diuisor was 452 and the Quotient 301: these do I multiply together, and there amounteth 136052: to which if I adde the 228 that remained, then will it be 136280, which was the whole sum to be diuided: and therefore I perceiue that I haue well done.

Master. This is the surest way, to examine diuision by Multiplication: and contrariwise the surest proofe of Multiplication is by diuision.

And therefore (according to my promisse) now will I shew you how you may proue Multiplication by diuision.

When you haue ended Multiplication, and would know whether you haue well done or not, set the grosse summe that amounteth, of the Multiplication ouermost, and diuide it by the Multiplier: and if the quotient be the same number that should be multiplied, then haue you well wrought, else not, as in that example where we multiplied 264 by 29, the grosse summe was 7656. Proofe of Multiplication by Diuision.

Now if you will know whether that Multiplication be true, you shall diuide that 7656 by the multiplier, 29, and you shall perceiue that the quotient will be 264, and that is a token that you haue well wrought.

Scholar.

Scholar. By your patience I will proue that, and first set downe the grosse summe and the multiplier, not after the rule of Multiplication, but after the rule of Diuision, for now that number is become the diuisor, that was before the Multiplier, I should set them therefore thus:

7656
29 .

Then shall I seeke how many times, 2 in 7, that may be 3 times, and 1 remaineth: but then may not 9 be found so often in 16, therefore must I take a lesser Quotient, that is to say 2: then say I, twice 2 maketh 4, which I take out of 7, and there remaineth 3, then do I cancell 7 and 2 and over 7 I write 3, and in the Quotient I set 2: so the figures stand thus:

3
7656 (2
29

Then say I forth, two times 9 make 18, which I abate out of 36, and there resteth 18: then cancell I 3, and over him set 1, and likewise I cancell 6 and 9, and over them I set 8: so that thus stand the Figures.

1
38
7656 (2
29

Then I set forward the diuisor by one place, and seeke a new Quotient, that is to say, how many times 2 are in 18, which I finde to be 9 times: but then can I not finde 9 so many times in 5, therefore I take a lesser Quotient, as to say 8: but yet that is to great: for if I take 8 times 2 out of 18, there remaineth

remaineth but 2, and I cannot find 8 times 9 in 25: therefore yet I take a lesse quotient, that is 7, which is also too great, for if I take 7 times 2 out of 18, there resteth 4, but now I cannot take 7 times 9 out of 45, therefore yet I seeke a lesser quotient, as to say 6, then say I, 6 times 2 make 12, that I take out of 18, & there remaineth 6, so I cancell 18, and the 2, and write 6 ouer 8 thus:

```
 16
 18
7656 (26
 299
  2
```

Then say I forth 6 times 9 maketh 54, that take I out of 65, and there remaineth 11, and the figures stand thus:

```
   1
 16
 181
7656(26
 299
  2
```

Then must I set forth the Diuisor againe & seeke a new quotient, which will be 4: for though I may finde 2 in 11, 5 times, & 1 remaine, yet I cannot find 9 so often in 6, therefore I set the Figures thus:

```
   1
 181
7656 (264
2999
 22
```

And the 4 in the quotient I multiply into the Figures of the diuisor saying, foure times 2 makes 8 which I take out of 11, and there rests 3 therefore I cancell the 11. & the 2, and set 3 ouer the first place of 11, thus:

```
 1
 163
 181
7656 (264
2999
 22
```

And then doe I say forth, 4 times 9 maketh 36, which I take from 36, and there remaineth nothing.

so that the Quotient of this Diuision also (where 7656 is diuided by 99) is 264: Which doth declare, that if 264 be multiplied by 29, the summe will be 7656. And thus I perceiue now how both Multiplication is proued by Diuision, and Diuision also by Multiplication.

Master. Now haue I ended the fiue common kinds of Arithmeticke. For (as touching Mediation, Duplation, Triplation, and such other) they are no seuerall kinds of Arithmeticke, but are contained vnder the other. For Mediation is contained vnder Diuision, and is nothing else but diuiding by 2: and so are Duplation and Triplation contained vnder Multiplication: for Duplation is nothing else but multiplying by 2, and Triplation is multiplying by 3, of which I will onely propose an example for the rules you haue heard already.

An example of Mediation.

If you would mediate or diuide into 2, this summe 4531010, you shall set 2 for the diuisor, and worke as you learned before, as thus:

4531010(
2

Then I finde 2 in 4 two times, therefore my Quotient must be 2: so I cancell 4 and 2, and remoue the Diuisor forward thus, as the worke requireth, and as before in Diuision hath béene declared.

4531010(2265505
2222222

Which mediation or diuision by 2 being finished

finished, you shall haue for your Quotient 2265505, which is the halfe of 4531010, as you may trie by Duplation; for double that Quotient, or multiply it by 2, and the same number will amount. Duplation

I will no longer tarry about these, seeing they are but members of the other kinds. But here now (according to my promise) I will teach you certaine easie formes both of Multiplication and of Diuision. And first of Multiplication.

If you would therfore multiply any summe by 10, you shall need to do no more but adde a cypher before his first place: as for example, 36 multiplied by 10, make 360. Easie formes of Multiplication.

Likewise if you would multiply any summe by 100, put two ciphers at his beginning. So if you would multiply any summe by 1000, adde three Cyphers to the beginning of it.

Scholar. This do I well perceiue, and also the reason of it.

Master. I will omit all reasons till our next meeting, when I shall tel you the reason of all other parts of Arithmetick also: and as to our matter now, look, as I haue told you, that you both remember it, and also often practise it.

And now you haue learned how to multiply easily by 10, 100, 1000: and of like manner may you do with any other of like sort.

But now if you will multiply by 20, 30, 40, and so forth, or by 200, 300, and such like, where there is one Cypher in the first place,

or many orderly in the first places, you shall take away those Cyphers and multiply the summe onely by the other figure, or figures, (if they be many) and then at the beginning of the summe that amounteth, you shall set so many Cyphers as you tooke away.

Example of 2873, which I would multiply by 300. First I omit the 2 Cyphers from the Multiplier, and I multiply the summe by thrée onely that is left, and it amounteth to 8619: before which I put the two Cyphers that I before omitted or tooke away, and then is it 861900. And that is the summe that amounteth when 2873 is multiplied by 300.

Scholar. And if there were two or more figures beside the Cyphers, I must onely take away the Cyphers, and multiply by the other figures, as I learned before: as if I would multiply 93648 by 25000, I should take away the thrée Cyphers, and multiply the same by 25, and then at the beginning of that total summe should I adde the 3 Cyphers againe.

Master. Euen so: but if it chance the number that should be multiplied, or both the summes, as well the number that should be multiplied, as the Multiplier, to haue Ciphers in their first places, euermore omit the ciphers & worke by the rest. But remember to restore as many Ciphers to the amounting summe as you bated before, as in this example: 30200 shall be multiplied by 206, I shall onely take away two Cyphers from the greater

ter number, and then multiply 302 by 206, and afterward adde the two Cyphers againe. But if I would multiply the same 30200 by 2060, I shall not onely take away the two Cyphers from the number that should be multiplied, but also I may take away the one Cypher from the Multiplier, and then must I adde 3 Cyphers to the summe that amounteth: but take héed that you take away no Cypher that commeth after any signifying figure, as in the last example, you may not take away that in the fourth place of the higher number, neither that in the third place of the Multiplier: howbeit, yet thus you may do: If one Cypher or more come in the midst of your summes you may multiply by the other Figures, and ouerskip them: but so, that you giue euery figure his due place: as thus, I will multiply 3026 by 2004, therefore I set them thus:

3026
2004
12104
52

And thus I do multiply them. First 4 times 6 make 24, I set the 4 vnder the first place, and kéepe the 2 still in my minde. Then say I againe, 4 times 2 maketh 8, and the 2 that is in my minde maketh 10. I set downe the Cypher 0, and kéepe the article 1 in my minde: Then 4 times 0 is 0, and the 1 in my minde maketh 1, I set downe the figure 1, and say againe, 4 times 3 is 12, I set downe 2, and kéeping the 1 still in my minde (hauing no more places of the vpper number to multi-

tiply it withall) I put it downe next 2 in the fifth place.

But now when I come to the next place (being a Cypher o) I let it go, because it multiplieth nothing & likewise the second cipher.

But then, when I come to the 2, and multiply it into the 6 of the ouer number, you must take heed (according as I taught you in Multiplication) that the first number amounting of the Multiplication be set right vnder the Multiplier, and the other orderly toward the left hand, according as you may see in this example, which being finished, with the addition thereof gathered together, will stand as this example sheweth.

	3026
	2004
	12104
	6052
	6064104

Which is indeed wrought so much the sooner and shorter by ouerskipping of the 2 Cyphers: which otherwise (if the same Example were wrought at length) it would haue had 2 workings more, as by the same example here also set downe doth appeare.

	3026
	2004
	12104
	0000
	0000
	6052
	6064104

Scholar. Sir I thanke you, for I see great ease in this way of Multiplication: (and if you can shew me such like in diuision) you shall greatly further me.

Master.

Master. Yes, I will teach you some easie wayes in Diuision also, and first this: If you would diuide any summe by 10, you shall onely with your pen make a square line betwéene the first figure of your summe and the second, and then haue you done: for the whole number that followeth the line, standeth for the Quotient, and the figure that is before the line, is the remainer: as for example, 3648 diuided by 10 364 |8 Where 364 is the Quotient, and betokeneth that so many times are 10, in 3648, and the 8 after the line is the remainer, which cannot be diuided into 10, but by breaking it into fractions, wherewith I will not meddle yet.

Easie formes of Diuision.

And so likewise if you would diuide any summe by 100 with your pen, you shall cut away the two first figures, and if you would diuide by 1000, you must cut away the thrée first figures, and so of any other diuisor, whose last figure is 1, and the other ciphers, looke how many ciphers the diuisor hath, and so many figures at the beginning shall you cut away with the square line, and they stand alwayes for the remainer, because they are lesse then the diuisor, and cannot be diuided by it, and the other Figures that are behinde the line stand for the Quotient.

But now if your Diuisor haue any other figure in his last place then 1, and in all his other places haue cyphers, looke how many

Cyphers they be, cut away so many of the first figures of the number that should be diuided, and diuide the rest that followeth the line by that Figure that is in the last place, as if it were the whole diuisor.

Example of 64284, which I would diuide by 300, here must I cut away the two first figures (for so many Cyphers my diuisor hath) and must diuide the rest by 3, which is the figure in the last place of the Diuisor. First therefore I part away the two first figures, and the summe standeth thus:

642 | 84 (2
3 |

Then do I diuide 642 by 3. and the quotient is 214: for in 6 I find twice 3, and in 4 once, and 1 remaining, which I with the 2 next before doth make 12, wherein I finde 3 foure times: and this is a ready way to turne shillings into pounds: for sith one pound doth containe 20 shillings, I must diuide the whole number of shillings by 20. Therefore easily to do it, I see that my diuisor hath one Cypher, and therfore I cut away one figure from the beginning of the whole summe of shillings and then I do mediate or diuide by 2 the other figures or summe that followeth.

Scholar. I will put an example.

If you would diuide 64287 shillings by 20: that is to say, if I would turne so many shillings into pounds, I must cut away the first figure, that is 7, and diuide the rest, that is 6428 by 2, so shall the quotient be 3214,

whereby

whereby I know that 64287 shillings make 3214 pounds, and 7 shillings remaining.

Master. Now proue by Multiplication whether you haue well done or no.

Scholar. The quotient is 3214, which I do multiply by the diuisor 2, and it doth amount to 6428.

Master. Hereby you may perceiue not onely that you haue well done, but also how by diuision you may turne shillings easily into pounds: and contrariwise by Multiplication you may turne pounds into shillings.

But here shall you see amongst diuers men diuers formes of such diuision: but if you marke what I haue told you, you shall perceiue easily all the wayes. For some men do not cut away so many of the first figures of the summe that they would diuide, as there are Cyphers in the first places of the diuisor: but they set all their Cyphers orderly vnder the first places of the number that they would diuide; and then with the other figure or figures (if there be many) they diuide the rest of their summe.

Another manner of the Abridgement.

Example. If they would diuide 725931 by 3400, they do set their summes thus.

7259	31
34	00

And then do they diuide orderly till they come to the Cyphers: for there they stay and end their worke, as in this example.

They seeke how often 3 may be found in 7, which is two times, and 1 remaining: there-

therefore they set 2 in the Quotient, and cancell 3, and 7, and ouer 7 they set the 1 that remaineth thus:

1
725931 (2
34 00

Then doe I go forth saying, two times 4 maketh 8, which they take out of 12, and there remaineth 4, thus:

14
725931 (2
34 00

Then remoue they the diuisor forward, and seeke how often 3 may bee found in 4, which is but once, and 1 remaineth, then set they 1 in the Quotient, & cancell 3 and 4, and ouer them they set that 1, thus.

1
14
725931 (21
344 00
3

Then take they once 4 out of 15, and there resteth 11. Or else more easily: Take once 4 out of 5, and there resteth 1: so they cancell the 4 and 5, and set 1 ouer them thus.

1
141
725931 (21
344 00
3

Then set they forth the diuisor againe and seek how many times 3 are in 11, which they find three times, and 2 remaining: so they set 3 in the Quotient, and cancell 11 and 3, and ouer them set 2 thus:

12
141
725931 (213
34400
33

Then doe they multi-

ply

ply 4 by 3, which maketh 12, that withdraw they out of 29 and there resteth 17, of which the 7 must be set ouer the 9, and the 1 ouer the 2, thus:

```
     1
    12
   1417
  725931 (213
  343400
    33
```

And now are the two Cyphers next ensuing, so that the Diuisor can no more be set forward, and therefore is the Diuision ended, and the Remainer is 1731.

Now the Quotient which is 213, doth declare, that if you diuide 725931 by 3400, you shall finde it therein 213 times, and there remaineth 1731: so shall you finde it, if you worke as I taught you, by cutting away the 2 first figures, because of the 2 Cyphers.

But this must you marke (as you may perceiue by this last example) that if there be left any other Remainer in the summe that was behinde the squire line, that the Remainer, must be set to the latter end of the first Remainer, which was cut away with the squire line: as if you would diuide 725931 by 3400, after the forme that I taught you, then would your summes appeare thus. Note.

```
     |1
    12|
   141|7
  7259|31 (213
  3444
   33
```

So that 17 which remaineth after the line, must be set to the 31 (that was cut away

way with the line) in higher places, as you see here: where that 17 with the 31, do make 1731.

Scholar. Sir, is there no other forme of Diuision in practise but this?

Master. Yes verily, there are other formes in practise, but because I loue breuity, I will declare onely one, which I first learned of, and is practised by that worthy Mathematician, my ancient and especiall louing friend, Master Henry Brigges, wherein not any one figure is defaced or cancelled. As if I should diuide 72 by 6, first place them thus:

6) 72.

Then if you please you may write the diuisor in a loose paper that it may more easily without cancelling or defacing of the worke be applyed to, and remoued from the diuidend at pleasure; then apply your diuisor 6 to 7, the first figure of the diuidend, and inquire how oft it may bee had in 7, and seeing 6 is but once in 7, set 1 in the quotient line, thus:

Write the Diuisor in a loose paper, to remoue at pleasure.

6) 72 (1

Then multiply the diuisor 6, by the quotient 1, and set the product 6 vnder 7 thus:

6) 72 (1
6

Then draw a line vnder 6, and subduct

6 out of 7, ſetting the remainer 1 vnder 6 thus:

6)72(1
6
——
1

Then bring downe the next figure of the diuidend, and ſet it with the Remainer 1 vnder the line thus:

6)72(1
6
12

And bring the moueable diuiſor 6 vnder the 2, and as before enquire how oft 6 is in 12, and finding it to be twice in 12, ſet 2 in the quotient thus:

6)72(12
6
——
12
12
——
0

And multiply 6 by that new quotient 2, ſetting the product 12 vnder the other 12, and ſubducting it out of the vpper number, there reſteth nothing. And ſince the vnites of this product do ſtand vnder the vnites of the diuidend, the diuiſion is ended: otherwiſe you ſhould procéed as before, bringing downe the next figure; remouing the diuiſor, diuiding, multiplying, ſubducting, &c.

Scholar. This is very eaſie, but if there be greater numbers propounded, is the operation the ſame?

Maſter. If the numbers be neuer ſo great, the work is the ſame without any difference, as ſhall appeare by this example.

Diuide

Diuide 7890 by 33.

First set them thus, 33) 7890 (2

then bring the Diuisor vnder 78, and sée how oft it is there found, which is twice, and therefore set 2 in the quotient, by which multiply the Diuisor 33, and set the product 66 vnder 78, and subduct it out of it thus.

```
33) 7890 (239 3/33
     66
    ---
    129
     99
    ---
    300
    297
    ---
      3
```

Then bring the next figure 9 downe, and set it with the Remainer 12, it maketh 129, and remouing the Diuisor 33 thereto, enquire how often 33 is contained in 129, and I finde it but thrice, (though at the first it made a shew of more) therefore set 3 in the Quotient, and multiplying 33 by 3, set the product vnder 129, subducting that product out of the number aboue, and procéed as before.

Then shall you finde the diuisor 9 times in the Remainer, therefore setting 9 in the Quotient, multiply, and subduct as before, and at the last you shall finde onely 3 remaining, which must be set aboue a line after the Quotient, and the Diuisor vnder, as aboue appeareth.

Scholar. Is there no more difficulty in the

the

the whole Rule?

Master. Not any, although your number be neuer so great, as before I haue said.

¶ And here will I make an end of Diuision, (sauing that I doe request you to exercise your selfe well herein by many summes, till you haue attained some expertnesse therein.)

For the reasons and conclusions thereof are so many, and so auailable for all sorts of men whatsoeuer; that if I should speake of the infinite vses thereof, I should rather lacke words then matter. And therfore recommending it to your iudgement hereafter, vpon your further trauell into the Art, I will here end this Treatise, representing vnto you one example or simple question of Diuision and Multiplication, in stead of many, which is this.

There are foure brasse Peeces. The first of them at a shot spendeth 9 pounds of powder, the second spendeth 5 pounds, the third 4 pounds, and the fourth 2 pounds. They are all appointed against the battery of a hold, and there is allowed by the Master Gunner 700 pounds of powder to be spent by these foure Peeces in this assault. The question is twofold: The first how many shot each Peece shal iustly make about with this 700 pounds of powder? And lastly, how many pounds of powder ought iustly to be allowed to each Peece for his true proportion?

A question of shooting in Ordnance.

Scholar.

Scholar. Why Sir, you make me smile, to beare me in hand, that these two demands may be simply resolued by Multiplication and Diuision.

Master. Truly that they may, and that you may by and by worke your selfe with a little labour: first adde together their quantities of powder that is, 9 pounds, 5 pounds, 4 pounds and 2 pounds, all which make 20: Diuide the 700 pounds of powder by that 20, and your quotient giueth 35, as here appeareth, which sheweth for most certainty that they shall make iust 35 shootes about.

x
700
200(35
2

Scholar. Sir, all this haue I done, and I see it is so, but whether it be true or not, I cannot tell.

Master. To try the truth of the same, multiply the first peece that spends 9 pounds by 35, & you shall see his allowance, which is 315 pounds of powder. Multiply also the second peece that spends 5 pounds by 35, & you shall find 175 pounds his allowance: then 4 by 35 and you shall find 140 pounds his allowance. Lastly, multiply 2 by 35 and you shall finde 70 pounds his allowance. All which foure particular summs you shall adde together by Addition, as here appeareth, and it maketh iust 700 pounds, and so is the question truly absolued.

	315
	175
	140
	70
	700

Schollar:

Schollar. Truly sir, these excellent conclusions do wonderfully more and more make me in loue with the Art.

Master. It is an Art, that the further you trauell, the more you thirst to go on forward. Such a Fountaine, that the more you draw, the more it springs: and to speake absolutely in a word (excepting the study of Diuinity, which is the saluation of our Soules) there is no study in the World comparable to this, for delight in wonderfull and godly exercise: For the skill hereof is well knowne immediately to haue flowed from the wisedome of God into the heart of man, whom he hath created the chiefe image and instrument of his praise and glory.

Schollar. The desire of knowledge doth greatly incourage me to be studious herein: and therefore I pray you cease not to instruct me further in the vse hereof.

Master. With a good will. And now therefore for the further vse of these two latter, that is, Multiplication and Diuision; I will briefly shew you the feat of Reduction.

Reduction.

Reduction.

Reduction what it is.

REduction is, by which all summes of grosse denomination may be turned into summes of more subtile denomination. And contrariwise, all summes of subtile denomination, may be brought to summes of grosser denomination.

Grosse denomination.

Schollar. What call you grosse denomination, and subtile denomination?

Subtile denomination.

Master. That I call a grosse denomination, which doth containe under it many other subtiler or smaller: as a pound, (in respect to shillings) is a grosse denomination: for it is greater then shillings, & containeth many of them. And shillings (in comparison to pounds) are a subtile denomination, for because they are lesser then pounds, and many of them are contained in one of the other: and so likewise of other things: whatsoever thing is compared to other, if it be greater, and containeth many of them, it is a grosser denomination: but if it be lesser (so that many of them are in the other) then are they called the subtile denominations: whereby you may perceive that one denomination may be called a grosse denomination, and also a subtile, (that is to say, a great and a smal) in divers comparisons. For shillings compared to pounds, are a subtile or small denomination: but compared to pence, they

they are a grosse or great denomination.

Schollar. Now I understand the name, I pray you teach me the vse.

Master. The vse is easily learned, if you remember what you haue learned before. For if you will reduce any summe of a grosse denomination into a summe of a smaller or subtiler Denomination, you must consider how many of that subtiler Denomination do make one of the grosser Denomination, and by that number or Numerator do you multiply the summe: as if you would reduce 20 pounds into shillings, you must consider that in a pound are included 20 shillings, therefore multiply the one 20 by the other 20, and there will amount 400, whereby you may know that in 20 pounds, are contained 400 shillings. Likewise, if you would reduce 30 shillings into pence, considering that in a shilling are 12 pence, you must multiply 30 by 12, and it will be 360, whereby you may finde that in 30 shillings are contained 360 pence. And thus may you reduce any grosse denomination into a more subtiler, by multiplication, if you know how many of the lesser do make the greater: of which thing I will anon giue you a briefe Table for the most accustomed kindes of Money, Weight, Measures, and Time, and such like: whereby you may know how often each subtile denomination is contai- To reduce grosse denominatió to subtile.

To reduce subtile denomination to grosse.

ned in the grosser, when you shall néed it for the foresaid kinde of Reduction. And also the same shall serue you, if you would reduce any summe of a subtiler denomination, into a summe of a grosser denomination. For in such Reduction you must consider (as in the other forme) how many of the smaller doe make the greater: and by that number you must diuide the other summe, and the Quotient will declare how many of the greater denomination are comprehended in that summe; as for example: If you would know how many shillings are contained in 3240 d, consider that 12 pence do make 1 s: you must diuide that 3240 by 12, and your Quotient will be 270, whereby you know that so many shillings are in 3240 d. But if you would know further how many pounds are in these 270 shillings, séeing that euery pound containeth 20 shillings: diuide that 270 by 20, and it will be 13, and 10 remaining, whereby you may know that in 3240 d. (or 270 shillings) are 13 pounds and 10 shillings. For euermore the remainer must be named by the name or denomination of that summe that was diuided, which in this place were shillings. And thus may you do with any other kindes of Denominations.

¶ Wherefore to the intent you may haue certaine light or knowledge in most common coynes, weights, and measures (which is the chiefe and principallest thing in trafficke to be knowne)

knowne) I haue in each reduction as they come in order, set downe certaine instructions incident thereunto. And first I haue hereunto added this Table, wherein is comprehended, not onely our currant and common coynes, but also the most part of the vsuall coynes of Christendome, with their iust weights and value currant in this Realme of England, intending at the latter end of my Addition to this Booke, to write of the ordinary money vsed in diuers places, and their common values currant for trafficke, with the manner of their exchanges from place to place, &c.

A Table of the names and now valuation of the most vsuall Gold-coynes throughout *Christendome, with their seuerall weight of Pence and Grains: and what they are worth* of currant English money this present yeare 1630.

The names and titles of the Gold.	The weight in Pence.	Grains.	The value in Shil.	pence.
Great Soueraigne.	10	0	33	0
Double Souer. K.H.	8	1	22	0
Double Soue. of Q.E	7	7	22	0
Royall.	4	23	16	6
Halfe Royall	2	11 d.	8	3
Old Noble	4	6	14	8
Halfe Noble	2	3	7	4
Angel	3	8	11	0
Halfe Angel	1	16	5	6
Salute	2	5	6	11 ob
2 parts of Salute	1	11	4	7
George Noble	3	0	9	9 ob
Halfe George Noble	1	12	4	11 q
First Crowne K.H.	2	9	6	11 ob
Base Crowne K.H.	2	0	5	6
Soue. H K. best	3	24	11	8 ob q
Soueraigne K.H.	4	0	11	0
Edward Soue.	3	15 d	11	0
Elizabeth Souer.	3	15 d	11	0
Elizabeth Crowne	1	19	5	6
Halfe Crowne	0	19 d	2	9
Unite	0	12	22	0

Double

Double Crowne	3	6	11	0
Britaine Crowne.	1	15	5	6
Thistle Crowne.	1	7	4	4 ob q
Halfe Crowne.	0	19 d	2	9
Crosse dagger.	3	6 d	11	0
Halfe Crosse dagger	1	15	5	6
Rose Royall.		21	33	0
Spurre Royall.	4	10 d	16	0
The Angel.	2	23 d	11	0
Halfe Angel.	1	11 d	5	6

¶ *All the seuerall peeces of Gold heretofore mentioned are set downe according to their valuation by the Kings Maiesties Proclamation for Gold. Dated the 23 of Number, 16:1.*

A Table of forraigne gold coyne, according to their ancient valuation and seuerall weight in pence and graines.

The names and titles of the Gold.	The weight in Pence,	Graines	The value in Shil.	Pence.
Unicorne of Scot.	2	10	6	0
Scotish Crowne.	2	5	6	0
French Noble.	4	16	13	4
All sorts of French Crownes.	2	5	6	0
Flanders Riders.	2	6	6	6
Gelders Riders.	2	2	3	6
Philips Royall.	2	10	10	0
Philips Crowne.	2	5	5	0
Collen Gilden.	2	2	4	8
New And Gild	2	2	5	0

The names and titles of the Gold.	The weight in Pence.	Grains.	The value in Shil.	pence.
Flaunders Noble.	4	10	22	0
Halfe Flan. Noble.	2	6	5	0
Flan. Angell best.	3	6	9	0
Flaund Royall or key	3	10	10	0
Carolus Gilden	1	13	3	6
Flaunders Royall.	2	6	5	0
Saxon Gilden.	2	2	4	8
Flaunder Crowne.	2	5	6	0
Philips Gilden.	2	3	4	2
Halfe Phil. Gilden.	1	1	2	1
Golden Lyon.	2	16	7	8
3 parts of gold Lyon	0	21	2	5
2/3 parts of gold Lyon	1	19	4	11
Dauids Gilden.	2	2	4	0
Horne Gilden.	1	12	4	11
Old Andre Gilden	2	3	4	10
Crusa long Crosse	2	6	6	0
Crusa short Crosse.	2	6	6	2
Milreyes.	4	20	1	4
Halfe Milreyes.	2	10	6	8
Portigue 1 ounce.	2	16	68	0
Golden Castilo.	2	23	8	10
Ducket of Aragon.	2	6	6	6
Hungary Ducket.	2	7	6	4
Double Pistolet.	4	9	11	8
Single Pistolet.	2	4 d	5	10
Ducket of Floren.	2	5	6	4
Double Ducket.	4	11	13	0
Single Ducket.	2	6	6	6
Double duc. of Rom	4	13	12	8

It is to be vnderstood (gentle Reader) that whereas in these tables, the weight is called by the name of a penny, it is not ment a penny of siluer money, but a penny of Gold-smiths weight, which containeth 24 Barly-corns. Concerning which, see Troy weight in folio 133.

So if a man haue not the weight wherewith to weigh any peece of gold, he may do it with Barly-cornes, being dry, and as it is said, fol. 133.

The prices of Gold which the bringers *in of forraine Gold shall receiue at the Mint,* according to the Kings Maiesties *Proclamation. Dated the* 14 of May, Anno 1612.

FOr an ounce of *French* crowns, being 22 Karacts fine.	3 li, 6 s.
For euery ounce of *Spanish* Pistolets, being 21 Karacts, 3 graines and a halfe fine.	3 li, 6 s.
For Duckets of *Spaine*, being 23 Karacts, 1 graine fine at least the ounce.	3 li, 8 s, 8 d.
For Milreas Crusado long crosse. Crusado short crosse, the ounce.	3 li, 6 s, 2 d
For *Hungary* Duckets being 23 Karacts, 1 graine fine at least the ounce	3 li, 9 a s.
For the Checkeen of *Venice*, being 23 Karects, 1 graine fine at least the ounce:	3 li, 10 s.

For

For *Barbary* Gold, being 23 Karects and digraine fine, at the least the ounce.	3 li, 9 s.

And if the said Barbary Gold bee of lesse fineness, abatement to be made according to the rate.

For Sultaines being 23 Karects, 1 grain fine at least the ounce.	3 li. 8 s. 8 d.
For all other Gold, being 22 Karects fine the ounce	3 li, 6 s.

And being finer, a greater price according to that rate, and being courser, a lesse, so that the bringer in supply the lesse fine, with the more fine, in such sort, that in the totall it makes good the same rate of 22 Karects fine.

The price of Siluer, which the bringers in of forraine Siluer shall receiue at the Mint, according to the *Kings Maiesties aforesaid* Proclamation.

For the Ounce of *Spanish* siluer money of *Ciuill.*	5 s.
For the Ounce of *Mexico* money.	4 s, 10 d,

For

For Ingots of Siluer, being 11 Ounces, 2 d. weight fine, according to the Standard of England, the Ounce. ——— } 5 s.

¶ *And for other Siluer of more finenesse, a better price according to that rate, and for courser a lesse: so that the bringer in supply the lesse fine with the more fine, in such sort, that in the totall it makes good the said rate of* 11 *Ounces, 2 pence weight fine, according to the Standard of England.*

Of Siluer Coynes currant in this REALME.

The *Edward* Crowne of 5 s.
The *Edward* halfe Crowne of 2 s. 6 d.
The *Edward* shillings, halfe shilling, and the three pence.
Phillip and *Maries* shilling, and halfe shil.
The *Mary* Groat, and *Mary* two pence.
Queene *Elizabeth* shilling. 9 d. 6. d. 4. d. 3 d. 2 d. 1 d. three farthings, and halfe penny.

HEre would I now expresse the values of sundrie other Coynes of diuers Countries, but for three causes I now refraine. The first and chiefest is, because they are not currant by the Statutes of this Realme. Another cause is, by reason they are so vncertaine, that they be neuer long at one rate.

rate. And againe, they are so different in so many places, that it were matter enough for a great Booke to speake sufficiently of them all. Howbeit, because you shall not be altogether ignorant of them, I will shew you the values of some that are most in vse, and first of France.

French Coynes.

The most common money are Deniers Soulx, and Frankes: 12 Deniers makes 1 shilling, 20 Soulx make 1 Franke: so that as you see these 3 kindes are like in the rate to pence, shillings and pounds with vs; but that this is the difference, that their Denyer is but the ninth part of our penny, and so their Soulx (commonly called *Scuses*) go 9 to our shilling, and 9 of their Frankes to an English pound of money. So that 3 of their Frankes make a Noble. And by those three you may practice how to reduce French money into English money, according as I haue set forth here following.

2160	Deniers make	240 d, or 20 s.
3240		360 d, or 30 s.
8352		929 d, or 2 li 17 s 4 d

2160 *Soulx* make 240 shillings. And so of other in like rate. As for the rest of their Coynes, I omit them till hereafter, that you haue some vnderstanding in broken numbers.

Flanders Coynes.

But now as for the Coynes of Flanders, they be so changeable, that you must know them from time to time: else you cannot reduce them into our money certainly: but yet

that

that you haue an example of their money to exercise you withall, you shall take those that be most common: as Stiuers both single and double, Groats Flemish, Carolus, and Gyldens. A Flemish Groat is a little aboue 3 Farthings English. A single Stiuer is 1 d. ob.q. halfe farthing. The double Stiuer Carolus is 4. d. ob. halfe farthing. Then there is also the Carolus Gylden, which is worth 20 stiuers. And the Flemish Noble is worth 3 Carolus Gyldens, and 12 Stiuers.

So that if you would conuert Flemish money, or any other kinde of money whatsoeuer it be, iustly into sterling, you must reduce it first into the smallest part of English money that is in that Coyne. As for example: If I would reduce 368 double stiuers into English money (considering that a double stiuer containeth 3 d. farthing) you shall first looke how many farthings be in the double stiuer, & you shall finde them 13: therefore multiply the summe of the stiuers by 13, and then haue you their value in farthings, which is 4784. Now, if you diuide that by 4, then there will appeare the number of pence: but better it were to diuide it by 48 (for so many farthings are in 1 shilling) and then will the Quotient declare the summe of the shillings.

Likewise if you would reduce any summe of single stiuers into English money, you must multiply the summe first by 13, and then haue you reduced them into a certaine summe, that

is

is to wit, halfe farthings, which summe if you diuide by 8, then will amount the summe of pence: or if you diuide it by 96, the summe of shillings will appeare.

But marke this in all Diuision: when ye do reduce to bring one Denomination into another, if there be any Remainer after the Diuision, that must be named by the Denomination of the grosse summe that was diuided. As for example, I would bring 254 farthings into pence, therefore I diuide that 254 by 4 (for so many farthings make a peny) and the Quotient is 63, which is the summe of the pence, and then remaineth yet 2, which are farthings still, as one may proue by diuiding. And this must be marked in all Diuision, namely, when it is done for Reduction.

Note well.

¶ Touching Danske Money, they haue their Soulx, whereof 20 is a Liuer: which is 2 shillings sterling. They haue also their Grash, whereof 80 make a Gilden, which is 4 shillings sterling. They haue also Dollors, & their common or old Dollor is 35 Grash. New Dollors they haue, which be diuers, some valued at 24 Grash, some at 26, and some at 30. And thus much I thought good to adde to the Author, touching Danske Money.

Danske Money.

Concerning Spanish Money, whereof the most common are Cornadoes, Marucides, Ryals, and Duckets: sixe Cornadoes make a Marucide, 4 Marucides make a Ryall, and

Spanish money.

11 Ryals

11 Ryals make one Ducket, so the Ducket containeth 374 Marucides, which is about 5 shillings, 10 pence, sterling. Therefore if you would conuert 124 pounds, 5 shillings, sterling into Duckets, consider that pence is the least value or Denomination named in this question: therefore reduce 124 pounds, 5 shillings into pence, and it maketh 29820 pence: which if you diuide by pence that a Ducket is worth, (which is 70,) you shall haue for your Quotient 426 Duckets, your desire.

In Venice they haue bettes, Souldyes, Lieures 5 bettes make an English penny, 60 bettes a shilling, which is 2 Souldyes, and 20 Souldyes a Lieure of Venice which is a pound sterling. Venige money.

Thus much haue I said of Money: Now will I shew you in like sort, the distinction of weights. Weights.

After a Statute made Anno 11. *H.* 7. *there ought to be but one sort of weight. As* 24 *Barley-cornes d*[illegible]*, and taken out of the middest of the Eares, do make a penny weight,* 20 *of those penny weights make an ounce; and* 12 *ounces a pound of Troy weight, by which is weighed Bread, Gold, Siluer, Pearle, Silke, and such like.* But commonly there is vsed another weight called Haberdupoise; in which 16 ounces make a pound. Therefore when you would reduce ounces into pounds, you must consider whether your weight be Troy weight or Haberdupoise: and if it be Troy weight, you must Troy weight. A penny weight. An ounce. A Pound Troy. Haberdupois weight.

diuide

diuide your ounces by 12, to bring them to pounds, but if it be Haberdupoise, you must diuide them by 16. Now againe, there be greater weights, which are called a hundred, halfe a hundred, and a quarterne, and also a halfe quarterne, &c.

A hundred weight.

Schollar. Why? so there may be reckoned 20 pound, 40 pound, 200 pound, and such innumerable.

Master. All these are numbers of weight, but they haue not common weights made to their rate, as the other haue. And againe these that I did name, are not iust in number as they seeme by their name: for an hundred is not iust 100, but is a 112 pound. And so the halfe hundred is 56: the quarter 28, and the halfe quarter 14. And these be the common weights used in most things that are sold by weight.

Wooll Weights. Todde. Stone.

Howbeit there are in some things other names, as in Wooll, 20 pound is not called a quarterne, but a Todde: and 14 pound is not named halfe a quarterne, but a Stone, and the 7 pound halfe a stone. Other names because they differ in many places, and agree in few, I let them passe.

Sacke of wooll.

But a sacke of Wooll by the Statutes, is limited to be 26 stone.

Cheese weights.

¶ Now in Cheese, though it be sold by the hundred, and by the stone in some places, yet the very weights of it are Cloues, and weyes. So that a cloue containeth 8 pound, and

a wey 32 Cloues, which is 256 pound, that is 12 score and 16 pound and so much weigheth the wey of Suffolke Cheese, and the like is or should be the Barrel of Suffolke butter.

The Wey of Essex Cheese containeth six score and sixteene pound: and so much is also the Barrel of Essex butter.

Moreouer this weight is vsed by the Apothecaries in their Physicall composition, and mixture of medecine, wherein the least is a graine,

And	*20 Graines*	make	*A Scruple*	thus characterd.	℈
	3 Scruples		*A Drachme or Dragme*		ʒ
	8 Dragmes		*An Ounce*		℥
	16 Ounces		*A Pound.*		lb.

The Apothecaries weights.

Now of weights are made other measures both for graine & liquor. For a pound in Troy weight, maketh a pint in measure, so that 8 pound or 8 pints do make a gallon: halfe a gallon is named a pottle, and halfe a pottle is called a quart, which containeth two pints: Now aboue a Gallon the next measure is a Firkin: then the Tertian a kilderkin, or halfe Barrel, & a Barrel. And by these measures are sold commonly Ale, Beere, Wine, & Oyle, Butter and Soape, Salmon, Herrings, and Eeles.

Measures for liquor. A pint. Gallon. Pottle. Quart. Ferkin. Tertian. Kilderkin. Barrell.

But as these be vnlike things, so the measures of their vessels do differ, for the measures of them all are as followeth.

Of Ale	*the Firkin*	*containeth*	8	*Gallons.*
	the kilderk.		16	
	the barrell.		32	

Ale measure.

Beer measures.

Of Beer	*the Firkin*	*containeth*	9	*Gallons.*
	the kilderk.		18	
	the barrell.		36	

Sope measures.

Sope measures, both Firkin, Kilderkin, and Barrell should be equall to Ale measure.

Moreouer the Statutes do limite the weight of euery of those three vessels being empty.

A Barrell	*to weigh empty*	26	*pounds.*
Halfe a Barrell		13	
A Firkin		$6\frac{1}{2}$	

Herrings.

Herrings also sold by the same measures that Ale and Soape be sold by.

Herrings are sold by the tale, 120 to the hundred, ten thousand to the last.

Salmon & Eeles.

Salmon and Eeles haue a greater measure.

Salmon & Eeles	*the Butte*	*holdeth*	84	*gallons.*
	the Barrell		42	
	halfe Bar.		21	
	the Firkin		$10\frac{1}{2}$	

Howbeit, some Statutes did limite Eele vessels equall with Herring vessels.

Wine measures.

Now as for wine vessels they are seldome smaller then Hogsheads, which are of 63 Gallons: Euery Hogshead, is two Barrels: yet there are many other wine vessels, but of them all see this Table and marke the measures one by another.

Of

	the Runlet		18½	
	the Barrell		31½	
Of wine	*the Hogſh.*	*hold-*	63	*Gal-*
and oyle	*the Tertian*	*eth*	84	*lons.*
	the Pipe		126	
	the Tunne		252	

But you ſhall marke that there be other kindes of Tertians: for there be Tertians, (that is to ſay) Th rds of Pipes, of Hogſheds, and of Barrels, as well of other things as of wine. Tertians.

Alſo Malmſeyes, and Sacke, &c. the halfe Tun is not called a Pipe, but rather a Butte. A Butte.

And thus much haue I thought meet to tell you at this time.

Scholar. And is that alwayes true?

Maſter. I haue told you how it ſhould be, but how it is, I may not ſay: how they do differ daily from their iuſt meaſure, that Gaugiers can tell you better then I. But I will let this paſſe now, and ſpeak briefly of the other meaſure.

And as of weights there did ſpring the liquid meaſures (whereof I ſpake laſt) ſo of the ſame ſpringeth dry meaſures, as Pecks, Buſhels, Quarters, and ſuch like, wherby are meaſured Corne and like graines, alſo Salt, Lime, Coals, and other like. And this is the order and quantity of them. Drie meaſures.

A Pecke is the meaſure of two Gallons.

A Bushell. A Quarter. A Wey.

A Bushell	containeth	*foure Pecks.*
A Quarter	containeth	*eight Bushels.*
A Wey	containeth	*six quarters.*

These are the common names & measures, but in diuers places there be diuers sorts.

Strike. Cornocke

The Bushell in many places is two bushels, but then is that Bushell there called a Strike: and in some places halfe a quarter is called a Cornocke. But those diuersities are too many to tell you briefly them all: and againe, sith they are against the law and statutes, I count them vnmeet to be vsed.

Measure to mete length, bredth, & thicknesse.

But now remaineth yet another kinde of measures, whereby men mete length, breadth, and thicknesse, and those are, an Inch, a Foot, and such other: whose names and quantities this Table sheweth.

An Inch. Foot.

3 *Graines of Barley in length make an inch.*

12 *Inches*	*make*	*a Foot.*
3 *Foot*	*make*	*a Yard.*
3 *Foot and* 9 *Inches*	*make*	*an Ell.*
5 *Yards and a halfe*	*make*	*a Pearch.*

Yard. Elle. Pearch.

1 *Pearch in breadth, and* 40 *in length, doe make a rodde of Land, which some call a rood, some a yard land, some a Farthendele.*

2 *Farthendels*	*make*	*halfe an acre of ground.*
4 *Farthendels*	*make*	*an Acre.*

Acre.

¶ *More,* 40 *rods in length do make a furlong,* 8 *furlongs make an English mile, which containeth* 320 *Perches.*

So that an English mile, grounded vpon the *Statute*, is in length 1760 yards, 5280 foot,

foot, and 63360 Inches.

Some what greater then the Italian mile of 1000 paces, and 5 foot to a pace.

Here might I tell you many things else touching measures, and also how to reduce strange measures to our measures, but because it cannot be wel done without the knowledge of Fractions, which as yet you haue not learned, I will let them passe till another time, that I haue taught you the knowledge of broken numbers.

Scholar. But yet sir of the parts of time, I pray you tell me somewhat. The parts of time.

Master. You know that a natural day hath 24 houres, and euery houre hath 60 minutes. It néedeth not to tell you, that 7 dayes make a weeke, and 4 weekes make a common moneth, and 13 moneths make a yeare, lacking one day, and certaine houres and minutes: but of that I shall instruct you hereafter. A day. An houre, Weeke, Moneth, yeare.

Here will I make an end of Reduction for this time, which though it be counted no kinde severall of Arithmetick, you see it is no lesse needfull to be knowne, or easier to be done, then any of the other.

Scholar. Marry sir, it séemeth vnto me much harder then any other sort, for it requireth the knowledge of so many things: but now sir when you sée time, I am readie to learne forth, as much of Reduction as you haue taught mée, I remember; but and if I doe at any time forget, I shall haue re-

course to the Tables which you set forth for me.

Master. So do you: for it will not be remembred without exercise. But in as much as you understand so much as wee haue intreated of, I will now instruct you in Progression.

Progression.

What progression is.

ALthough vntill this day the most part of writers haue defined progression as a compendious kinde of Addition, yet truly it is not so: for Progression (as the very nature of the word doth informe any man) is a going forward and proceeding in numbers, and that regularly and orderly, whose place is aptly chosen to be very neare, or rather next after the exposition of the foure principal parts of Arithmeticke, for in it after a most easie manner, are all the foure former parts exercised and practised: and not onely Addition, as customably is done. Which custome hath beene the cause, why it hath so specially beene named a kinde of Addition, and defined to be a quicke and briefe Addition of diuers summes, proceeding by some certaine and reasonable order.

You shall also vnderstand there are infinite kinds of Progressions, but for you (as yet) two are sufficient to be exercised in, of which the one I call Arithmeticall, and the other Geometricall.

Arithmeticall Progression.

Arithmeticall Progression is a rehearsing or placing down of many numbers, number after number, in such sort, that betweene euery two next numbers rehearsed or placed down, the difference, diuersity, or excesse, be equall and alike.

Scholar. Sir, I thank you for that you haue both opened vnto me what Progreſſion is truly, and alſo why it is here placed.

But I pray you with an example make plaine your definition.

Maſter. Examples cannot want, ſeeing all reaſonable creatures naturally vſe the order of one kinde of Arithmeticall Progreſſion (which therfore is alſo named natural) whenſoeuer they diſtinctly do count or number any multitude by one, ſaying, 1, 2, 3, 4, 5, 6, whereby the proceeding from number to number, & euery one ſurmounting and exceeding his fellow next before by a like quantity (which here is 1) declareth the ſame to be Arithmeticall Progreſſion. And for the more plainneſſe, I ſet it downe in this manner.

The common exceſſe. 1 1 1 1 1

The Progreſſion. 1 2 3 4 5 6

Scholar. This is moſt euident. And I think that I am able to tell you now of any Progreſſion Arithmeticall propounded, what is that common exceſſe or difference whereby it proceedeth, if this order be kept in it.

Maſter. What ſay you of 3, 6, 9, 12, 15?

Scholar. They exceed each other by 3: And that may I ſet down in ſuch euident order, as you did your example of natural Progreſſion, in this wiſe.

The

John Pelgrave 1640

Jo: Pelgra

The common excesse. 3 3 3 3

The Progression. 3 6 9 12 15

Master. And doe you not also now perceiue, that the whole Table of Multiplication may be made by the order of Progression Arithmeticall? either if you will begin at the first number of any of them on the left hand, and so procéed right ouerthwart: or at any of the first numbers of the vpper row, and go directly downward.

Scholar. I pray you let me consider the thing a little, and I will answer you.

1	2	3	4	5	6	7	8	9	10
2	4	6	8	10	12	14	16	18	20
3	6	9	12	15	18	21	24	27	30
4	8	12	16	20	24	28	32	36	40
5	10	15	20	25	30	35	40	45	50
6	12	18	24	30	36	42	48	54	60
7	14	21	28	35	42	49	56	63	70
8	16	24	32	40	48	56	64	72	80
9	18	27	36	45	54	63	72	81	90
10	20	30	40	50	60	70	80	90	100

By this triall I perceiue it now very well, for the common excesse or difference betwéene any two next, is continually as much as the first number of euery row, either from the left hand ouerthwart taken, or from any of

of the uppermost overthwart rowes downeward.

Master. Now then, if of any such Progression, you would speedily know the totall summe much quicklier then by common Rules of Addition: first tell how many numbers there are (which numbers here we call places, or parcels) and if they be odde, write their summe downe by it selfe: as in this example, 2,4,6,8,10,12,14, where the numbers are 7, as you may see: therefore set downe 7 in a place alone, then adde together the first number and the last, as in this example: Adde 2 to 14, and that maketh 16, take halfe of it, and multiply by the 7 which you noted for the number of the same places, and the summe that amounteth, is the summe of all those figures added together: As in this example, 8 multiplied by 7, maketh 56, and that is the summe of all those figures.

To know the totall summe of an Arithmeticall Progressiō

Scholar. That will I worke by another example. I would know how much this summe is, 5,8,11,14,17,20,23,26,29. I tell the places and there are 9, that I note. Then I put the first number 5, and the last 29 together, and they make 34, I take the halfe of it, that is 17, and multiply by 9, and it maketh 153. That you say is the summe of all the numbers.

Master. So shall you finde it if you try it.

Scholar. How shal I try it?

Master. By your common Addition, for it

if you adde all the parcels together, you ſhall ſée the ſame ſumme amount, if you did worke well. And that manner of Addition tryeth all kinds of ſumming any Progreſſion.

Scholar. Then can I ſumme any Progreſſion, if the number of the parcels be odde. But what if they be euen: as in this example, 1,2,3,4,5,6,7,8?

Maſter. When the number of the parcels is euen, then note that alſo as you did before, and likewiſe adde the firſt ſumme to the laſt, and by the halfe of the number of the places, do you multiply it: as in our example, the parcels are 4, that I note: then adding the firſt ſumme to the laſt, there amounteth 9, that do I multiply by halfe of the parcels, that is, by 4, and it maketh 36, which is the ſumme of the parcels.

But if you wil take one Rule for theſe both, do thus: Multiply the halfe of the one by the other whole, and the ſumme will amount all one. For ſometime it chanceth that the number of the parcels be odde, ſo their halfe cannot be taken: and that ſometime it chanceth the Addition of the firſt number and the laſt, do bring forth an odde number, ſo that halfe of it cannot be taken: but they will neuer be both odde.

A generall rule.

Scholar. Then I perceiue this, if there be no more belonging to it.

Maſter. As accuſtomably it hath béene taught, this hath béene the chiefe and onely

exerciſe

exercise in Progression used. But that you may perceiue how diuers wayes, and to how great profit so simple a thing (as this Arithmeticall Progression is) may bée considered and used, I will here propound you sixe propositions: of which foure of them were inuented by a friend of mine, and neuer before thus published: and the two first were neuer to my knowledge written of but by thrée Men.

Schollar. This doth greatly encourage me to be attentiue unto your words, séeing I shall not onely be instructed at your hands in the common knowne Rules of this excellent Art, but besides that, so abundantly in other new rules enformed, as my very entrance shall séeme to passe a great many mens further study, and longer continuance. Therefore Sir, I beséech you, let me know your sixe Propositions.

Master. These they are.

To know the last number without proceeding by continuall Addition, till you come unto it, so that the common excesse, the first number, and the number of the places be knowne.

2 *The first number of the Progression and the last being knowne, with the common excesse to finde the number of the places.*

3 *The excesse being giuen, and the first or last, to know the quantity of any middle number, whose place is giuen from the first or last.*

4 *The totall summe being giuen, and the first and last, to finde out the number of the places.*

The

The totall ſumme of any Arithmeticall Progreſſion being giuen, and the firſt and laſt, to finde out the common exceſſe. 5

The totall ſumme being giuen, and the mutuall exceſſe, with the number of the places, to giue the firſt or laſt number of the ſame Progreſſion. 6

Many moe conſiderations could I propound you in theſe *Arithmeticall Progreſſions*, but theſe are ſufficient, to giue you occaſion to thinke, that Rules of knowledge and *Arts* are infinitely capable of enlargement.

*S*cholar. Happy were I, if I did but well vnderſtand that which is already inuented and written: And yet in my ſimple fantaſie, theſe things offer themſelues (in manner) to be ſtudied for about Progreſſion, therefore I pray you to procéed to the Rules anſwering to theſe propoſitions.

Maſter. I will orderly for euery of theſe ſixe propoſitions giue you Rules, and with euery one an example, vnleſſe the plainneſſe and eaſineſſe néed no farther exemplifying.

For the Solution of the firſt multiply the exceſſe by a number leſſe by 1 then the number of the places and the off-come adde to the firſt number, ſo you ſhall haue the laſt number, which is ſought for. 1. Propoſition.

As for example. If there were ſeuen places in a Progreſſion Arithmeticall, whoſe continuall encreaſe or mutuall exceſſe were 4, and the firſt number were 5, and I would know what the laſt and ſeuenth number is: I multiply 6, which is one leſſe then 7, (the

number

number of the places) by 4, thereof commeth 24, which I adde to 5, that maketh 29, and that is the last number which I desire to know. And this you may straightway proue by continuall proceeding from 5, till the seuenth place, encreasing euery one by 4, as thus.

5 9 13 17 21 25 29.

Lo here, the last, being also the seuenth, is 29.

Scholar. I perceiue already one good property in this Rule, which in all works is to be desired: that is, it will ease one from great labour, if a Progression were propounded of a hundred or two hundred places, or moe: And also it is very easie to worke, and most necessary for the totall summe finding, in a very long Progression.

Master. It is true, and therefore now let me see if you can answer me this question by this proposition.

A Merchant buyeth 50 pounds of Spices, & agreeth to pay for the first pound 4 pence, for the second 7 pence, for the third 10 pence, for the fourth 13 pence, &c.

The question is, how much he must pay for the last pound, and then how much the 50 pound commeth to?

Scholar. According to the proposition, I multiply 49 (which is lesse by one then the number of the places) by the excesse, which is

3,

3, to the product 147, I adde the firſt number which is 4 it maketh 151 pence, the price of the laſt pound. Now I adde 4, the price of the firſt pound, to 151 the price of the laſt pound, it maketh 155, which I multiply by halfe the number of the places, which is 25 the product 3875 pence is the totall ſumme or price of the 50 pounds of Spices, as appeareth.

49	Places 1 leſſe.	151	laſt
3	exceſſe	4	firſt
147		155	
4	firſt	25	halfe places
151	the laſt	775	
		310	
		3875	totall ſumme which amounteth to
		li ſ d	
		16——2——11	

Maſter. It is truly wrought.

Scholar. Then I intreat you to proceed to your ſecond propoſition.

Maſter. *The ſecond Rule is this. From the laſt ſubtract the firſt, the remainder diuide by the common exceſſe, to the* Quotient *adde* 1, *and you haue the number of the places, which you would know: As in this Progreſſion.* 2. Propoſition.

6 11 16 21 26 31

If I know onely 6 and 31, and that they encreaſe

encrease by 5, then according to the rule, from 31 I subtract 6, there remaineth 25: which 25 I diuide by 5, (the common excesse) the Quotient commeth forth 5, to which I adde 1 that maketh 6: and so many are the places, as you see.

Scholar. This Rule is so easie, that I were much to blame, if I could not remember it.

3 Proposition. Master. *The third Proposition may alwayes thus be solued. Multiply the* excesse *by a number lesse by one, then the distance of the place is from the* first, *or the* last number giuen: *the of-come adde to the* first, *if the* distance *be reckoned from the* first, and *the* first *also knowne, or subtract from the* last, *if the distance be from the* last counted, *and the* last giuen *also, and that which commeth forth, either in that* Addition to the first, *or* subtraction from the last *is the number* sought. As for example, *I propound you this* Progression

8 15 22 29 36 43 50 57

And for the apt considering the manner of this question, I will note ouer euery place his distance from the first, and vnder euery place his distance inclusiuely from the last, thus.

1	2	3	4	5	6	7	8
8	15	22	29	36	43	50	57
8	7	6	5	4	3	2	1

Now if the excesse whereby this Progression

sion standeth be knowne to be 7, and the first number given, being 8, and I would know what number standeth under 4, that is to say, in the fourth place. I multiply 7 by 3, (which is lesse by 1 then the number of the place proponed) that yeeldeth 21, to which I adde 8, the first number so commeth 29: which I say to belong to the fourth place, as ye see in the example it also doth: or if in the third place from the last, you would know what number in this example should stand, the last number being knowne to be 57, and the common excesse 7, then by 2 which is lesse by 1 than the place propounded, I multiply 7, that giueth 14, which appertaineth to the third place inclusiuely reckoned from the last, and so my example giueth you.

Scholar. I perceiue right good vse of this rule: for if I had forgotten what the first number were, and remember still but the last, the common excesse, & the number of the places, then might I come by the knowledge of my first number againe.

And me thinketh, that it differeth not much from the first proposition, sauing that which you make here a middle number, there was made the last: and also in this point it differeth, that in it the last was onely sought, and no consideration had in numbring the places from the last, as here I marke in your numbers noted vnder your Progression.

Master. And thinke you not, the middle number

numbers of progression standing of a hundred or three hundred places or moe, may as much cumber a man to come to the knowledge of them by continuall encreasing from the first by the common excesse, or abating from the last continually the common excesse as the very small numbers in a shorter progression would doe?

Scholar. Yes sir, that I think right well, & therefore I am glad of this new framed proposition, and the manner of the working of it.

4 Proposition. Master. *The rule of the fourth is this. Adde the first and the last together, and by the of-come diuide the totall summe. Double the quotient, and that will be the number of the places.*

Scholar. Then if in a Progression, whose summe were 207, and the first number 12, & the last 57, if I adde 57 and 12 together, that maketh *69*, and by it I diuide 207, the quotient will be 3, which I double: and so I haue *6*, and so many must be the number of the places that this Progression standeth on.

Master. Whether it be so or no, how wil you trie?

Scholar. Halfe *6*, which is 3, being multiplied by *69*, must make 207, the totall summe: if *6* be the number of the places. For so the whole worke of your rule in summing any Arithmeticall Progression did enforme me. I will then multiply *69* by 3, thus.

	69
	3
It commeth forth iustly.	207

Master.

Master. I must much herein commend your promptnesse both in memory and in well applying your rule: although in manifest words it did containe no such matter.

Scholar. Sir I pray you heare me frame one example more.

Master. I am well pleased, so that ye bée short, for you make mée more longer here then willingly I would haue béen: but I cannot perceiue how I could haue omitted any thing as yet, without your great lack thereof.

Scholar. If I had receiued 85 pounds of certaine men, but of how many I haue forgotten, yet I remember that the first gaue me 7 pound, and the last 27 pound, and euery payment after other did rise by a like sum. And the man for whom I receiued this mony, conditioned with me, that of euery payment I should haue twelue pence for my labour: now vnlesse I can by Art finde the truth of this case, I am like to lose the most part of my reward.

A question of money.

Master. I perceiue you can handsomely frame an example which should concern your owne gaine: I pray you let me sée how you would do iustice in this point.

Scholar. I adde the first and the last together, that maketh 34: by which I diuide 85, thus:

	1	
	27	17
	85	(2—
	34	34

Why how now? Sir, here is a remnant of 17, in which 34 cannot bée

 had:

had: so that now I am in the briers for doubling of my Quotient, and farewell then both my Iustice, and a good lumpe of my gaines.

Master. Ye are neuer the farther from the matter, though it fall into a Fraction. For you shall vnderstand, that the Fraction which of any such worke proceedeth, is euer halfe of one such, as the vnites of the Quotient before are. And that you may trie, if you double that which so remaineth, for then it will be equall to your Diuisor, as if ye double 17 (the remnant) it maketh 34, & your diuisor also was 34 this noteth the remainder to be halfe of one.

Scholar. Now I am glad of this hard example. For with it I haue a generall rule for the Fraction that may hap in this worke So that the Quotient being two and a halfe, I double that, it maketh 5, therefore should my gaine be 5 shillings. And to be sure (by your leaue) I will trie it, for I will multiply halfe of 34 (which is the first and last number ioyned together) by 5, thus

	17
	5
It is most true (I see) that I shall lose nothing by the former working.	85

Master. The fifth proportion hath this rule appertaining vnto it: By the fourth rule finde the number of the places, that being done, from the last [illegible] the first. & the residue diuide by a number [illegible] the number of the places, and the [illegible] excesse which is sought for.

5 Proportion.

An

An example hereof ſhall be this : If ye had disburſed 685 pounds to a certaine number of men, you neither can tell how many they were, or how much the ones mony exceeded his next before, but you are ſure that the exceſſe was equall between euery two next: and alſo you remember that the firſt had 19, and the laſt 118 pounds, how would you finde the number of the name and the exceſſe, continually obſerued in the ſucceſsion of their payments. Example.

Scholar. Your rule doth plainly bid, firſt to finde the number of the places, which I will do according to the fourth rule : I adde 19, and 118 together thus.

```
118
 19
---
137
```

By this 137, I diuide 685 thus.

```
 13
 685 (5
 137
```

Seeing there is no fraction but a whole number, being 5, I double that, and then muſt the number of the places be 10. Now from the laſt I ſubtract the firſt, as 19 from 118, thus: And ſo remaineth 99.

```
118
 19
---
 99
```

This 99 I diuide by a number leſſe by one then the number of the places, and ſeeing the places were 10, I diuide 99 by 9, thus:

```
99
99 (11
```

The Quotient is I, and ſo was the exceſſe, if I haue followed

your rule right.

Master. You haue wrought euery part of this question both well in order, and truly in the practise of your rules.

Scholar. I will then set it downe also formably, so that the number of the places, the excesse and the totall summe may straight appeare, as your first example stood.

The common excesse. The progression.

11	11	11	11	11	11	11	11	11	11
19	30	41	52	63	74	85	96	107	118

That the places be 10, and that from the first to the last, the common excesse is 11, I perceiue most euidently: but whether the totall summe be 685. I haue not yet proued, which I will now doe: I adde 19 and 118 together, that maketh 137: I multiply that by halfe the number of the places, thus.

137
5
685

All things agrée most exactly, so that I am perfect enough in these rules, if I forget them not againe.

Master. Use maketh all things perfect.

6 Proposition.

Your sixth rule is this. By the number of the places diuide the totall summe, double the quotient, and that will bee the first and last ioyned in one summe. Then by a number lesse by 1, then the number of the places, multiply the excesse, that of-come subtract from the first doubled,

doubled quotient, and the halfe of the reſidue is the firſt number. The laſt number you may diuerſly finde out, as by the firſt of our ſix rules, or by ſubtracting this firſt number from the ſumme which here contained both the firſt and laſt ioyntly, (or thirdly) by continuall adding the exceſſe.

Scholar. I pray you make this ſomewhat more plaine with an example.

Maſter. If euery moneth in the yeere (counting them now as 13) you gained clearly 40 ſhillings more then you did the moneth next going before, and at the yeeres end you finde the whole gaine 5720 ſhillings, but ye remember not how much either the gaine of the firſt moneth or the laſt was, by this rule it may be tried out. Example of gaine.

Scholar. So that here ye ſeeme to apply the 13 moneths to thirteene places, the 40 ſhillings euery one more then the other next before it, to be the common exceſſe, and 5720 ſhillings to the totall ſumme.

Maſter. It is true: by 13 then I diuide 5720 in this manner.

$$\begin{array}{l} \quad\cancel{1} \\ \cancel{1}\cancel{5} \\ \cancel{5}\cancel{7}\cancel{2}\cancel{0}\,(440 \\ \cancel{1}\cancel{3}\cancel{3}\cancel{3} \\ \quad\cancel{1}\cancel{1} \end{array}$$

I double this quotient, ſo haue I 880 for the firſt and the laſt ſumme ioyned together; by 12 which is leſſe by one then the num-

ber of the places; I multiply 40 (the common exceſſe) ſo commeth 480	40
	12
This 480 I ſubtract from 880, ſo	80
remaineth 400: halfe whereof is the	
firſt number which wee deſired to	4
know, that is 200.	480

And as for the laſt number, I can giue you it thrée wayes. As by the firſt of my ſix rules I multiply, the exceſſe by a number leſſe by 1 then the number of the places, as 40 by 12 that giueth 480, which I adde to the firſt, being 200, ſo ſhall the laſt be 680.

The ſame ſumme commeth forth, if ye ſubtract 200 from 880.

And thirdly, if I begin at 200, and ſo procéed, encreaſing by 40, I ſhall at the thirteenth place haue 680, as thus:

200	240	280	320	360	400	440
480	520	560	600		640	680

Scholar. I thanke you moſt heartily for theſe ſixe rules. Now if it be your pleaſure I would heare and learne ſomewhat of Progreſsion Geometricall.

Maſter. There are yet very many rules and propoſitions, which fall into this Arithmeticall Progreſsion.

And for the vſe and practiſe of them, I will propone vnto you certaine pleaſant and neceſſary queſtions of Arithmeticall Progreſsion, and to the performance of their workings, ſuch

ſuch neceſſary rules and documents, as are requiſite for the better vnderſtanding of them, or any ſuch like.

A certaine Mercer ſold 20 yards of Veluet to be paid in 12 weekes, by Arithmeticall proportion: that is to wit, to receiue the firſt weeke 6 ſhillings, the ſecond weeke 12 ſhillings, the third weeke 18 and ſo forth, encreaſing the number of weeks by 6 ſhillings, till the twelfth and laſt weeke were expired. The queſtion is how many pounds hee had for 20 yards of veluet.

A queſtion of Veluet.

To the performance of this queſtion, and ſuch other the like, I ſet forth the 12 payments in ſuch ſort, as for exāple, heere appeareth.

6
12
18
24
30
36
42
48
54
60
66
72

Then touching the adding together of theſe ſummes, without the aid of Addition, according to the rules I taught you in Progreſsion Arithmeticall, I note the number of the places, which are 12, then adding the laſt number of the Progreſsion, which is 72, and the firſt number together, make 78; and multiplying 78 by halfe the number of the places, which is 6, amounteth to 468 ſhillings, and in pounds maketh 23 pounds 8 ſhillings. And ſo much hath the Mercer for his 20 yards of Veluet, which is nigh about 23 ſhillings, 5 pence, a yard.

Scholar. I vnderſtand this worke very well, but is there any proofe for the iuſtifying hereof,

hereof, as you haue of other workes?

Master. The workes of it selfe (being so perfectly wrought) that in your proceeding and going forward from number to number, each number exceeding his fellow by an equall or like quantity, is all that is demanded for iustifying of the same: yet notwithstanding, because your request is reasonable, I will propone an example for the proofe hereof.

The proofe of the last question.

A certaine man is bound to pay for 20 yards of velvet, the summe of 23 pound 8 shillings, and it is to be paid weekely, in 12 weekes or termes by Arithmeticall Progression. The question is therefore to know with what number the same Progression is to be begun and continued in such equall proportion Arithmeticall, that in 12 weekes the same may iustly be accomplished.

For the resolution whereof, and of all such other like, reduce 23 pound 8 shillings, all into shillings, which maketh 468 shillings.

A generall rule.

Then adde 1 vnto 12, the number of the termes, it maketh 13, which 13 you shall multiply by halfe the number of the termes, which is 6 it maketh 78; then diuide 468 by 78, and you shall finde 6 in the quotient, which is the true number that shall begin and continue the said Progression. That is to say, the first weeke 6 shillings, the second 12 shillings, and the third week 6 shillings more, which is 18 shillings, and so euery weeke as they rise, 6

shillings

ſhillings more then the weeke before, as is manifeſt in the queſtion aforeſaid.

A Farme is to be ſold to be payed by the weekes in a yeare, the firſt weeke to pay 4 ſhillings, the ſecond weeke 8 ſhillings, the third weeke 12 ſhillings and ſo forth, increaſing each number by 4, till the number of 52 (which are the number of weeks in a yeare be expired.) The queſtion is, what the price of the Farme commeth to? A queſtion of a Farme

Scholar. I doubt not, but by that you haue already taught me, to end this queſtion very well; wherefore I ſet forth the Progreſſion with his exceſſe 52 times.

Maſter. Nay ſtay a while: And here for your further eaſe (to abridge you of great labour that appeareth to fall out in this queſtion, and ſo may do in any other the like) if a queſtion were proponed of 100 or 200 places or moe, and that this queſtion, nor any other the like can be ended, vnleſſe you know abſolutely what the laſt number of the Progreſſion at the 52 place is, or ought to bée) I will giue you a generall rule how to know the laſt number of any Progreſſion Arithmeticall, as well as if you had ordinarily procéeded by continuall Addition, till you had come to the laſt worke, which is this.

Multiply the exceſſe by a number leſſe by one then the number of the places, and thereto put the firſt number of the Progreſſion, and you ſhall haue your deſire. A generall rule.

Scholar.

Scholar. This rule is well worth the noting: for if I understand you aright, I consider that my excesse is 4, which I multiply by 51, which is one lesse then the number of the places, and it maketh 204, whereunto I adde the first number of the Progression, which is 4, and then it is 208, which you say is, or should be the last number of the Progression.

Master. This is a most approued truth, if there were neuer so many places.

Scholar. This rule is so easie, that I were much to blame if I do not remember it. For by the benefit hereof, I haue such an ease and light into this excellent Art, that my first entrance doth seeme to passe a great many mens further study, and longer continuance.

Master. Many moe considerations could I propound you in these *Arithmeticall Progressions*; but these are sufficient for a taste, to giue you occasion to thinke that *Rules* of knowledge and *Arts*, are infinite capable of enlargement.

Scholar. Happy were I, if I did but well understand that which is already inuented and written. But these things, in my simple fantasie, offer themselues to be greatly beneficiall unto the aids of Progression. Therefore now I will go forward with your question.

Now considering that the 52 and last place is 208, I adde thereunto the first number

number of the Progreſsion, which is 4, it maketh 212, which I multiply by halfe the number of the places, which is 26, and it amounteth to 5512 ſhillings. And ſo much is the totall ſumme or addition of this Progreſſion: which maketh 275 pounds, 12 ſhillings, as appeareth here by my Tables.

Maſter. I like well your labour, and commend you for your diligence; I will here propone one example more, and therewithall for this time will end Progreſsion Arithmeticall.

A certaine man bought 20 Ells of Holland, to be paid in 17 weekes or termes by Progreſſion Arithmetical. And the firſt weeke to pay 1 ſhilling 8 pence, the ſecond weeke 3 ſhillings, 4 pence, the third weeke 5 ſhillings, the fourth weeke 6 ſhillings 8 pence, and ſo forth, each weeke ſucceeding 20 pence more then the weeke before. The queſtion is, what the ſumme of his 20 Elles commeth to? A queſtion of Holland

Scholar. Becauſe here is mention made both of ſhillings and pence, I feare there is ſome harder matter contained herein, then in the other before: therefore I pray you worke it your ſelfe, and I will diligently marke your labour.

Maſter. There is no more to be done in this, then in the other before; but becauſe your requeſt is ſo reaſonable, be attentiue vnto me.

Firſt by the generall Rules I ſeeke to finde out the laſt number of the 17 place, what

this

this Progreſſion ought to bée. Therefore here in my Tables multiplying the exceſſe 20 by 16, which is one leſſe then the number of the termes or places, and it commeth to be 320; and thereunto adding the firſt number of the Progreſſion, which is 20 pence, all is 340 pence, or 28 ſhillings 4 pence: for ſo much ought the laſt number of the payments to be.

Then finally, to know what the whole 17 places amount unto, I adde the firſt number of the Progreſſion and the laſt together, which make 360. Now becauſe 17 is an odde number, whoſe halfe cannot be taken, I take the halfe of 360, which is 180, and multiplying 180 by 17, commeth to 3060 pence, which maketh as you ſée by Diuiſion 12 pound, 15 ſhillings. And ſo much is the buyer to pay for his 20 Elles of Holland. Which 3060 pence if you diuide by 20, the number of Elles that was bought, you ſhall finde 12 ſhillings 9 pence, and ſo much payed he for an Elle one with another.

The proofe.

A queſtion of debt.

A certaine man doth owe 12 pound, 15 ſhil. to bee payed in 27 weekes or termes Arithmeticall Progreſſion. The queſtion is, to know with what number he ſhall begin and continue the Progreſſion in ſuch equall proportion, as the ſame may be truly payed

paid and satisfied in 17 *weeks.*

The answer.

First I reduce 12 pounds 15 shillings, all into pence, which as you see here in many Tables, make 3060 pence, that I let stand by a while.

Then I adde 1 to 17, the number of the places or terms, which maketh 18, which I should multiply by halfe the number of the weeks or termes, which is 8 $\frac{1}{2}$ which 8 $\frac{1}{2}$ multiplied by 18 cannot well be done, unlesse you were acquainted with Fractions or broken numbers, therefore you shall let that passe and multiply 17. by the halfe of 18: which is 9, (for that is all one with the Multiplication of 8 $\frac{1}{2}$: and the Multiplication of 9 into 17 maketh as you see 153, with which number you shall divide the 3060 pence beforesaid, and the Quotient bringeth forth 20 pence, which is the first number or payment to begin the Progression withall: and so each week succeeding to rise 20 pence more then the weeke before, and thereby in 17 weekes shall 12 pound 15 shillings be paid: as before was sufficiently declared. Thus much for Progression Arithmeticall.

*S*cholar. Certainly Sir, I know not how to render you condigne thanks for these benefits shewed me, which me thinketh are so easie, delightfull, and pleasant, that I count my selfe happy to be in your company.

Master.

Master. I am glad you delight so well herein, which is an Art of wonderfull dexterity to all sorts of men of what degrée or profession soever they be. And now will I procéed to Progression Geometricall wherein I will be more briefe, both because I haue béene so long in this part of Arithmeticall Progression, and also for that it would require the knowledge of Roots and surd numbers, (whereof ye haue learned nothing) if I should frame the like propositions in them as I haue done in these. Therefore I will onely teach you two practises about it, and so end the considerations and works of these Progressions.

Progression Geometricall.

Progression Geometricall is when the numbers increase by a like proportion, that is, if the second number containe the first, 2, 3, or 4 times, and so forth: then the third containeth the second so many times also: and so the fourth the third, and the fift the fourth, wherefore I set these 3 examples

3	6	12	24	48
1	3	9	27	81
2	10	50		250

Here in the first example you see, that euery number containeth the other (that goeth next before him) two times: and in the second example three times, and in the third example fiue times. Now if you will know how to finde easily the summe of any such number, do thus: Consider by what numbers, they be multiplied,

multiplied, whether by 2, 3, 4, 5, or any other, and by the ſame number multiply the laſt ſumme in the Progreſſion. To finde the totall ſumme in any Geometricall Progreſſion.

*S*cholar. I pray you worke it by this example, 2, 8, 32, 128, 512, 2048, which I haue framed by proceeding from 2, and continuall multiply by 4.

Maſter. Then muſt I multiply the laſt ſumme (which is 2048) by 4 alſo, and it will be 8192. Now muſt I abate from this ſum the firſt number of the Progreſſion, which here is 2, then reſteth 8190: which ſumme I muſt diuide by 1 leſſe then was the number that I multiplyed by. Seeing then I multiplyed by 4, I muſt diuide by 3, ſo diuiding 8190 by 3, the Quotient will be 2730, which is the ſumme of all the Progreſſion. And now to proue whether you can do the ſame, I giue you theſe numbers to adde by this rule 3, 15, 75, 375, 1875, 9375, 46875.

*S*cholar. I cannot well tell by what number this Progreſſion doth increaſe.

Maſter. In any ſuch doubt do thus: Diuide the ſecond number by the firſt, and the quotient will ſhew you the number that engendreth the Progreſſion.

Scholar. Then is that number in this example 5, for ſo many times is 3 in 15.

Maſter. So is it. Now worke as I taught.

Scholar. The laſt number is 46875, which I multiply by 5, and it yeeldeth 323475, from

which I abate the last number of the Progression, that is 3, and there resteth 2 3 4 3 7 2, which I diuide by 4, for that is one lesse then 5, and the Quotient is 5 8 5 9 3, which is the whole summe of the Progression.

Master. If you remember well this, you haue learned the Art of Progression both Arithmeticall, and also Geometricall, which you may proue either by subtracting of each number alone from the summe, and so will there nothing remaine: or else by adding together of all the parcels, for so will the same summe amount.

A question of Satten.

A Mercer hath 12 yards of Satten, which he valueth at 16 shillings the yard, and selleth the same 12 yards to another man to be paid as followeth: That is to wit, for the first yard to haue one shilling, for the second yard two shillings, for the third yard foure shillings, for the fourth yard 8 shillings, &c. doubling each number following, till the twelfth and last yard. The question is, who hath made the better bargaine of the buyer or the seller.

First you may set downe 12, the number of the yards as you see here in this Example. And against each number the number of shillings due to be paid as the order of Duplation or Multiplication by two teacheth.

Then resorting to the adding up or summing of this progressiō, where I consider that the increase of this sum proceeded by the Multiplication of 2, & therefore after I haue drawne a

8	
1	1
2	2
4	3
8	4
16	5
32	6
64	7
128	8
256	9
512	10
1024	11
2048	12
4095	

a line vnder the 12, I worke and multiply the last summe by 2 also, & it yéeldeth 4096: from whence I abate the first number of the Progression, which is 1, and then resteth 4095: which I should diuide by one lesse then I did multiply by, but séeing it is 1, I néed not to diuide it: for 1 (as I haue said before) doth neither multiply nor diuide, therefore I take that summe 4095 for the whole summe of the shillings, which by Reduction amounteth to 204 pounds 15 shillings, and so much hath the Mercer for his twelue yards of Satten: which is 17 pound, 1 shilling, 3 pence a yard. But I thinke you will buy none so deare.

Scholar. No Sir, by the grace of God this yeare.

A question of an horse

Master. *Then what say you to this question? If I sold vnto you an horse hauing 4 shoes, and in euery shoe 6 nayles, with this condition, that you shall pay for the first nayle one* ob *: for the second nayle two* ob *: for the third nayle foure* ob *: and so forth, doubling untill the end of all the nayles.* Now I aske you how much would the price of the horse come vnto?

Scholar. First to know the number of the nayles, I must multiply 6 by 4, and it maketh 24. Then will I do thus: I will write the number of the nayles euery one in order from 1 to 24, and against each number of the nayles the summe of halfe pence duly, as the order of Duplation or Multiplication by 2 teacheth, and as in the next figure following appeareth.

Then do I resort to the Rule of summing by the Progression, where I consider that the increase of this summe proceedeth by the Multiplication of 2, as the last example did. And therefore multiplying the last summe by 2 also, and it yeeldeth 16777216, from which I abate the first number which is 1, and then resteth 16777215, which I should diuide by one lesse then I

also

Lord John Clanbrasill

Lord Joh: Clanbrasill

Lord John Clanbrasill

1 | 1 did multiply: but seeing
2 | 2 that it is 1, I need not to
4 | 3 divide it, for 1 (as you
8 | 4 have before said) doth nei-
16 | 5 ther multiply nor divide,
32 | 6 therefore I doe take the
64 | 7 number 16777215 for
128 | 8 the whole summe of the
256 | 9 half pence, which by Redu-
512 | 10 ction I finde to be 699050
1024 | 11 shillings, and 7 pence, halfe
2048 | 12 penie: that is 34952
4096 | 13 pounds, 10 shillings, 7
8192 | 14 pence, ob.
16384 | 15 Master. That is well done,
32768 | 16 but I think you wil buy no
65536 | 17 horse of the price.
131072 | 18 Schol. No sir, if I be wise.
262144 | 19 Master. Well then an-
524288 | 20 swer mee to this questi-
1048576 | 21 on.
2097152 | 22 *A Lord delivered to a*
4194304 | 23 *bricklayer a certaine number*
8388608 | 24 *of loads of Bricke, whereof he*
16777216 *willed him to make twelve walles, of such sort, that the first wall should receive two thirdels of the whole number, and the second two thirdels of that which was left; and so every other, two thirdels of that that remained: and so did the Brick-layer: and when the 12 walls were made, there remaineth one load of Bricke.*

A question of Bricks.

N 3 Now

Now I aske you, how many load went to each wall, and how many load was in the whole?

Scholar. Why Sir, it is impossible for mee to tell.

Master. Nay, it is very easie, if you marke it well. Marke well that I said, that every wall should receive two thirdels of the summe that was left. Now take away two thirdels from any summe, and you must néeds grant that that which remaineth, is one thirdell of the summe last before: Example of 9, from which if you take two thirdels, there will remaine thrée, which is one thirdle of 9. Likewise from thrée bate two thirdels, and there will remaine 1.

Scholar. This is true, and now I perceive the least wall had but two load of brick.

Master. And by the same reason may you know how many load every wall had, according as this figure following doth shew, and likewise what the whole summe of brickes was, for if you make 12 summes, multiplying by 3, still from the last remainer, as you may sée here on the left side of the Table, there will appeare all the remainers of the whole wall: and if you multiply the last of those 12 summes by 2 also, then will that be the summe of the loads which was delivered to the Bricklayer.

The

	1	12	2	
The remainer after every wall.	3	11	6	*Loads due to each wall.*
	9	10	18	
	27	9	54	
	81	8	162	
	243	7	486	
	729	6	1458	
	2187	5	4374	
	6561	4	13122	
	19683	3	39366	
	59049	2	118098	
	177147	1	354294	
Summe of the loads		531440		*delivered.*

Againe, if you double every Remainer, as you may sée at the right side of this Table, those numbers will shew the summe of loads that went to each wall, wherby you may perceive that each wall was thrée times so great as the next lesser.

Scholar. Lo now it appeareth easie enough. Now surely I sée that Arithmeticke is a right excellent Art.

Master. You will say so when you know more of the use of it: For this is nothing in comparison to other points that may bée wrought by it.

Scholar. Then I beséech you cease not to instruct mée further in this wonderfull cunning.

The Golden Rule, or Rule of Proportion direct, called the Rule of Three.

Maſter.

The Rule of propor-tion.

BY order of the Science (as Men haue taught it) there ſhould follow next the extraction of Roots of number, which becauſe it is ſomewhat hard for you yet, I will let it paſſe for a while, and will teach you the feate of the Rule of Proportion, which for his excellency is called the Golden Rule.

The Gol-den Rule.

Whoſe vſe is, by three numbers knowne to finde out any other vnknowne, which you deſire to know, as thus.

Queſtion of boar-dings.

If you pay for your board for three moneths ſixteene ſhillings, how much ſhall you pay for eight moneths?

To know this and all ſuch like queſtions, you ſhall conſider which two of your numbers bee of one denomination, and ſet thoſe two the one ouer the other, ſo that the vndermoſt be it that the queſtion is of: as in my queſtion 3 and 8, be both of one denomination, for they both be moneths; and becauſe 8 is the number that the queſtion is asked of, I ſet the one ouer the other, and 8 vndermoſt thus, with ſuch a crooked draught of lines. Then doe I ſet

3
8

the

the other number which is 16, againſt 3 at the right ſide of the line, thus.

3 —— 16
8 ——

And now to know my queſtion, this muſt I do: I muſt multiply the lowermoſt on the left ſide, by that on the right ſide, & the ſumme that amounteth, I muſt diuide by the higheſt on the left ſide: or in plainer words, thus, I ſhall multiply the number of which the queſtion is asked (which is called the third number) by the number of another denomination (which is called the ſecond, & the ſumme that amounteth, muſt I diuide by the ſumme of like denomination (which is called the firſt) Then for the knowledge of this queſtion, I multiply 8 into 16, and there amounteth 128, which I diuide by 3, & it yéeldeth 42 ſhillings, and 2 ſhillings remaineth, which I turne into pence, and they be 24 pence, of which third part is 8 pence, ſo the third part of 128 ſhillings, is 42 ſhillings, 8 pence, which ſum I write at the right hand of the figure againſt 8 thus. Note.

The third number. *The ſecond number.* *The firſt number.*

3 —— 16 ſhillings,
8 —— 42 ſhil. 8 pence.

Hereby I know that if three months boarding, do come to 16 ſhillings, that 8 moneths boarding will come to 42 ſhillings, 8 pence, & likewiſe of any other like queſtion.

But here muſt you marke, that the firſt number and the third be of one denomination and alſo the ſecond and the fourth, for which

you

you ſéeke: or elſe be of ſuch denominations, that you in working may bring them into one; As if a man ſhould aske me this queſtion.

Queſtion of expences.

Twelue weekes iourneying coſt me 14 *French crowns at* 6 *ſhillings the peece, how many pounds is that in one yeare?* Here you ſée no two numbers of one denomination, but yet in working you may turne them into like denomination: as thus, turne the one yeare into 52 weekes, and the fourth ſumme will bée French Crowns, by the order of the working. Then to know this queſtion, multiply the third ſumme 52, by the ſecond 14 and the ſumme will be 728: that diuide by your firſt number, 12, and the Quotient will bée 60. Crownes, and 8 Crownes remaining: which if you turne into ſhillings, they will be 48 ſhillings, which if you diuide by your firſt number 12, the quotient will be 4, which ſignifieth 4 ſhillings: put thoſe 60 French Crownes, which make 18 pounds with the 4 ſhillings, for the ſum that anſwereth to the queſtion, and it is the iuſt expences of a yeare: And the worke will be thus.

12	14
52	60 4 ſ.

A generall rule.

And take this euermore for a generall rule touching this whole Art, that the doubtfull or vnknowne number that you would be reſolued of, ſhall alwaies be ſet in the third place. Note alſo the firſt number & the third, muſt euer be of one nature & denomination, or elſe muſt in working be brought

to

to like denomination, and then of necessity must the other number be in the second place.

Remember also that the place of the first number is highest on the left side, and the place of the second, right against it on the right side; the place of the third number is vnder the first, as by those examples you haue seene.

Scholar. This I trust I can do.

Master. But and if the question be asked thus: *In* 8 *weeks I spend* 40 *shillings, how long will* 105 *shillings serue me?* Here you see that 8 weeks answers himselfe, and saith 40 shillings. But how long time 105 shillings will serue you know not. Therefore you shall set 105 in the third place, according as I told you euen now. And the first place must alwayes be of the same nature or Denomination that the third is of, which here is 40. Then must 8 needs be that other: Now multiply 105 by 1, and it will be 840, which if you diuide by 40, it will yeeld 21, which is the fourth number, and sheweth how many weekes 105 shillings will serue, if you spend 40 shillings in 8 weekes.

The figure of this question is this: as if you should say: if 40 shillings serue for 8 weekes: 105 will serue for 21 weekes

Shillings.	*Weeks.*
40	8
105	21

Other diuersities there be of working by this rule, but I had rather that you would learne this one well, then at the beginning

to

to trouble your minde with many formes of working, sith this way can do as much as all the other, and hereafter you shall learne the other, more conueniently.

Note. ¶ And for your further aid and instruction, to make you better acquainted with this Golden Rule, I haue here proponed sixe questions, and their answers, which I thinke most conuenient and meet to preferre the desirous to perfect vnderstanding. The first foure are all branches of one question sprung out of the best tree (for a yong learner to taste of) that groweth in this *Ground of Arts*: for that no manner of question in the Rule of Three whatsoeuer it be, can be proponed, but it must be comprehended vnder the reason or stile of one of these foure.

The Questions.

If 15 *elles of cloth cost* 7 pound 10 shillings, *what comes* 27 *elles to at that rate?* Answer, 13 pound 10 shillings.

If 27 *elles cost* 13 pound 10 shillings, *what are* 15 *ells worth?* Answer 7 pound 10 shillings

If 27 *ells cost* 13 pound 10 shillings: *how many ells shall I haue for* 7 pound 10 shillings? Answer: 15 *Ells*.

If I sell 15 *ells for* 7 pound 10 shillings: *how many ells are to be deliuered for* 13 pound 10 shillings? Answer, 27 *ells*.

If 8 *pound of any thing cost* 16 shillings 6 pence,

pence: *what money is to be receiued for* 49 pound? Answer: 5 pound, 1 shilling od $\frac{1}{4}$.

If 4 pound *of any thing cost* 17 pence: *what money will* 8765 *pound of that commodity cost?* Answer: 155 pound, 4 shillings, 3 pence, q.

Of all which questions, I omit the worke of purpose, that you shall whet your wit thereby at conuenient leisure, to clime each branch, and gather the fruit of them, and do minde now, before we make an end of this Rule, to giue you some instructions of the backer rule of three, whose order is quite contrary to this that you haue learned.

Scholar. I thanke you heartily for the sixe Questions, which I will God-willing practise at conuenient times: I pray you proceed therefore to the Backer or Reuerse Rule.

John Fountainbleu

London 1602

The

The Golden Rule, or Rule of Proportion Backward, or reuerse.

Master.

IN the former Rule euermore looke how much the third number is greater then the first, so much the fourth number is greater then the second. And contrariwise: looke how much the first summe is greater then the third (if it do chance so) so much is the second summe greater then the fourth.

Note this well.

The backer or reuerse rule of three.

But in this rule, there is a contrary order, as this: That the greater the third summe is aboue the first, the lesser the fourth summe is beneath the second: and this rule therefore you may call the Backer or Reuerse Rule, as in example.

Question of cloth.

If I haue bought 30 yards of cloth of two yards breadth, and would haue Canuas of three yards broad to line it withall, how many yards should I need?

Scholar. Why, there is none so broad.

Master. I do not care for that, I do put this example onely for your easie understanding: for if I should put the example in other measures, it would be harder to understand. But now to the matter: If you would know this question, set your numbers as you

did

did before: but you shall multiply now the first number by the second, and that ariseth thereof, you shall diuide by the third: which thing if you do here, I meane if you multiply 30 by 2, it will be 60: which summe if you diuide by 3, there will appeare 20: whereby I know, that if 30 yards of cloth of two yards bread, should be lined with canuas of three yards broad, 20 yards of canuas would suffice, as this figure sheweth. Note.

Breadth.	*Length.*
2	30
3	20

And now because ye found fault with my Example, how say you, perceiue you this?

Scholar. Yes Sir, I suppose.

Master. *Then answer me to this question: how many Elles of Canuas of Elle breadth, will serue to line 20 yards of Say, of three quarters broad?*

Scholar. In good faith Sir I cannot tell, for I know not how to bring the summes to like Denominations.

Master. Then will I tell you, sith there is mention here of quarters, and again euery one of the measures both elles and yards may bee parted into quarters, part them so both in the breadth and length, and then put forth the question by quarters.

Scholar. Then I shall say thus. How many quarters of canuas of 5 quarters broad, will line 80 quarters of 3 quarters broad?

Master.

Maſter. Now anſwer to the queſtion.

Scholar. Firſt I will ſet them downe in their forme thus: for 5 is ioyned with the queſtion, & is therefore the third number: then is 3 the number of the ſame Denomination, I meane becauſe they be both referred to breadth.

Breadth. Length.
3 80
5

Now I multiply 80 by 3, and it is 240, which I diuide by 5, and it yéeldeth 48. Then ſay I that 48 quarters of 5 quarters broad, will ſuffice to line 80 quarters of three quarters broad.

Maſter. Turne the quarters againe into ells and yards.

Scholar. Then I ſay, that 9 ells and thrée quarters of a yard of ell-broad, will ſerue to line 20 yards of thrée quarters broad, as this figure ſheweth.

Breadth. Length.
3 80
5 48

¶ Maſter. *Now what ſay you to this queſtion: I lent my friend* 400 pound *for* 7 *moneths, how much money ought he to lend mee againe for* 12 *moneths to recompence my curteſie ſhewed him? can you anſwer to this?*

Scholar. Yes Sir, I ſuppoſe, for I will ſet downe my Numbers thus: where I multiply 7 into 400, and it maketh 2800, which I diuide by 12, and it yéeldeth

Moneths. Pounds.
7 400
12

yéeldeth 233 pound, and there is 4 pound remaining of my Diuision, what shal I do therwith?

Master. Turne the same 4 pound into shillings, & then diuide it by 12 as you did before.

Scholar. Well Sir, it shall be done: so haue I 16 shillings, for my Quotient, and yet remaineth 8 shillings vpon my Diuision.

Master. You must also reduce that 8 shillings into pence, which maketh 96, and diuide that also by your Diuisor.

Scholar. So haue I done, and I finde 8 pence for my quotient, and nothing is left.

Master. This must you alwayes do when any thing remaineth vpon your Diuision, whether it be money, weight, measure, or any kind of thing whatsoeuer. This Rule is so profitable for all estates of men, that for this rule onely (if there were no more but it) all men were bound highly to estéeme Arithmeticke.

By this rule may a Captain in warre, work many things, as Master Digges in his Stratiocos doth declare: Only now in this my simple addition, for a taste & incouragement, I wil inlarge the Author with a question or two more wishing you and euery my Countrimen or Gentlemen whatsoeuer, that by nature be any thing giuen to Military affaires, to be familiar and acquainted with this Excellent Art, the which he shall finde not onely at the Sea, but also in the Campe and Field-seruice, aboundantly to aid him, either in fortification, pay-

ing of Souldiers wages, charges of Ordnance, Powder, Shot, Munitions, and Inſtruments, whatſoeuer, as for example.

Queſtion of an Armie.

If it ſhould chance a Captaine which hath 40000 Souldiers to be incloſed with his Enemy, that he could haue no freſh purueyance of victuals, and that the victuals which hee had would ſerue that Armie but onely three moneths, how many men ſhould he diſmiſſe to make the victuall to ſuffice the reſidue eight moneths?

Scholar. As you taught me, I ſet the numbers thus, ſaying: If thrée moneths ſuffice 40000, to how many will eight ſuffice?

Moneths	*Men.*
3	40000
8	

To know this; I multiply the firſt number 3 into the ſecond 40000, and it yéeldeth 120000, which ſumme I diuide by 8, and there will be in the quotient 15000, which if I doe ſubtract from 40000, the remainer will declare that he muſt diſmiſſe 25000 as this figure ſheweth.

Moneths,	*Men,*
3	40000
8	15000

A queſtion of a Fort.

¶ Maſter. *Now anſwer mee to this queſtion: If 136 Maſons in a moneth bee able to build a Fort to preſerue the Souldiers from the Enemy, and ſuch expedition requireth that I would haue the ſame finiſhed in eight dayes: how*

how many workmen say you is there to be appointed.

Scholar. As you taught mee; I set the numbers thus saying:

If 28 dayes require 136 Masons, what number of men by the like proportion will 8 dayes require?

28 —— 136
8

To know this, I multiply the first number 28 into 136; and it yeeldeth me 3808: which I diuide by 8 and my Quotient is 476: which is the iust number of Masons that shall supply this worke. And now me thinke these questions are very easie.

Master. Truly if you take delectation herein, you shall finde this Art not onely easie, but wonderfull pleasant and profitable. Now therefore one question more I will propose, and so leaue off this Rule in whole numbers, untill we come to the use of it in broken numbers: for had you the understanding of broken numbers perfectly, not onely in this Rule, but in all other, the question that in the sight or appearance seemeth to bee 100 times more harder to absolue, may thereby be wrought as soone, or sooner then this.

Scholar. Your words do greatly incourage me to be studious to attaine whole numbers: but might I once attaine to be a practitioner in broken numbers, I should thinke my selfe happy.

Master. *What say you then to this question? If 48 Ioyners in two dayes make 200 light horsemen staues (esteeming they worke but 12 houres a day) and such need requireth that 384 Ioyners are set to the finishing of these 200 staues; in what time say you, will they make them vp?*

Scholar. I see here that I must turne my 2 dayes into houres. And so doing, I set my numbers thus:

48 — 24

384

Saying, if 48 men are 24 houres, 384 men will make an end quickly. For it is grounded vpon an old Prouerbe. *Many hands make quicke speed.*

I multiply 48 into 24, and it amounteth to 1152, which I diuide by 384, and my quotient is three houres, which is my desire.

Note. *I take this for a note worthy the marking, either in the Rule of Three, forward or backward, when the two numbers are multiplied together, the Product is of the same nature and denomination that the second number is of.*

The

The double Rule of Proportion direct.

Master.

WEll, sith you perceiue now the vse of this Rule, I wil shew other which insue of the same, & first the *Double Rule* which is so called, because there is in it double working, by which thing onely it differeth from this. The double Rule.

Scholar. Then by an example I would vnderstand it well enough.

Master. *So shall you, and let this be the example: If the carriage of* 100 *weight (that is* 112 *pound)* 30 *miles do cost* 12 *pence, how much will the carriage of* 500 *weight cost, being carried* 100 *miles?* Question of cariage.

Scholar. I pray you shew me the working of it.

Master. You must make two workings of it: the first thus: If C weight cost 12 pence, how much will fiue hundred weight cost? Set your figures thus:

Weight.	*Pence*
1	12
5	

And multiply 5 by 12, and thereof amounteth 60, which if you diuide by one, the quotient will be still 60, that is the price of 500 weight for 30 miles.

Then begin the second worke, saying: If

30 miles coſt 60 pence. how much will 100 miles coſt? Set your figures thus:

Miles	*Pence*
30	60
100	

Then multiply 100 by 60, whereof amounteth 6000, which being diuided by 30, will yéeld 200 pence. Then you may ſay, that ſo many pence ſhall coſt the carriage of 500 pound weight 100 miles, after the rate of 12 pence for the 100 carried 30 miles.

Scholar. Now I perceiue it alſo.

¶ Maſter. Theſe and ſuch other like queſtions of the double Rule of Three, are to bee anſwered much ſooner, at one only working by the Rule of proportion compoſed of fiue numbers, which anon I will ſhew you, and then when you haue the vſe thereof, you may vſe which way you think good.

Scholar. Sir, I thanke you much for your courteſie, and I long now till this Rule be ended, that I may ſée how I ſhall behaue my ſelfe with that new Rule of fiue numbers: for that I haue euer ſince you taught me hitherto in the Golden Rule both forward and backward, wrought but with three numbers onely.

Maſter. But yet awhile we will go on forward with this Rule of Three, therefore anſwer to this queſtion.

Queſtion of ſowing. Thirty buſhels of wheat ſowed, yeelded in one yeare 360, how many will 80 buſhels yeeld in 7 yeares? I meane ſowing euery

yeare

yeare of those seuen, still 80 bushels?

Scholar. First I say, that if 30 bushels will yeeld 360 in one yeere, then 80 bushels will yeeld 960 in one yeere. Then for the second worke I say. If one yeare yeeld 960, then 7 yeeres will yeeld 6720: as these two figures do shew.

Seed.	*Encrease*	*Yeere.*	*Encrease.*
30	360	1	960
80	960	7	6720

But now Sir, if I set forth 30 *bushels of Corne to another man for* 7 *yeares, agreeing so that hee shall sow euery yeare the whole increase of the Corne, and I at the end of these* 7 *yeeres to haue the halfe of the whole increase: I would know how many bushels will there amount to my part, supposing the increase to be after the rate of the last question, for* 30 *bushels in one yeare to yeeld* 360? Question of Corne.

Master. In such a question you must haue so many seuerall workings as there bee yeares, as for example: in the first yeare 30 bushels yeelds 360: then to know the yeelding of the second yeare, I must say, If 30 yeeld 360, how many yeeldeth 360? Worke by your Rule, and you shall finde 4320. Then say for the third yeare. If 30 yeeld 360, how many will 4320 yeeld? You shall haue 51840, and so euery yeare multiplying the whole increase by 360, and diuiding

it by 30, the increaſe of the next yeere will amount, as theſe 7 figures following do orderly declare: where I haue ſet 7 letters for the 7 yeeres, of which the firſt is ſet without Art, becauſe that is the increaſe which you do preſuppoſe: and the laſt number of each other doth ſhew the increaſe of that yeere that it ſtandeth for, which the letters do declare, ſo that the increaſe of the ſeuenth yeere, is 1074954240 buſhels: how many quarters that is, and alſo how many waies you may by Reduction ſoone finde.

a
30 — 360

b
30 360
360 4320

c
30 360
4320 51840

d
30 360
51840 622080

e
30 360
622080 7464960

f
30 360
7464960 89579520

g
30 360
89576520 1074954240

Queſtion of mowing.

Now with one queſtion more I will proue you. *If ſix Mowers do mow 45 acres in fiue daies, how many mowers will mow 300 Acres in ſix dayes?*

Scholar.

Scholar. If 45 acres require 6 Mowers, then 300 Acres require 40. Now againe, if fiue daies require 40 Mowers, then 6 dayes neede but 33 Mowers.

Master. Why do you not make mention of the 2 that remaineth in the last Diuision? for the last part of the question is wrought by the Backer rule, where the first number 5 is multiplied into the second, that is 40, whereof amounteth 200, which if you diuide by the third number 6, the Quotient will be 33, as you said: but then will there remaine 2 which cannot well be diuided into 6 parts: howbeit you may understand by the 6 part of 2, the third part of one Mans worke, which you must put to the 33: or else you must say that 33 Workemen will end all the 300 acres in 6 daies saue 2 mens worke for one day or 2 dayes worke for one man. But such broken numbers called Fractions, you shall hereafter more better perceiue, when I shall wholly instruct you of them.

¶ Master. *Yet one question more of field matters I will propone, and so I will make an end of this double Rule of Three.*

Scholar. With all my heart. Sir I thanke you, and I will dispatch it as soone as I can, because I would faine see the order of the next Rule of 5 numbers.

Master. *If a Captaine ouer a band of men did set 300 Pioners a worke, which in eight houres did cast a trench of 200 Rods: I demand how many labourers* Question of entrenchings

labourers will be able with a like trench in three houres, to intrench a camp of 3400 *Rods.*

Scholar. I thinke I am now in the Backe-house ditch: for I know not well which way to go about it. And besides that, truly I think I shall neuer come to preferment that way, my growth is so small.

Master. You know not how God may raise you hereafter by knowledge and seruice into the fauour of your Prince, for the auaile of your Countrey.

Example for nauigation: Sir *Francis Drake*, a man greatly honoured for his knowledge, was not the tallest man, and yet hath made as great an aduenture for the honour of his Prince and Countrey, as euer Englishman did.

Scholar. Sir, I thanke you for your good incouragement. My minde, though I be little, is as desirous of knowledge, as any other: I haue pondred now a little of it, and thus I set forth the worke.

Rod	*Men*
200	300
3400	

Saying, if 200 Rod require 300 men, what shall 3400 rods require? I multiply 3400 by 300, and it yeeldeth 1020000, which I diuide by 200, and my quotient is 5100 men.

Then must I say for the second worke, if in 8 houres 5100 men, be able to discharge it, how many shall performe the same in three houres; Now if I would worke by the

Golden

Golden Rule of Proportion forward, I should finde a lesse number of men: because three houres is lesse then 8 houres: but because reason teacheth me, that the lesser the time is, wherein the trench must be made, the more Labourers I ought to haue, thereupon I vse now the Backer Rule, as in example. And I haue in my Quotient 13600. So many Pioners must I haue to intrench the Campe in three houres.

Master. You haue answered the question very artificially: And truly I commend you for your diligence and apt vnderstanding: and now according to my promise, I will (in whole numbers) giue you a little taste of the Rule of Proportion, compounded of fiue numbers.

The

The Rule of Proportion, composed of 5 Numbers.

The first part of the rule of proportiō cōpound, direct.

THis Rule of Proportion composed, is distinct for most needfull questions, into severall parts or workings: And there belongeth unto it alwayes five numbers, whereof in this rule being the first part, the second number and the fifth, are alwayes of one nature and like denomination, which Rule is to be wrought thus: you must multiply the first number by the second, and that shall be your Divisor: Then againe, multiply the other 3 numbers, the one by the other, and their product shall be your dividend.

And now according to my promise, we will first worke the question of weight and carriage, which I delivered you in the double rule of three, to be absolved by this rule, which was this.

If the carriage of 1 *C weight* 30 *miles cost* 12 *pence, what will the carriage of* 5 *C weight stand me in being carried* 100 *miles?*

C. weight. Miles. Pence. C. weight. Miles.

1 —— 30 —— 12 —— 5 —— 100

Now marke well how these five numbers stand: Then multiply the first number by the second, as 30 by 1, which maketh but

30,

30. that number keepe for your Diuisor. Then multiply the other three numbers, the one into the other: that is to wit, 12 by 5 which maketh 60: Lastly 60 by 100 which as you see here in our Tables, ariseth to 6000, which 6000 you shall diuide by the product of the two first numbers, which here is 30. And you see there is found 200 pence, which is the duty that you ought to pay for the carriage of 500 weight 100 miles, after the rate of 12 pence a hundred, and agreeth with the conclusion of the double Rule of three.

Scholar. Sir I thanke you, it is euen so.

Master. *Yet note this for a generality in this rule, looke what nature or denomination your middle number is of (which here are pence) and of the like denomination or nature is alwayes your quotient.* Note this.

Schollar. Well now and if it please you, by your patience, I will see how I can end the question, next following of 30 Bushels of wheat sowed, which in one yeare yeelded 360 how many then will 80 Bushels yeeld in 7 yeare sowing euerie yeare of those 7 still 80 bushels, and according to your reasons I set my numbers thus.

Bush. Yeare. Bush. Bush. Yeare.
30 — 1 — 360 — 80 — 7

80
28800
7
201600

When

Where I multiply 30 by 1, and it maketh 30, my diuisor: then multiplying the other thrée numbers the one into the other, as here appeareth in my tables, they make 201600, which I diuide by 30: & my quotient is 6720 bushels, my desire, for so much also it came to at two workings by the Rule of three.

Master. Yet one question more I will propound vnto you, and so leaue this Rule, till it please God hereafter, that I may make you worke it in broken numbers.

Question of Interest.

What comes the interest of 258 pound, *for fiue moneths to, after the rate of* 8 pound *taken in the* 100 pound, *for 12 moneths?*

Scholar. Sir, this is yet within the compasse of some reasonable vsance. Therefore to minister equity in this case, I will sée how I can worke the same, which I set downe thus, praying you if I haue not done wel, to shew me mine errour.

li *moneths*. li li *mon.*
100–12–8–258–5

Master. Procéed, you haue done very well.

Scholar. Then I doubt not by the grace of God but to end it: I multiply 100 by 12, it yéeldeth 1200, and the thrée other numbers multiplied together produce 10320, which I diuide by 1200: and my quotient is 8 pounds. Then according as you haue taught me heretofore, I turne the 720 pound that I left, into shillings: and diuiding it by the first number, my Quotient is 12. shillings,

lings. So I answer, that the loane of 258 pounds for 5 moneths, after the rate of 8 pound in the 100 pound for a yeare, comes to 8 pound 12 shillings.

Master. You say true, I commend your diligence: now behold the manner of the second part of this rule.

The

The backer Rule, or the second part of the Rule of Proportion compound.

Master.

IN the second part of this Rule of Proportion composed, the third number is like unto the first. And the Rule is to be wrought thus: you shall now, contrary to the last rule, multiply the third number and the fourth together, and that Product shall be your Divisor. Then multiply the fifth by the second, and the Product thereof by the first: and that is the number that shall be divided. For example, I propound this question, for a proofe of my last question of interest.

The proof of the last question.

A Merchant hath received 8 pound 12 shillings, *for interest of certaine money for* 5 *moneths tearme, which he received after the rate of* 8 pound *in the* 100, *for a yeere. The question is now, how much money was delivered to raise this interest.*

Behold therefore the manner, how the question is set forth.

li	moneths.	li	moneths.	li	s.
100	12	8	5	8	12

Scholar. Sir I perceive it very well: and

according

according to the doctrine which you prescribed for the working thereof: if please you now it is set downe, I thinke I can follow the worke.

Master. Nay stay a while, and before you worke, marke well how I deliuer a reason for the perfect understanding of this Rule, which is thus: *If* 8 pound *in* 12 *moneths do yeeld me* 100 pound, *to take* 8 pound 12 shillings *for* 5 *moneths, must needs yeeld a great deale more.* Note.

So upon the knowledge that I haue in this Art, the first part of this rule is answerable to the rule of three forward: and this latter part accordeth to the rule of three backward.

Scholar. Sir, I yeeld you most heartie thanks for these your last instructions, they haue giuen mee great light into these two rules, whereby I may the better by deliberation conceiue how to use them hereafter when occasion shall requIre.

Master. You say well, go to now if you will, and trie your cunning in the question: But this note take with you by the way, in as much as here is mention made of shillings: turne all your money as you worke into shillings for your more ease in working. Note.

Scholar. If it please you to behold me a little, I will quickly end it: for I haue but my first, my second, and my last number to be multiplies together for my diuidend: And my third

P into

into my fourth for my Diuiſor.

```
li    Moneths.       Moneths.  li   ſ.
100—12——8———————5——8—12
20          20          20
——          ——          ——
2000        160         172
12          5
——          ——
4000        800
2000
——
24000
172
——————
48000  4128000(
168000     8  00
24000
——————4128000
```

Which 4128000 I diuide by 800, and my Quotient is 5160 ſhillings, which in pounds yéeldeth 258, my deſire.

Maſter. I will here for this time in whole numbers end this Rule, and I will inſtruct you in the Rules of Fellowſhip. You may at your conuenient leaſure for your exerciſe worke the ſame by the Rule of Three at twice. And for your aid and incouragement therein, I ſet downe héere a proffer how to apply it.

A

A				B
Moneths.	li.	s	*Pound.*	li.
5	8	12	8	100
12	4	12 $\frac{4}{5}$	412 $\frac{4}{5}$	258 li

The Rule of Fellowſhip.

The Rule of Fellowſhip without time.

BVt now will I ſhew you of the Rule of Fellowſhip or Company, which hath ſundry operations, according to the diuers number of the Company. This Rule is ſometime without difference of time, and ſometimes there is in it difference of time. Firſt I will ſpeake of that without difference of time, of which let this

Company made a
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60
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king
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be the
ſumme
you

ſhall ſet the portion of each man one after another, and then worke by the Golden Rule, and the fourth ſumme will ſhew you each mans gaines: as in example.

30
50
60
160
———
240

The parcels of theſe foure Merchants make in one ſumme 240 pounds: ſet that in the firſt place, the gaines in the ſecond, and the firſt mans portion of ſtocke in the third place, thus:

240 3000
30

Now multiply the ſecond by the third, and it will be 90000, which you ſhall divide by 240, and there will appeare 375 pounds thus:

And that is the gaines for the firſt man.

240 3000
30 375

Now for the ſecond man, ſet the 50 pound that he brought, in the third place, and worke as before: and his part will be 625 pound: as this figure ſheweth.

240 3000
50 625

Likewiſe for the third man, ſet his money which was 60 pounds and his part of gaines will bee 750 pounds, as here appeareth.

And so for the fourth man; if you set his sum which is 100 pound his gains will bee 1250 pound, as the work will declare.

240 — 3000
60 — 750

Scholar. This I perceiue: but is there any way to examine whether I haue well done or no?

240 — 3000
100 — 1250

Note this common proofe.

Master. For the triall hereof, adde together all their foure portions, and if their addition make the whole summe of their gaines, then is the worke well done.

Scholar. That will I try by and by, the foure parcels are these which added together make 3000, which is the full summe of money that they gained, whereby I know the worke is well done.

375
625
750
1250
3000

Master. Well, now another example will I put to you, not of gaines, but of losse: for one reason serueth for both.

A question of losse.

If three Merchants in one ship, and of one fellowship, had bought Merchandise, so that the first had laid out 200 pound, *the second* 300 *pound, the third* 500 *pound, and it chanced by tempest that they did cast ouer board into the Sea Merchandise of the value of* 100 pound, *how much should each man beare in this losse?*

Scholar.

Scholar. If I shall do in this, as you did in the other question, then must I ioyne their three portions together 200,300, 500, which maketh 1000. Then say I, if 1000 lose 100 then shall 200 lose 20, and 300 shall lose 30, and 500 shall lose 50, as by the three figures it doth appeare plaine.

1000	100	1000	100
200	20	300	30

1000	100
500	50

Master. Well sith now you haue done these, I will propound a question of more importance, which shall make you not only the abler to understand this Rule, but also it will greatly aid you in the next Rule of Fellowship with time, if such need be that your money be of diuers denominations.

For this may not be forgotten in all such questions: if the number be of diuers kinds, you must by reduction bring it into one kind, that is to say, to the least value that is named in the question. And likewise shall you do, if the time be of diuers kinds, as some yeares, some moneths, weekes, and dayes, you shall make all moneths, weekes, or dayes, according as the least name of time in the question is, as for example.

First in diuersity of mony. Three companions bought 2000 sheep & paid for them 241 pound 13—4 *pence, of which summe one paide* 101 A question of sheepe.

pound, 10 shillings. *The second* 82 pound, 17 shillings, 10 pence. *And the third payed* 57 pound 5 shillings 6 pence, *How many sheepe must each of them haue?* Answer: *The first shall haue* 840. *The second* 686. *And the third* 474. *And that must you worke thus.*

First, considering that your money is of diuers denominations, you shall (by Reduction) bring it all into the smallest denomination which is in it, that is to say, pence, and so will the totall summe be 58000 pence. Solution.

Now if you turne each mans money into pence also, the first mans summe will be 24360 pence: The second mans money will be 13746 pence.

Now to know how many sheepe euery man shall haue, let the whole summe of money, that is, 58000 pence be set in the first place, and in the second place set the number of sheepe, and then orderly in the third place set each mans money, and then multiplying the third and the second summes together, and diuiding that that amounteth by the first, there will appeare the number of sheepe that each man ought to haue: as these three figures do shew.

a

58000 — 2000
24360 — 840

b

58000 — 2000
19894 — 686

c

58000 — 2000
13746 — 474

Scholar. Why do you ſet the money in the firſt place, ſeeing in the queſtion you ſay 2000 ſheepe coſt 58000 pence, and not thus: 58000 coſt 2000 ſheepe.

Maſter. You remember I taught you at the beginning of the Golden Rule, that the firſt and third numbers muſt be of one name, and of like things: and euermore the number that the queſtion is asked of muſt be ſet in the third place.

Now is the queſtion plainly this: If foure men bought 2000 ſheepe for 58000 pence, how many ſheepe ſhall each man haue?

But ſeeing in this queſtion, there ought more reſpect to be had to the ſumme of money then to the ſumme of the perſons (for in the ſumme of money is their proportion toward the ſheep, and not in the number of perſons)

If 58000 pence bought 2000 ſheepe, how many did 24360 buy? Againe, how many did 19894 pence buy? And how many bought 13746 pence?

Scholar. I perceiue it reaſonably, and ſo
ſhall

ſhall I do in all queſtions.

Maſter. Euen ſo. But for eaſineſſe of the worke, marke this: Whenſoeuer the firſt and ſecond numbers haue ciphers in the firſt places, you may both in the Multiplication and in the Diuiſion leaue out thoſe cyphers, ſo that you leaue out like many out of both ſummes, as in this queſtion, the firſt number 58000 hath thrée cyphers, and ſo hath the ſecond, that is 2000: therefore caſt away their cyphers, and ſo will the firſt number be 58, and the ſecond 2: ſet them in their places, and worke according to the Rule, and you ſhall perceiue that will be all one, ſauing that this is the ſhorter and eaſier way, as theſe three figures do ſhew.

Note.

a

58 — 2
24360 — 840

b

58 — 2
19894 — 686

c

58 — 2
13746 — 474

And this you ſée is both eaſier, and alſo the more certaine way to know the anſwer to this queſtion.

Scholar. Truth it is as you ſay: but Sir me ſéemeth I might aske a further queſtion here, not onely how many ſheepe each man ſhould haue, but alſo what euery ſheepe coſt.

Maſter. That queſtion doth not onely be-

long

long to this Rule, but may also be discussed by Diuision, especially if the questions number be one onely, as thus: Diuide the total summe 58000 pence by 2000 (or 58 by 2, omitting the ciphers, and the quotient will be 29 pence that is 2 shillings 5 pence. Howbeit, by this Rule you may do it, and best when the number of the question doth exceed 1; as if I should aske this question,

2000 sheepe cost 58000 pence, how much do 20 cost? Then shall I set my figure as before.

2000 — 58000
20 —

And doing after the Rule, there will amount 580 pence, that is, 2 pound 8 shilling 4 pence the price of one score: but if you will vse that easie way that I did teach you now, you may change the first and second number thus.

2 — 58
20 —

Thus do you perceiue the vse of the Rule without Time.

Scholar. All this I vnderstand very well: I pray you now instruct me in the Rule of Fellowship with Time.

The

The Rule of Fellowship with time.

Master.

The Rule of Fellowship with time.

TO the intent you may as well perceiue the same Rule with diuersity of time, I propose this example.

Foure Merchants made a common stocke, which at the yeares end was increased to 35145 pound. *Now to know what shall be each mans portion of gaine, you must know each mans stocke, and time of continuance.*

Question of a bank.

The first man of these foure layd in 669, which he did take from the stocke againe at the end of 10 moneths. The second man laid in 810 pound, for eight moneths. The third layd in 900 pound, for seuen moneths. And the fourth layed in 1040 pound, for 12 moneths.

Note. A generall Rule.

This question shall you examine as you did the other before, sauing that whereas in the third place of the figure you did set each mans summe alone, heere you shall set the same being multiplied by the number of their time: and likewise in the first place of the figure you shall set the number which amounteth of their whole summes so multiplied by their time, and added into one

whole

whole ſumme, as thus.

The firſt mans ſumme is 669 pounds, which I multiply by 10 (that was the number of his time) and it maketh 6690. The ſecond mans ſumme 810 pound, multiplied by 8 (which was his time) maketh 6480. The third mans ſumme 900 pound, multiplyed by 7 (for that was his time) yéeldeth 6300. The fourth mans ſumme was 1040 pound, and his time 12: multiply the one by the other, and it will be 12480.

The foure ſummes thus multiplied by their time, muſt be ſet orderly in the third place of the figure, and in the firſt place muſt be ſet the whole ſumme of all foure, which is 31950; and the gaine muſt be in the ſecond place, which is 35145. Now to end the queſtion, I ſay firſt, If 31950 did get 35145, what did 6690 get? Anſwer, 7359 pounds as by this figure appeareth.

a

31950 35145

6690 7359

Likewiſe, the ſecond man had to his part 7128 pound, the third muſt haue 6930 pounds, and the fourth man ſhall haue for his part 13728 pound, as theſe figures do partly declare.

b

31950 35145
6480 7128

c

31950 35145
6300 6930

d

31950 35145
12480 13728

Scholar. This I like very well: but what proofe is there of this worke?

Another proofe.

Maſter. The ſame that I taught you for the other: howbeit, there is uſed both for this work and the other alſo, this manner of proofe, to adde all the portions together, and it may agrée to the whole ſumme, then ſéemeth your worke well done: but this is no ſure proofe.

Scholar. Yet will I proue in this example: The foure parcels are theſe, which if I adde together, there will amount 35145, and that was the whole ſumme, whereby I perceiue the worke is well done.

7359
7128
6930
13728
35145

Maſter. If it fall out otherwiſe, be ſure it is not well.

Scholar. Then do I underſtand this work alſo very well: but what haue I now to learne?

Maſter. There are many other excellent parts

parts behinde, of which I will not as now make mention, because that without the knowledge of Fractions they cannot bee duely taught, and much lesse understood. Therefore will I propose to you two or three questions more (that thereby you may better perceiue the vse of this Rule and all other the like) and so make an end for this time.

¶ *Three partners, by some ill aduenture sustained the losse of* 160 pound, *whereof the first laid into the common stocke* 200 pound, *for ten moneths. The second laid in* 350 pounds, *and the third* 100 pound, *but for how long the two latter, is vnknowne. But breaking off their partnership, the first found himselfe a loser* 80 pound, *the second* 56 pound, *and the third* 24 pound. *The question is for how long time was the money of the two latter in company.* A question of losse.

For the solution hereof, and of such other like, you must also multiply the first mans 200 pound, that he put into the stocke by his time of continuance, which was 10 moneths, and it maketh 2000: wherefore now I affirme, if his mony that lost 80 pound, multiplied by his time make 2000; what shall his mony make that lost 56 pound, and his that lost 24 pound, which two numbers I commit to the triall of the rule of Three, at two workings thus:

If 80 giue 2000, what giueth 56? And againe, if 80 giue 2000, what giueth 24?

80	2000	80	2000
56	1400	24	600

To conclude, if you now diuide 1400, the second mans portion by 350, which was his stocke that he laid into Company, you shall finde in your quotient 4 moneths, and for so long time did the second man put his money into the common stocke.

Lastly, if you diuide the third mans new laying in, which was 600 by 100, which was his stocke that hee put into Company: the Quotient declareth his time of continuance, which was 6 moneths. And thus is the question resolued.

Scholar. Sir, I haue attentiuely beheld your working, and the more we trauell herein, the more me thinke I am in loue with this excellent Art.

Master. Then what say you to this Question?

A question of Canon. *There is in a Cathedrall Church* 20 *Canons, and* 30 *Vicars, these may spend by yeare* 2600 pound. *but euery Canon must haue to his part fiue times so much as euery Vicar hath: how much is euery mans portion, say you?*

Scholar. I pray you make the answer your selfe also, so shall I perceiue best the meanes to answer to such other like.

Master. In this question, you must doe as in those beforesaid, that haue diuersity of time,

time, for here is diuersity of portions. Therefore shall you multiply the number of the persons by their difference of portion: (as you did in the other by time:) Then must you multiply the 20, (which is the number of Canons) by 5, (for that is the number of their portion) so will it be 100. Then 30, (that is the number of Vicars) by 1, (that is the number of their portion) and it will be 30: put these two summes together, and they make 130. Then say thus: If 130 spend 2600 pounds, what may 100 spend? The Rule sheweth 1000 pounds.

Againe for Vicars: If 130 spend 2600 pound, what may 30 spend? Answer, 600 pound, as these figures shew.

130	2600	130	2600
100	2000	30	600

But if euery Canon should haue so often times 4 pound, as the Vicar should haue 3 pound, then should I multiply 20 by 4, (that were 80) and 30 by 3, (that were 90) and then both were 170. Then should the figures be set as followeth.

	li ſ d		li ſ d
170	2600	170	26000
80	1223-10,7	90	1376-9.5

But this sort is too hard for you, by rea-

son of the fractions, therefore I will let it rest to that place.

And by this rule you see what the 20 Canons may spend, which summe if you diuide by 20, you shall see each Canons portion: and so of the Vicars, if you diuide their summe by 30, the quotient will declare euery Vicars portion.

June 1st

J. F. James Driscott

was Born 3rd Day

June 1793 & Baptized

June 25th 1793 How

Old is the Ch[illegible] each

1 year. m. 2 Days

15 Born & Baptized 24

Days

15

The second Dialogue.

The accounting by Counters.

Master.

NOW that you haue learned the common kinde of Arithmeticke with the pen, you shall see the same Art in Counters: *which feat doth not onely serue for them that cannot write and reade, but also for them that can do both, but haue not at sometime their pen or tables ready with them.*

This sort hath two formes commonly. The one by lines, and the other without lines. In that that hath lines, the lines do stand for the order of places: and in that that hath no lines, there must be set in their stead so many Counters as shall néed for each line one, and they shall supply the stead of lines.

Scholar. By examples I should better perceiue your meaning.

Master. For example of the lines, loe here you sée six lines, which stand for six places, so that the

—100000—

—10000—

—1000—

—100—

—10—

—1—

Numera-
tion by
Counters.

nethermost standeth for the first place, and the next aboue it for the second, and so vpward till you come to the highest, which is the sixth line, and standeth for the sixth place.

Now what is the value of every place or line you may perceiue by the figure which I haue set on them, which is according as you learned before in Numeration of figures by the pen: for the first place is the place of vnites or ones, and every counter set in that line, betokeneth but one: and the second line is the place of 10, for every counter there standeth for 10: the third line the place of hundreds, the fourth of thousands, and so forth.

Schollar. Sir, I do perceiue that the same order is here of lines, as was in the other figures by places, so that you shall not need longer to stand about Numeration, except there be any other difference.

Master. If you do vnderstand it, then how will you set 1543?

Schollar. Thus as I suppose.

Master. You haue set the places truly, but your figures be not meet for this vse: for the meetest Figures in this behalfe, is the Figure of a counter round, as you see here, where I haue expressed that same summe.

1
5
4
3

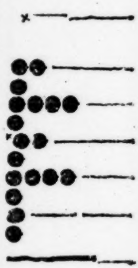
Schollar. So that you haue not one Figure

for

for 2, nor 3, nor 4, and ſo forth, but as many Digits as you haue, ſo many Counters you ſet in the loweſt line, and for euery 10 you ſet one in the ſecond line, and ſo of other. But I know not by what reaſon you ſet that one Counter for 500 betwéene two lines.

Maſter. You ſhall remember this, that whenſoeuer you néed to ſet downe 5, 50, or 500, or 5000, or ſet forth any number whoſe Numerator is 5, you ſhall ſet one counter for it in the next place aboue the line that it hath his denomination of: as in this example of that 500, becauſe the numerator is 5, it muſt be ſet in a voyd ſpace, and becauſe the denomination is a hundred, I know that the place is the void place next aboue hundreds, that is to ſay, aboue the third line.

And further you ſhall marke, that in all working by this ſort, if you ſhall ſet downe any ſumme betwéene 4 and 10, for the firſt part of that number you ſhall ſet downe 5, and then ſo many counters more, as there reſt numbers aboue, 5. And this is true both of Digits and Articles. And for example, I will ſet downe this ſumme 297965, which ſumme if you marke well, you néed none other examples for to learne the numeration of this forme.



But this shall you marke, that as you did in other kinds of Arithmeticke, let a pricke in the places of thousands, in this worke you shall set a Starre, as you see before.

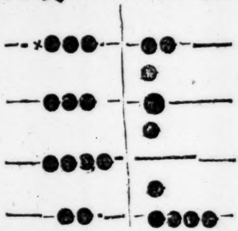
Schollar. Then I perceiue Numeration: But I pray you how shall I do in this Art to adde two summes or more together.

Addition.

Addition.

Maſter.

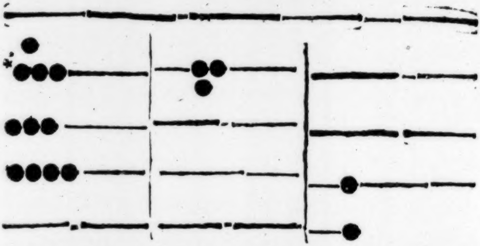
THe eaſieſt way in this is to adde but two ſummes at once together: Howbeit you may adde more, as I will tell you anon. Therefore when you will adde two ſummes you ſhall firſt ſet downe one of them, it forceth not which, and then by it drawe a line croſſe the other lines. And afterward ſet downe the other ſumme, ſo that the line may bee betweene them: as if you would adde 2659 to 8342, you ſet your ſummes as you ſee heere.



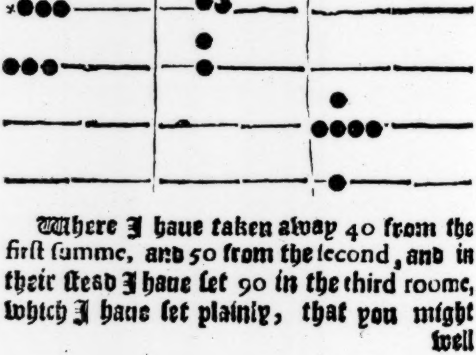
Addition of two ſummes.

And then if you liſt, you may then adde the one to the other in the ſame place: or elſe you may adde them both together in a new place; which way, becauſe it is moſt plaine, I will ſhew you firſt.

Therefore will I beginne at the vnites, which in the firſt ſumme is but 2, and in the ſecond ſumme 9, that maketh 11. Thoſe do I take vp, and for them I ſet 11 in the new roome, thus.



Then do I take vp all the Articles vnder a hundred, which in the firſt ſumme are 40, and in the ſecond ſumme 50, that maketh 90: or you may ſay better, that in the firſt ſumme, there are foure Articles of 10, and the ſecond ſumme 5, which maketh 9, but then take héed that you ſet them in their right lines, ſée héere.



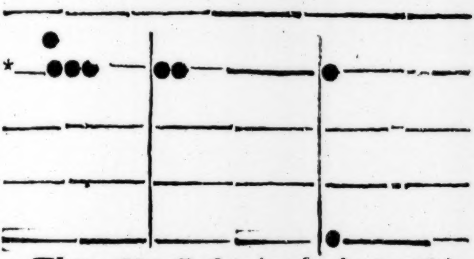
Where I haue taken away 40 from the firſt ſumme, and 50 from the ſecond, and in their ſtead I haue ſet 90 in the third roome, which I haue ſet plainly, that you might
well

well perceiue : howbeit ſeeing that 90 with the 10 that was in the third roome, already, doth make 100, I might better for thoſe 6 Counters ſet 1 in the third line thus:

For it is al in one ſumme, as you may ſee, but it is beſt neuer to ſet fiue Counters in any line, for that may be done with one Counter in a higher place.

Schollar. I iudge that good reaſon, for many are vnneedfull where one will ſerue.

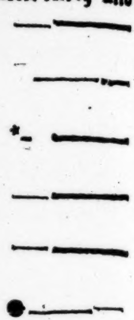
Maſter. Well, then will I adde forth of hundreds: I finde 3 in the firſt ſumme, and 6 in the ſecond, which maketh 900, them do I take vp, and ſet in the third roome, where is 100 already, to which I put 900, and it will be 1000: therefore I ſet one Counter in the fourth line for them all, as you ſee here.



Then adde I the thouſands together, which in the firſt ſumme are 8000, and in the

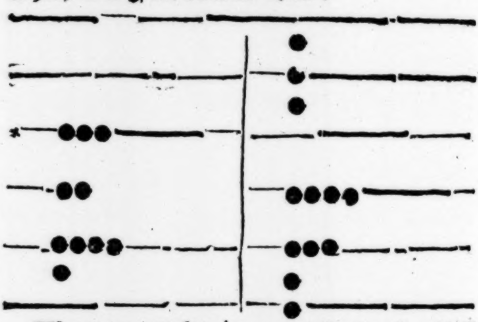
the second 2000, that maketh 10000. then do I take vp for those two places, and for them I set one Counter in the fifth line, and then it appeareth as you see to be 11001, for so many doth amount of the Addition of 8342 to 2659.

To adde summes together.



Schollar. Sir, this I do perceiue: but how shall I set one summe to another, not changing them to a third place?

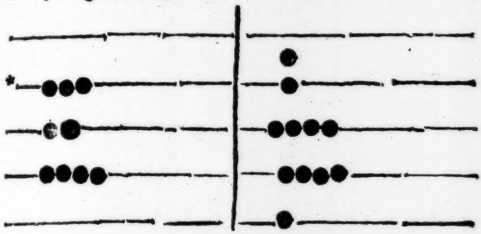
Master. Marke well how I do it. I will adde together 6 5 4 3 6 and 3 2 4 5, which first I set downe thus:



Then doe I beginne with the smallest denomination, which is 1 in the second summe, and set it in his place: then do I finde 5 in the first summe, and 5 in the second, which put together, sauing the two counters cannot be set in a void place of 5, but

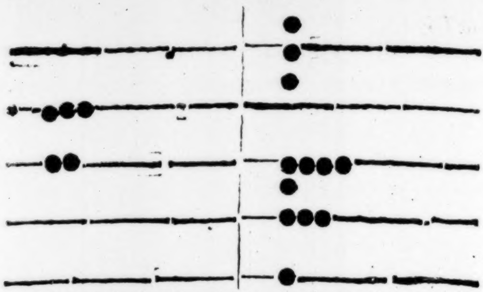
for

for them both I muſt ſet one in the ſecond line, which is the place of 10, therefore I take vp the fiue of the firſt ſumme, and the 5 of the ſecond, and for them I ſet one in the ſecond line, as you ſee here.



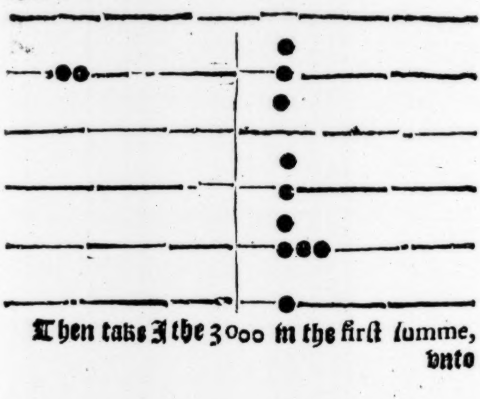
Then do I likewiſe take the 4 Counters of the firſt ſumme and ſecond line, (which maketh 40) and adde them to the 4 Counters of the ſame line in the ſecond ſumme, and it maketh 80: but as I ſaid, I may not conueniently ſet aboue 4 Counters in one line, therefore to thoſe 4 that I tooke vp in the firſt ſumme, I take one alſo of the ſecond ſumme, and then haue I taken vp 50: for which 5 Counters I ſet downe one in the ſpace ouer the ſecond line, as here doth appeare.

And



And then is there 80, as well with thoſe 4 counters, as if you had ſet downe the other 4 alſo.

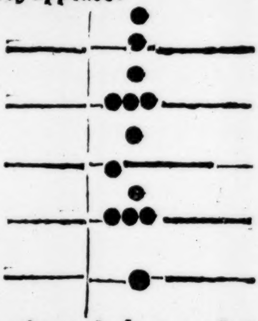
Now do I take the 200 in the firſt ſumme, and adde them to the 400 in the ſecond ſumme, and it maketh 600, therefore I take vp the two counters in the firſt ſumme, and thrée of them in the ſecond ſumme and for them 5, I ſet 1 in the ſpace aboue, thus:



Then take I the 3000 in the firſt ſumme, vnto

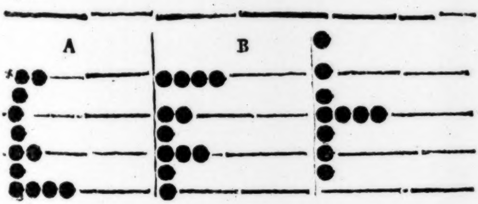
vnto which there are none in the ſecond ſumme agréeing, therefore I do onely remoue thoſe thrée counters from the firſt ſumme into the ſecond, as here doth appeare.

And you ſée the whole ſumme that amounteth of that Addition of 65436 with 3245, to bée 68681.

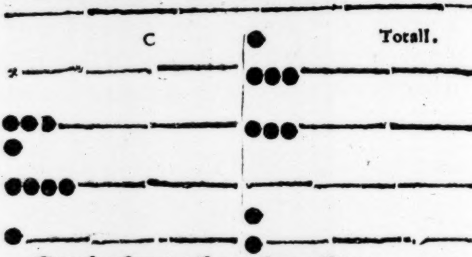


And if you haue marked theſe two examples well, you néed no further inſtruction in Addition of 2 onely ſummes: but if you haue more then two ſummes to adde, you may adde them thus:

Firſt adde two of them, and then adde the third and fourth, or more, if there bée ſo many: as if I would adde 2679, with 4286, and 1391. Firſt I adde the two firſt ſummes thus:



And then I adde the third thereto, thus.



And so of more if you haue them.

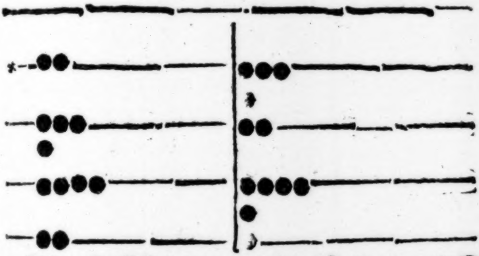
Schol. Now I think it best that you passe forth to Subtraction, except there be any way to examine this manner of Addition, then I thinke that were good to be knowne next.

Master. There is the same proofe here that is in the other Addition by the pen, I meane Subtraction; for that onely is a sure way, but considering that Subtraction must be first knowne, I will first teach you the Art of Subtraction, and that by this example.

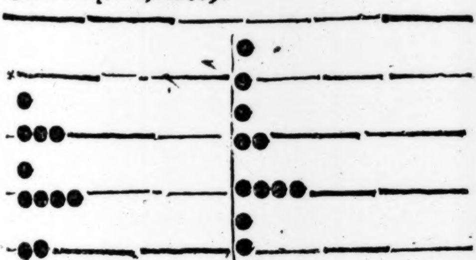
Subtra-

Subtraction.

I Would ſubtract 2892 *out of* 8746. *Theſe ſummes muſt I ſet downe as I did in Addition: but here it is beſt to ſet the leſſer number firſt, thus:*

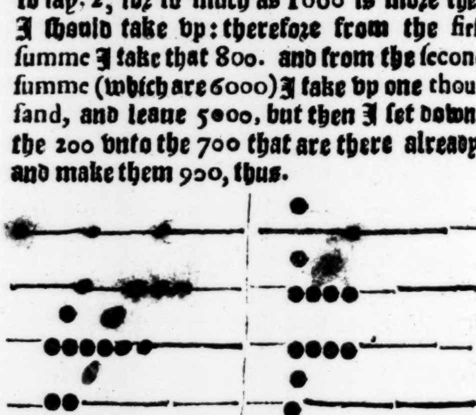


Then ſhall I begin to ſubtract the greateſt numbers firſt (contrary to the vſe of the pen) that is the thouſand in this example: therefore I find amongſt the thouſands 2, for which I withdraw ſo many from the ſecond ſumme (where are 8) and ſo remaineth there 6, as this example ſheweth.



Then

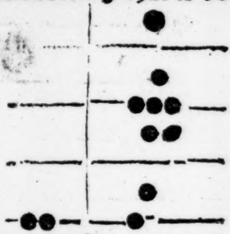
Then do I likewiſe with the hundreds, of which in the firſt ſumme I finde 8, and in the ſecond ſumme but 7, out of which I cannot take 2, therefore this muſt I do: I muſt looke how much my ſumme differeth from 10, which I find here to be 2, then muſt I abate for my ſumme of 800, one thouſand and ſet downe the exceſſe of hundreds, that is to ſay, 2, for ſo much as 1000 is more then I ſhould take vp: therefore from the firſt ſumme I take that 800. and from the ſecond ſumme (which are 6000) I take vp one thouſand, and leaue 5000, but then I ſet downe the 200 vnto the 700 that are there already, and make them 900, thus.



Then come I to the Articles of tennes, where in the firſt ſumme I finde 90, and in the ſecond ſumme but onely 40. Now conſidering that 90 cannot be abated from 40, I looke how much that 90 doth differ from the next ſum aboue it, that is, 100 (or elſe which is all to one effect) I looke how much 9 doth differ from 10 and I finde it to be 1: then in the ſtead of that 90, I do take from the ſe-

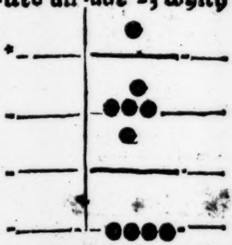
cond

cond ſumme 100: but conſidering that is 10 too much, I ſet downe 1 in the next line beneath for it, as you ſee here.



Hauing that here I haue ſet 1 Counter in the ſpace in ſtead of 5 in the next line.

And thus haue I ſubtracted all ſaue 2, which I muſt abate from 6 in the ſecond ſumme, and there will remaine 4, thus:

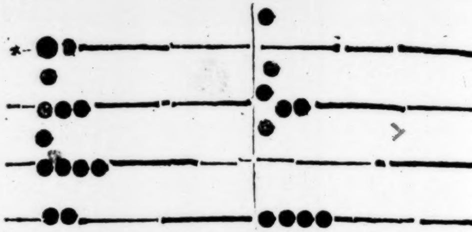


So that if I ſubtract 2892 from 8746 the remainer will bee 5854.

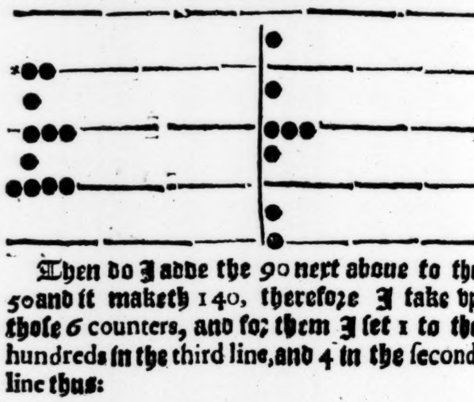
And that this is truly wrought, you may proue by Addition: for if you adde to this remainer the ſame ſumme that you did ſubtract then will the former ſumme 8746 amount againe.

Scholar. That will I proue, and firſt I ſet the ſumme that was ſubtracted, which was 2892, and then the remainer 5854, thus: A proofe of Subtraction.

R Then

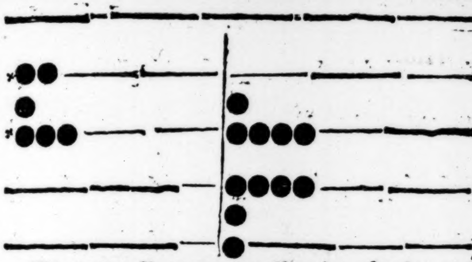


Then do I adde the first 2 to 4, which maketh 6: so take I vp 5 of those Counters, and in their stead I set 1 in the space: and one in the lowest line, as here appeareth.

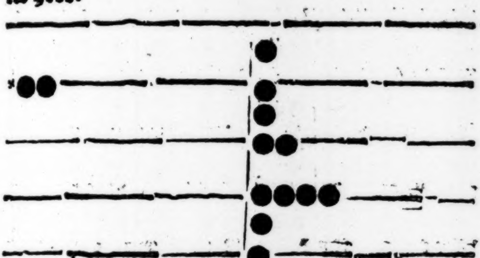


Then do I adde the 90 next aboue to the 50 and it maketh 140, therefore I take vp those 6 counters, and for them I set 1 to the hundreds in the third line, and 4 in the second line thus:

Then

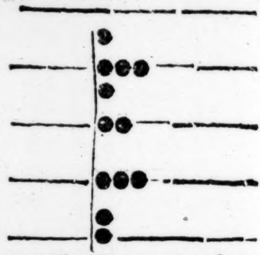


Then do I come to the hundreds, of which I finde 8 in the firſt ſumme, and 8 in the ſecond, that maketh 1600, therefore I take vp thoſe 8 counters, and in their ſtead I ſet 1 in the fourth line, and 1 in the ſpace next beneath, and in the third line, as you may ſee here.



Then is there left in the firſt ſumme but onely 2000, and in the ſecond 5000, which is 7000, which I ſhall take vp from thence, and ſet in the ſame line in the ſecond ſumme to the one that is there already: and there will the whole ſumme appeare as you may well ſee, to bee 8746, which was the firſt

groſſe ſumme, and therefore I do perceiue that I had well ſubtracted before.



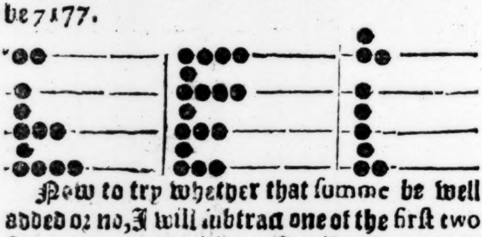
And thus may you ſée, how Subtraction may bée tried by Addition.

Scholar. I perceiue the ſame order here with counters, that I learned before in figures.

Maſter. Then let me ſee how you can try Addition by Subtraction.

Scholar. Firſt I will ſet forth this example of Addition, where I haue added 2189 to 4988. And the whole ſumme appeareth to be 7177.

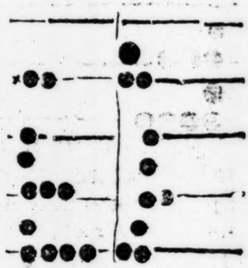
Proofe of Addition by ſubtraction.



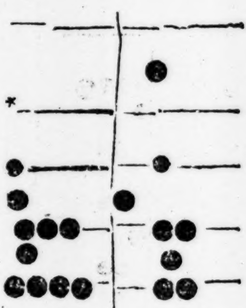
Now to try whether that ſumme be well added or no, I will ſubtract one of the firſt two ſummes from the third. And if I haue well done, the remainer will be like that other ſumme: as for example, I will ſubtract the firſt ſumme from the third, which I ſet thus in order.

Then

Then do I ſubtract 2000 of the firſt ſumme, from the ſecond ſumme, and then remaineth there 5000, thus.



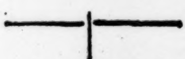
Then in the third line I ſubtract the 100 of the firſt from the ſecond ſumme, where is onely 100 alſo: and then in the third line, reſteth nothing, as you may ſee in this example following.



Then in the ſecond line with his ſpace over him I finde 80, which I ſhould ſubtract from the other ſumme: then ſeeing there are but onely 70, I muſt take it out of ſome higher ſumme, which is here onely 5000:

therefore I take vp 5000: and seeing that is too much by 4920, I set downe so many in the second roome; which with the 70 being there already, do make 4990 and then the summes do stand thus.

Yet remaineth therein the first summe 9, to be abated from the second summe, wherein that place of vnites doth appeare onely 7: then must I abate a higher summe, that is to say 10, but seeing that 10 is more then 9, (which I should abate) by 1, therefore shall I take vp one Counter from the second, and set downe the same in the first line, or lowermost line, as you see here.

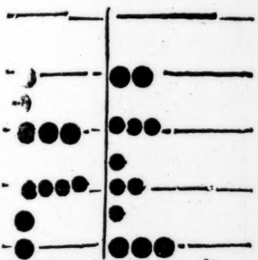


And so haue I ended this worke, and the summe appeareth to be the same which was the second summe of mine Addition, and therefore I perceiue I haue wel done.

Master.

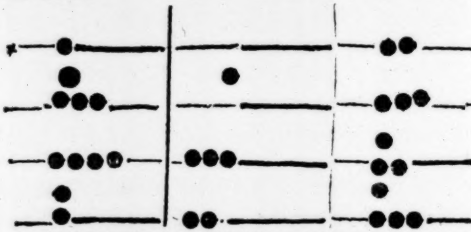
Another way of Addition.

Master. To stand longer about this, it is but folly: except that this you may also vnderstand, that many do begin to subtract with Counters, not at the highest summe, as I haue taught you, but at the nethermost, as they do vse to adde; and when the summe to be abated in any line appeareth greater then the other, then do they borrow out of the next higher roome, as for example.



I should abate 1846 from 2378, they set the sums thus:

First they take 6 which is the lower line, and his space from 8 in the same roomes in the second summe, and yet there remaineth two Counters in the lowest line. Then in the second line must 4 be subtracted from 7, and so remaineth there 3. Then 800 in the third line, & his space, from 300 of the second summe cannot be, therefore do they abate it from a higher roome, that is, from 1000, & because 1000 is too much by 200 therefore must I set downe 200 in the third line, after I haue taken vp 1000 from the fourth line. Then is there yet 1000 in the fourth line of the first summe which if I withdraw from the second summe, then do all Figures stand in order, thus: 532.



So that (as you see) it differeth not greatly whether you begin Subtraction at the higher lines, or at the lower.

Howbeit, as some men like that one way best, so some like the other: therefore you now knowing both, may use which you list.

Multi-

Multiplication.

BVt now touching Multiplication: you shall set your numbers into two roomes (as you did in those other kindes) but so that the multiplier be set in the first roome: then shall you begin with the highest numbers of the second roome, and multiply them first after this sort.

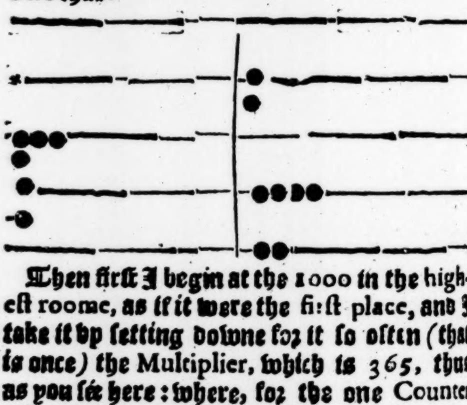
Take the overmost line in your first working as it were the lowest line, setting on it some moveable marke (as you list) and looke how many Counters be in him, take them vp, and for them set downe the whole multiplier so many times as you tooke vp counters: reckoning (I say) that line for the vnites. And when you haue done with the highest number, then come to the next line beneath, and doe so euen with it, and so with the next, till you haue done all. And if there be any number in a space, then for it shall you take the Multiplier 5 times, and then must you reckon that line for the Vnites, which is next beneath that space. Or else after a shorter way, ye shall take onely halfe the multiplier, but then shall you take the line next aboue the space for the line of vnites. But in each working, if by chance your multiplier be an odde number, so that

you

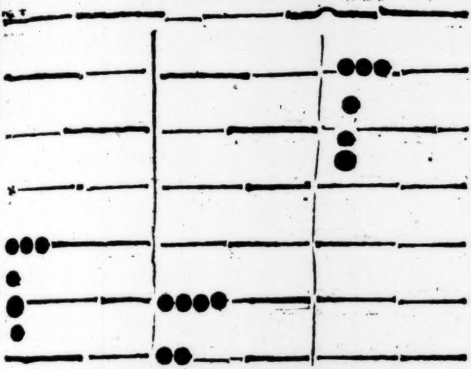
you cannot take the halfe of it iustly, then must you take the greater halfe, and set downe that, as if that it were the iust halfe: and further, you shall set one Counter in the space betweene that line, which you reckon for the line of vnites, or else onely remoue forward the same that is to be multiplied.

Scholar. If you set forth an example hereof, I thinke I shall perceiue you.

Master. Take this example: I would multiply 1542 by 365, therefore I set my numbers thus.

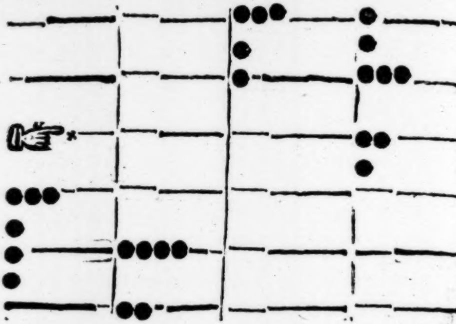


Then first I begin at the 1000 in the highest roome, as if it were the first place, and I take it vp setting downe for it so often (that is once) the Multiplier, which is 365, thus as you see here: where, for the one Counter taken vp from the fourth line, I haue set downe other sixe which make the summe of the multiplier, reckoning the fourth line, as if it were the first, which thing I haue marked by the starre set at the beginning.

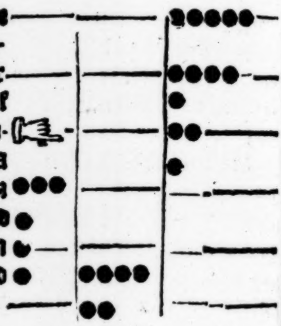


Scholar. I perceive well, for indeed this ſumme that you ſet downe, is 365000: for ſo much doth amount of 1000, multiplyed by 365.

Maſter. Well then go forth, in the next ſpace I finde one Counter, which I remove forward, but take it not up, but (as in ſuch a caſe I muſt) ſet downe the greater halfe of my multiplier (ſeeing it is an odde number) which is 182, and here I do ſtill let that fourth place ſtand as if it were the firſt, as in theſe examples you ſhall ſee.



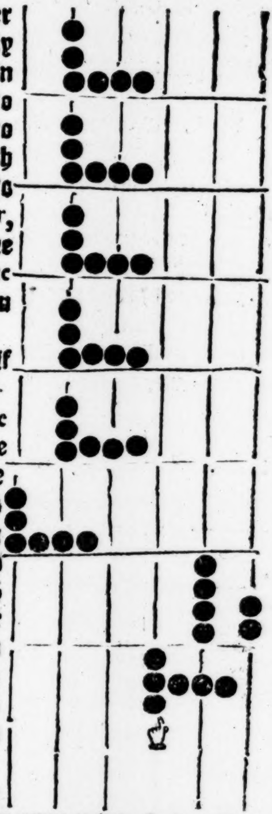
Where I haue set the Multiplicatiō with other but for the ease of your vnderstanding, I haue set a little line betwéen them. Now should they both in one summe stand thus.



How

Howbeit another forme to multiply such Counters in space, is this: first to remoue the finger to the next line beneath the space, and then to take vp the counter, and to set downe the Multiplier fiue times, as here you sée.

Another forme of Multiplication.

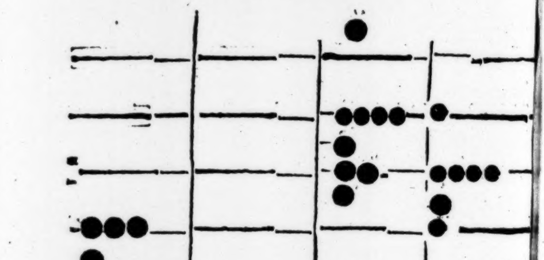


Which summes if you doe adde together into one summe you shall perceiue that it will bée the same that appeareth of the other working before, so that both sorts are to one intent: but as the other is shorter, so this is plainer to reason, for such as haue had small exercise in this Art.

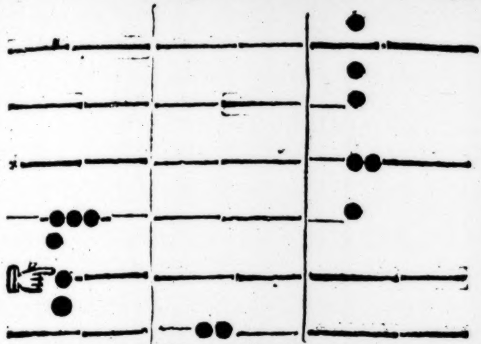
Notwithstanding you may adde them in your minde before you set them downe: as in this example you might

might haue said, fiue times 300 is 1500, and fiue times 60 is 300, also fiue times fiue is 25. which all put together, do make 1825, which you may at one time set downe if you list.

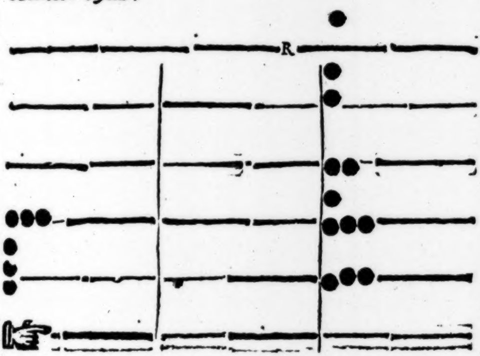
But now to go forth, I must remoue the hand to the next Counters which are in the second line, and there must I take vp those foure Counters, setting downe for them my multiplier foure times seuerally, or else I may gather the whole summe in my mind first, and then set it downe: as to say, foure times 300 is 1200: foure times 60 are 240: and foure times 5 make 20, that is in all 1460: that shall I set downe also, as here you see.



Which if I ioyne in one summe with the former numbers, it will appeare thus.



Then to end this Multiplication, I remoue the finger to the lowest line where are onely 2, then do I take vp, and in their stead do I set downe twice 365, that is 730, for which I set one in the space aboue the third line for 500, and 2 more in the third line with that one that is there already, and the rest in their order, and so haue I well ended the whole summe thus:



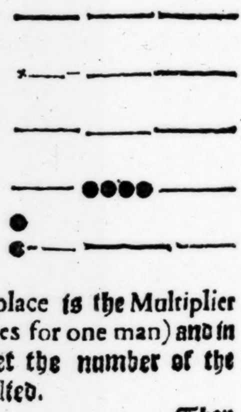
Whereby you ſee, that 1542 (which is the number of yeares ſince Chriſt his incarnation) being multiplied by 365, (which is the number of the dayes in one yeare) doth amount to 562830, which declareth the number of dayes ſince Chriſts Incarnation unto the end of 1542 yeares, beſide 385 dayes, and twelue houres for leape yeares.

Scholar. Now will I proue by another example, as this: 40 Laborers (after 6 pence the day for each man) haue wrought 28 daies, I would know what their wages doth amount unto.

Example of wages.

In this caſe muſt I worke doubly: firſt I muſt multiply the number of the Laborers, by the wages of a man for one day, ſo will the charge of euery day amount.

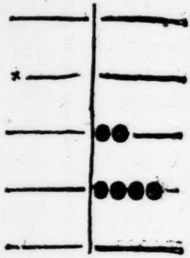
Then ſecondly ſhall I multiply the charge of one day by the whole number of dayes, and ſo will the whole ſumme appeare: Firſt therefore I ſhall ſet the ſummes thus.



Where in the firſt place is the Multiplier (that is one dayes wages for one man) and in the ſecond ſpace is ſet the number of the workmen to be multiplied.

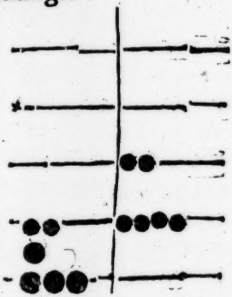
Then

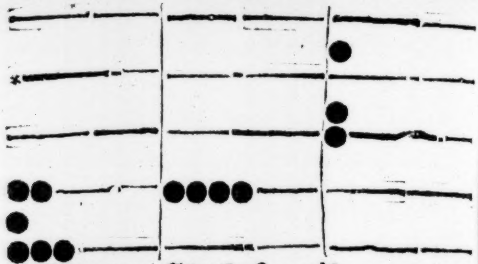
Then say: If 6 times 4 (reckoning that second line as the line of vnites) maketh 24, for which summe I should set two Counters in the third line, and 4 in the second, therefore do I set two in the third line, & let the foure stand still in the second line thus.



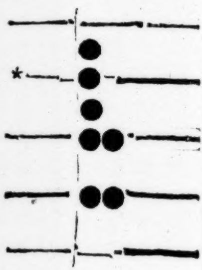
So appeareth the whole dayes wages to be 240 pence, that is 20 shillings.

Then do I multiply againe the same summe by the number of dayes, and first I set the numbers thus: then because there are Counters in diuers lines, I shall begin with the highest, and take them vp, setting for them the Multiplier so many times as I tooke vp Counters, that is twice, then will the summe stand thus.





Then come I to the ſecond line, and take vp thoſe 4 Counters ſetting for them the multiplier 4 times, ſo will the whole ſumme appeare thus:



So is the whole wages of 40 workemen for 28 dayes after 6 pence each day for a man, 6720 pence, that is, 560 ſhillings or 28 pound:

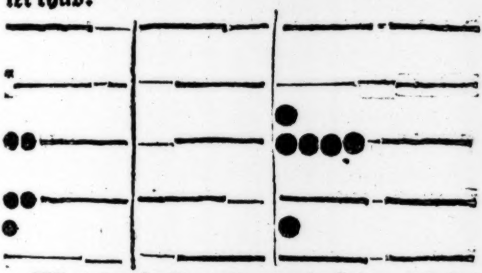
Maſter. Now if you would proue Multiplication: the ſureſt way is by Diuiſion: therefore will I ouerpaſſe it till I haue taught you the Art of Diuiſion, which you ſhall worke thus.

Diuiſion.

Diuision.

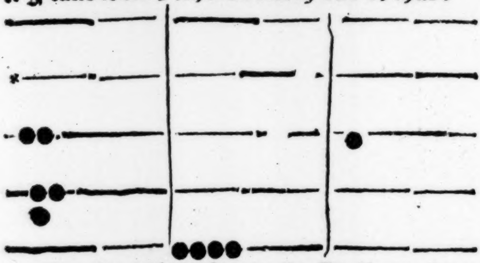
FIrst set downe the Diuisor, for feare of forgetting, and then set that number that shall bee diuided at the right side so farre from the Diuisor,, that the quotient may bee set betweene them: as for example.

If 225 sheepe cost 45 pound, what did every sheepe cost? To know this, I would diuide the whole summe, that is 45 pound, by 225, but that cannot be: therefore must I first reduce that 45 pound, into a lesser denomination, as into shillings, then I multiply 45 by 20, and it is 900: that summe shall I diuide by the number of sheepe, which is 225, these two numbers therefore I set thus: An example of sheepe.



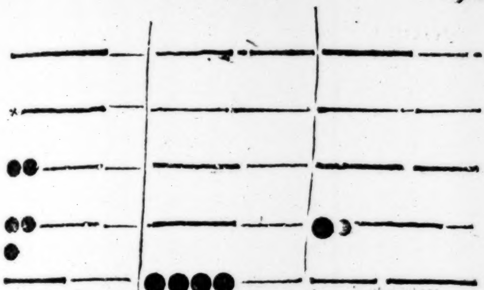
Then begin I at the highest line of the diuidend, and seeke how oft I may haue the

Diuiſor therein, and that I may do foure times: then ſay I, foure times 2 are 8, which if I take from 9, there reſteth but 1, thus:



And becauſe I found the Diuiſor 4 times in the diuidend, I haue ſet, as you ſée, 4 in the middle roome, which is the place of the quotient: but now muſt I take the reſt of the Diuiſor as often out of the remainer: therefore come I to the ſecond line of the diuiſor, ſaying two times 4 make 8, take 8 from 10, and there remaineth 2, thus:

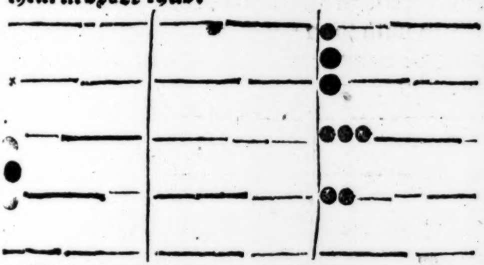
Then come I to the loweſt number, which is 5, and multiply it 4 times, ſo is it 20, that take I from 20, and there remaineth nothing, ſo that I ſée my quotient to be 4, which are in value ſhillings, for ſo was the diuidend: and thereby I know that if 225 ſheepe coſt 45 pound, euery ſheepe coſt 4 ſhillings.



Scholar. This can I do as you ſhall per-ceiue by the example. If 160 Souldiers do ſpend euery moneth 68 pound, what ſpendeth each man?

Examples of ſouldiers wages.

Firſt, becauſe I cannot diuide the 68 by 160, therefore I will turne the pounds into pence by multiplication, ſo ſhall there bee 16320 pence: now muſt I diuide the ſumme by the number of Souldiers, therefore I ſet them in order thus:

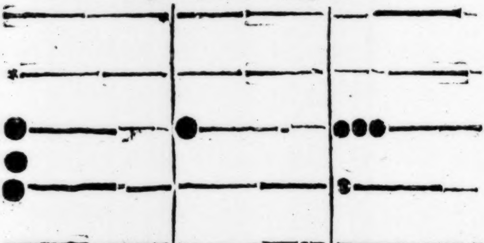


Then begin I at the higheſt place of the diuidend, ſeeking my Diuiſor there, which I finde once, therefore I ſet 1 in the nether line.

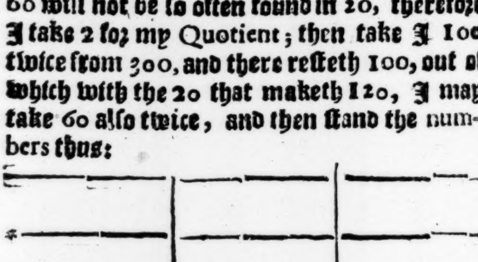
Master. Not in the neather line of the whole summe, but in the neather line of that worke, which is the third line.

Scholar. So standeth it with reason.

Master. Then thus do they stand.



Then seeke I againe the rest, how often I may finde my diuisor: and I see that in 300 I might finde 100 three times: but then the 60 will not be so often found in 20, therefore I take 2 for my Quotient; then take I 100 twice from 300, and there resteth 100, out of which with the 20 that maketh 120, I may take 60 also twice, and then stand the numbers thus:



Where

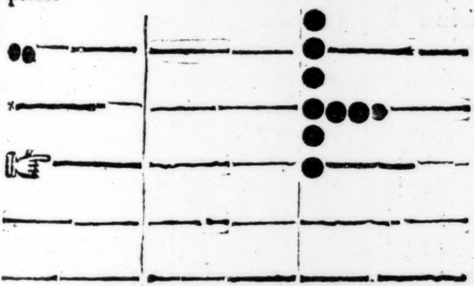
Where I haue set the quotient 2 in the lowest line: so is euery Souldiers portion 102 pence, that is 8 shillings o pence.

Master. But yet because you may iustly perceiue the reason of Diuision, it shall be good that you set your diuisor still against those numbers from which you do take it, as by this example I will declare.

If the purchase of 200 acres of ground did cost 260 pound, what did one acre cost?

First will I turne the pounds into pence, so will there be 69600 pence. Then in setting down these numbers, I shall do thus: An example of purchase.

First set the diuidend on the right hand as it ought, and then the diuisor on the left hand against those numbers from which I intend to take him first, as here you see where I haue set the diuisor two lines higher then his owne place.

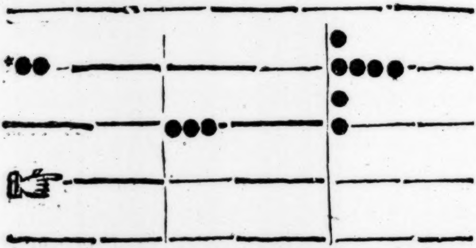


Scholar. This is like the order of diuision by the pen.

Master. Truth you say, and now must I

ſet the quotient of this worke in the third line, for that is the line of vnites in reſpect of the Diuiſor in this worke.

Then I ſee how often the Diuiſor may bee found in the diuidend, and that I find 3 times, then ſet I 3 in the third line for the quotient: and take away that 60000 from the diuidend and further I ſet the Diuiſor one line lower: as you ſee here.



And then ſeeke I how often the Diuiſor will be taken from the number againſt it, which will be foure times, and 1 remaining.

Scholar. But what if it chance that when the Diuiſor is ſo remoued, it cannot be once taken out of the diuidend againſt it?

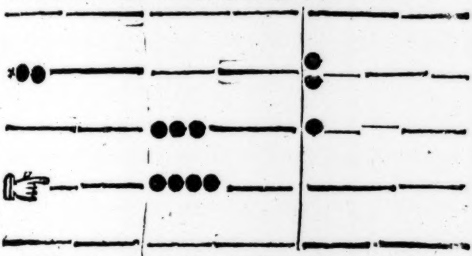
Maſter. Then muſt the Diuiſor be ſet in another line lower.

Scholar. So was it in Diuiſion by the pen, and therefore was there a Cipher ſet in the Quotient; but how ſhall that be noted here?

Maſter. Here needeth no token, for the lines do repreſent the places, onely looke that you ſet your Quotient in that place which

ſtandeth

standeth for vnites in respect of the Diuisor. But now to returne to the example: I finde the Diuisor foure times in the diuidend, and 1 remaining: for 4 times 2 make 8, which I take from 9, and there resteth 1, as this figure following sheweth: and in the middle space for the Quotient, I set 4 in the second line, which is in this worke the place of vnit.



Then remoue I the Diuisor to the next lower line, and seeke how often I may haue it in the Diuidend, which I may do here 8 times iust, and nothing remaine, as in this forme.

Where you may see ẏ the whole quotient is 348 pence, that is 29 shilings, wherby I know

that

that so much cost the purchase of one Acre.

Scholar. Now resteth the proofs of Multiplication, and also Diuision.

Master. Their best proofs are each one by the other, for Multiplication is proued by Diuision, and Diuision by Multiplication, as in the work by the pen you learned.

Scholar· If that be all, you shall not need to repeat againe that which was sufficiently taught already: and except you will teach me any other feat, here may you make an end of this Art, I suppose.

The reason of all the former rules.

Master. So will I do as touching whole number, and as for broken number, I will not trouble your wit with it, til you haue practised this so well that you be full perfect, so that you need not to doubt in any point that I haue taught you, and then may I boldly instruct you in the Art of fractions or broken numbers: wherin I will also shew you the reasons of all that you haue now learned. But yet before I make an end, I will shew you the order of common casting, wherein are both pence, shillings, and pounds, procéeding by no grounded reason, but onely by a receiued forme, and that diuersly, of diuers men: for the Merchants vse one forme, and Auditors another.

Merchants

Merchants vſe.

But firſt for Merchants forme, marke this example here, in which I haue expreſſed this ſumme 198 pounds, 19 ſhillings, 11 pence. *So that you may ſee that the loweſt line ſerueth for pence, the next aboue for ſhillings, the third for pounds, and the fourth for ſcores of pounds.*

Merchants Accompt.

And further you may ſée that the ſpace betwéene pence and ſhillings may receiue but one counter (as all other ſpaces likewiſe do) and that one ſtandeth in that place for 6 pence.

Likewiſe betwéene the ſhillings and the pounds one counter ſtands for 10 ſhillings.

And betwéene the pounds and 20 pounds, one Counter ſtandeth for 10 pounds.

But beſide thoſe, you may ſée at the left ſide of ſhillings, that one number ſtandeth alone, and betokeneth 5 ſhillings.

So againſt the pounds, that one Counter ſtandeth for 5 pound. And againſt the 20 pounds, the one Counter ſtandeth for fiue ſcore pounds, that is, 100 pounds: ſo that euery ſide Counter is fiue times ſo much as one of them againſt which he ſtandeth.

Auditors

Auditors Accompt.

Auditors Accompt.

NOw for the Accompt of Auditors, take this example.

Where I haue expreſſed the ſame ſumme 198 pound—19 ſhillings—11 pence.

But here you ſée the pence ſtand towards the right hand, and the other increaſing orderly towards the left hand.

Againe you may ſée, that Auditors will make two lines (yea and more) for pence, ſhillings, and all other values, if their ſummes extend thereto. Alſo you ſée that they ſet one Counter at the right end of each row, which ſo ſet there ſtandeth for fiue of that roome, and on the left corner of the row it ſtandeth for 10 of the ſame row.

But now if you would adde, or ſubtract after any of both thoſe ſorts, if you marke the order of the other ſeate which I taught you, you may eaſily do the ſame here without much teaching: for in Addition, you muſt firſt ſet downe one ſumme, and to the ſame ſet the other orderly, and in like manner, if you haue many. but in Subtraction, you muſt ſet downe firſt the greateſt ſumme, and from it muſt you

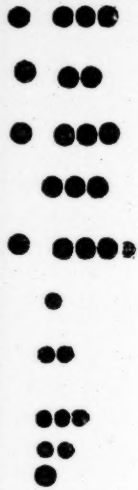
abate

abate the other, euery Denomination from his due place.

Scholar. I do not doubt but with a little practise I shall attaine these both: but how shall I multiply and diuide after these formes?

Master. You cannot duely do any of both by these sorts, therefore in such case you must resort to your other Arts.

Scholar. They that vse such accounts that it exceed 200 in the summe they set not 5 at the left hand of the scores of pounds, but they set all the hundreds in another farther row, and 500 at the left hand thereof, and the thousands they set in a farther row yet, and at the left side thereof they set the 5000, and in the space ouer, they set the 10000, and in a higher row 20000, which all I haue expressed in this example, which is 97869 pounds, 12 shillings, 9 pence, ob. q. Ninety seuen thousand, eight hundred threescore and nine pounds, twelue shillings and nine pence halfe penny farthing, for I had



not told you before, where neither how you ſhould ſet downe farthings, which (as you ſée here) muſt bée ſet in a voide place ſideling beneath the pence, for a farthing, one counter, ob. 2 counters, for ob, farthing 3 counters, and more there cannot be: for 4 farthings make 1 d, which muſt be ſet in his due place.

And if you deſire the ſame ſumme after Auditors manner, loe here it is.

But in this thing you ſhall take this for ſufficient, and the reſt you ſhall obſerue as you may ſée by the working of each ſort, for the diuers wits of men haue inuented diuers and ſundrie wayes, almoſt innumerable.

THE SECOND PART OF ARITHMETICKE, touching Fractions, briefely set forth.

Scholar.

Albeit I perceiue your manifold businesses doth so occupie or rather oppresse you, that you cannot as yet compleatly end the Treatise of *Fractions Arithmeticall, which you haue prepared, wherein not onely sundrie works of Geometry, Musicke, and Astronomy bee largely set forth, but also diuers conclusions and naturall works, touching* Arithmeticall fractions.

mixture.

mixtures of Metals and compositions of medicines, with other strange examples. Yet in the meane season, I cannot stay my most earnest desire, but importunely craue of you some briefe preparation toward the vse of Fractions, whereby at the least I may be able perfectly to understand the common works of them, and the vulgar vse of those rules, which without them cannot well be wrought.

Master. If my leasure were as great as my will is good, you should not néed to vse any importunate crauing, for the attaining of that thing, whereby I may bée perswaded that I shall any wayes profit the Commonwealth, or helpe the honest studies of any good members in the same: wherefore while mine attendance will permit me to walke and talke, I am well willing to helpe you as I may.

Therefore first to begin with the explication of this name Fraction, what take you it to be?

What a Fraction is

Scholar. Marry sir, I think a Fraction (as I haue heard it often named) to be a broken number, that is to say, to be no whole number but part of a number.

Master. A Fraction indéed is a broken number, and so consequently the part of another number: but that must be understood of such another number as cannot bée diuided into any other parts then Fractions: for although I may take the third part of 60, or the

the fourth part of it, and so of other parts diuersly, yet these parts be not properly, nor ought not to be called Fractions, because they may be expressed by whole numbers, for the third part of it is 20, the fourth part is 15, the twelfth part is 5, and so forth of other parts, all which be whole numbers.

Wherefore properly a Fraction expresseth the parts or part onely of a vnite, that is to say, that the number which is the whole or entire summe of any Fraction, may not bée greater then one: and therefore it followeth, that no one Fraction alone can bée so great, that it shall make 1, as by example I will declare, as soone as I haue taught you to know the forme how a Fraction is expressed or represented in writing. What a fraction is properly.

Nume-

Numeration.

The expressing of fractions.

BVt first to begin with expressing of a Fraction, *which is the numeration of it: you must vnderstand that a* Fraction *is represented by two numbers set one ouer the other, and a line drawne betweene them as thus,* $\frac{1}{3}$ $\frac{3}{4}$ $\frac{2}{5}$ $\frac{10}{17}$, *which foure fractions you must pronounce thus:* $\frac{1}{3}$ *one third part,* $\frac{3}{4}$ *three quarters,* $\frac{2}{5}$ *two fift parts,* $\frac{10}{17}$ *ten seuenteene parts.*

*S*cholar. I vnderstand the forme of their expression and pronunciation: but their meaning or valuation seemeth more obscure. Yet I thinke that by the two first fractions I vnderstand the valuation of the two latter fractions, and consequently of other.

Master. Value them then, that I may perceiue your taking of them.

*S*cholar. $\frac{2}{5}$ betokeneth two fift parts, that is to say, if one be diuided into 5 parts, that fraction doth expresse two of those 5 parts: $\frac{10}{17}$ doth signifie, that if one be diuided into 17 parts, I must take ten of them. And this I gather of the two first examples: for $\frac{1}{3}$, that is, one third part, doth easily declare, that if one thing be diuided into three parts, I must take out one of them: so $\frac{3}{4}$, that is three quarters, doth declare that one being diuided into foure quarters, I must take (for this Fraction) three

of

of those quarters.

If there be no more difficulty in their Numeration, then I pray you go forward to their Addition and Subtraction, and so to the other kindes of works. For I vnderstand that the same kinds of works be in fractions, that be in whole numbers.

Master. There are the same kindes of works in both, albeit the order of them is diuers, as I will anone declare: but yet more in Numeration before we leaue it. You must vnderstand that those two numbers which expresse a Fraction, haue seuerall names, the ouermost, which is aboue the line, is called the Numerator, and the other beneath the line, is called the Denominator.

Numerator.

Denominator.

Scholar. And what is the reason of their diuers names? For (in mine opinion) both be Numerators, seeing both they do expresse the numeration of the Fraction.

Master. You are deceiued: for one onely (which is the ouermost) doth expresse the Numeration, and the Denominator doth declare the number of parts, into which the vnite is diuided, as in this example, when I say: diuide a pound weight of Gold between foure men, so that the first man shall haue $\frac{2}{15}$, the second, $\frac{3}{15}$, the third $\frac{4}{15}$, and the fourth $\frac{6}{15}$.

Now do you perceiue that by the Denominator (which is one in all foure fractions) it is intended that the pound weights should be diuided into so many parts, I meane 15.

and by the foure seuerall Numerators is limited the diuers portion that each man should haue, that is, that when the whole is parted into 15, the first man shall haue two of those 15 parts: the second man three of them: the third man foure: and the fourth man six. And so may you see the seuerall offices (as it were) of those two numbers, I meane of the Numerator and the Denominator.

And hereby you perceiue that a man can haue no more parts of any thing then it was diuided into, neither yet aptly so many: so that it were vnaptly said: You shall haue $\frac{15}{15}$ that is 15, fifteene parts of any thing, seeing it were better said, you shall haue the whole thing.

Scholar. So doth it appeare reasonably, for the labour is vaine to diuide any thing, and then to apply the Diuision to no vse. And much lesse reasonable were it to say $\frac{16}{15}$: for if the whole be diuided into 15 parts onely, it is not possible to take 16 of them, that is to say, more then all together.

Master. This is true touching the proper and apt vse of the name of a Fraction: yet improperly (and after a vulgar acceptation for easinesse in worke) both those formes bee called Fractions, because they be written like fractions, although they be none indeed: for $\frac{15}{15}$, and generally all such other, where the Numerator and Denominator be equall, are not fractions, but the whole thing with all his parts: And so $\frac{14}{12}$ is not to be called a fracti-

on,

on, but a mixt number, of a whole number and a fraction, for it is as 1 $\frac{2}{12}$, that is one whole and $\frac{2}{12}$ parts, as shall be declared in Reduction. Therefore they do abuse the names that call them Fractions, where the Numerator is either equall, or greater then the Denominator.

An improper fraction of a mixt number.

Scholar. But is there any needfull cause, why they should so abuse the name?

Master. There is cause why they shall sometimes for easinesse in worke write some numbers after that sort like fractions but they néeded not to call them fractions, but (as they bee) whole numbers or mixt numbers, (that is, whole numbers with Fractions) expressed like fractions, or as improper fractions.

Now must you understand, that as no fraction properly can be greater then one, so in smalnesse under one the nature of fractions doth extend infinitely, as the nature of whole numbers is to increase above one infinitely, so that not onely one may be divided into infinite fractions or parts, but also every fraction may be divided into infinite Fractions or parts, which commonly be called Fractions of fractions, and they be expressed diversly: as for example, $\frac{3}{4}$ of $\frac{2}{3}$ of $\frac{1}{2}$, that is three quarters, of two third parts, of one halfe part. Whereby is signified, that if one be divided into two halfes, and the one halfe into three parts, and two of those

Fractions of fractions.

three parts be diuided ioyntly into foure quarters, this fraction of fractions doth represent three of those quarters.

Scholar. I pray you let me proue by an example in common money, whether I do rightly understand you or no. One Crowne which I take for an vnite, doth containe 60 pence, therefore the halfe of it is 30 pence, $\frac{2}{3}$ of that halfe is 20 pence, whereof $\frac{3}{4}$ is fifteene pence, so then 15 pence is $\frac{3}{4}$ of $\frac{2}{3}$ of $\frac{1}{2}$ of a Crowne: and so is 3 pence $\frac{3}{4}$ of $\frac{2}{3}$ of $\frac{1}{2}$ of a shilling.

Master. You perceiue this well enough: yet this note I giue you by the way, that the forme of expressing these fractions is voluntary, and hath no other reason than the will of the Diuisor, which forme many follow: for some expresse them thus, $\frac{3}{4}\frac{2}{3}\frac{1}{2}$ without any figure of distinction betwéene them, which forme also many follow. Some other do make lines betwéene euery fraction, and adde words of distinction, after this sort, $\frac{3}{4}$ of $\frac{2}{3}$ of $\frac{1}{2}$, which forme is best.

Some other expresse them thus in slope forme, to distinct them from Fractions of one whole number, for if they were set in one right line thus: $\frac{3}{4}, \frac{2}{3}, \frac{1}{2}$, then ought it to be pronounced, three quarters and two third parts, and a halfe, which maketh almost two whole vnites, lacking but one twelfth part. And so is it nothing agréeable

$\frac{3}{4}$

$\frac{2}{3}$

$\frac{1}{2}$

with

with the other fraction of fractions: wherefore it is a great overſight in certaine learned men, which do expreſſe them ſo confuſedly with ſuch ſeuerall fractions, that a man cannot know the one from the other.

Therefore ſome men (as Stifelius*) do expreſſe without a line, numbers of proportion being applied to* Addition *or* Subtraction, *becauſe they muſt be taken as two, where the line in* fractions *maketh them to be taken for one: For of the* Numerator *and* Denominator *is made one number.*

Scholar. Then I perceiue there be three ſeuerall varieties in fractions: firſt, when one onely fraction is ſet for one number, as $\frac{4}{5}$ that is, foure fift parts. The ſecond is, when there be ſet two or more ſeuerall fractions of one number, as $\frac{4}{9}$, $\frac{2}{5}$, that is, foure ninth parts, and two fifth parts. The third ſort is fractions, of fractions, as $\frac{4}{9}$ of $\frac{2}{5}$, that is, foure ninth parts of two fifth parts. Three ſeuerall varieties.

Maſter. You haue ſaid well, if you underſtand well your owne words.

Scholar. If it ſhall pleaſe you I will by an example in the parts of an old *E*nglish Angel, expreſſe my meaning.

Maſter. Let me heare you.

Scholar. The old Engliſh Angel did containe 7 ſhillings 6 pence, that is, 90 pence: Now $\frac{4}{5}$ of it is 72 pence. And of the ſame 90 pence, if I take $\frac{4}{9}$ and $\frac{2}{5}$, that is, foure nine parts, and two fift parts, $\frac{4}{9}$ is 40, and $\frac{2}{5}$ is 36, which both make 76: but if I take $\frac{4}{9}$ of $\frac{2}{5}$, that

is, foure nine parts of two fifth parts, séeing $\frac{2}{5}$ is but 36, then $\frac{4}{9}$ of 36 will yéeld but 16, for $\frac{1}{9}$ of 36 is but 4, and that taken foure times maketh 16.

Master. This is plainly expressed, and truly, and hereby (I doubt not) but you do perceiue, that as great a difference, as is betwéene 16 and 76, so much difference is betwéene those two fractions $\frac{4}{9}$ and $\frac{2}{5}$: and $\frac{4}{9}$ of $\frac{2}{5}$.

And now that you understand these varieties, I will procéed to the rest of the works: first, admonishing you, that there is another order to bée followed in Fractions, then there was in whole numbers: for in whole numbers this was the order: Numeration, Addition, Subtraction, Multiplication, Diuision, and Reduction: but in fractions (to follow the same aptnesse in procéeding from the easiest works to [illegible] harder) we must use this order of works, Numeration, Reduction, Addition, Subtraction, Multiplication and Diuision.

The order of works in fractions.

Scholar. That Addition and Subtraction should go together, and Diuision to follow Multiplication, naturall order doth perswade; but why Reduction should be first in order here next to Numeration, and Addition, and Subtraction in the middle, I desire to understand the reason.

Master. As in the Art of whole numbers, Order would reasonably begin with the easiest, and so go forward by degrées to the

hardest:

hardeſt: euen reaſon teacheth in fraction the like order. And conſider that Addition or Subtraction of fractions can very ſeldome bée wrought without Reduction: & contrariwiſe, Reduction may bée wrought without this forme of Addition or Subtraction: therefore was it orderly required, that Reductiō ſhould go before Addition and Subtraction, and this reaſon ſerueth for the placing of Reduction before the other.

Scholar. Then, if Reduction be the eaſieſt, I pray you declare the forme of it firſt by Rule, and then by example.

Maſter. Your requeſt is good.

Redu-

Reduction of Fractions.

Of Reduction of Fractions, there are fiue varieties.

THerefore will I now declare the diuersities of Reduction of fractions, which commonly hath fiue varieties, or formes.

First, when there be sundrie
1 *Fractions of one intire vnite, they must be reduced to one denomination, and also into one Fraction.*

2 *Secondly, when there be proponed fractions of fractions, they must be reduced likewise into one fraction: for otherwise they cannot be brought into one denomination.*

Thirdly, when an improper fraction is proponed,
3 *that is to say, a fraction in forme, which indeed, is greater then an vnite: it must be reduced into apt forme, expressing the vnite or vnites of it, and the proper fraction distinctly. And sometimes also it shall be needfull to conuert such a mixt number of vnites with fractions, into the forme of a fraction, that is, into an improper fraction: which two formes I esteeme but as one, because they worke one kinde of number.*

4 *Fourthly, there happeneth sometimes fractions to be written in great numbers, which might bee written in lesser numbers: therfore is there a meane to reduce such great numbers into their smallest termes.*

5 *Fifthly, when any fraction betokeneth the parts of a whole thing, which hath by common partition*

certaine

certaine parts, but none of like denomination with that Fraction, then may you reduce the said fractions into another, whose denomination shall expresse the common parts of that whole thing.

Scholar. This distinction in doctrine delighteth me much, but more with hope then present fruit: for as yet I do not understand scarcely the varieties, and much lesse the practise and use of their works.

Master. Reduction is an orderly alteration of numbers, out of one forme into another, which is never done orderly but for some needfull use, as in every of the said five severall formes, I will distinctly declare.

First therefore, when two, or more severall fractions of any vnite be proponed; as for example, $\frac{3}{16}$ and $\frac{4}{6}$, because it is hard to tell what proportion of the intire number those two Fractions do expresse, therefore was Reduction diuised, to be a meane whereby these severall fractions might be brought into one denomination and fraction. The first forme of Reduction

And in these fractions, this is the Art for bringing them to one denomination.

Multiply first the denominators together, and the totall thereof you shall set twice down vnder two severall lines for two new denominators, or rather for one common Denominator. Then multiply the Numerator of the first fraction, by the denominator of the second, and set the totall thereof for the Numerator ouer the first line. Likewise multiply the Numerator of the second fraction by the denominator How to reduce fractions, of diuers denominations into one denomination.

tor

tor of the first, and set that totall ouer the second line for the Numerator of that fraction: and so are these two first fractions of seuerall denominations, brought to one denomination.

Scholar. If I vnderstand you, as I thinke I do, my example shall declare the same. The fractions which you proponed were these, $\frac{3}{16}$, and $\frac{4}{6}$, whose denominators (being 16 and 6) I multiply together, and there amounteth 96, which I set vnder two lines thus: $\frac{}{96}\ \frac{}{96}$.

Then I multiply the Numerator of the first fraction by the Denominator of the second saying, 3 into 6 maketh 18, that I set ouer the first line for a new Numerator, and it will be thus $\frac{18}{96}$.

Likewise I multiply the Numerator of the second fraction by the Denominator of the first saying 4 times 16 maketh 64, that I set for the second Numerator, and the fraction will appeare thus, $\frac{64}{96}$.

So that both fractions brought to one denomination, must stand thus $\frac{18}{96}$ and $\frac{64}{96}$.

Master. You haue done well.

Scholar. I beséech you let me examine it after my accustomed forme, by common parts of coyne or other measure.

Master. Go to.

Scholar. I haue a péece of Gold which is accounted worth 8 shillings, and containeth 96 pence, whereof $\frac{1}{16}$, that is, the sixtéenth part, is 6 pence, and $\frac{3}{16}$ is 18 pence, that

is

is $\frac{18}{96}$. Againe $\frac{1}{6}$ of the same péece of gold is 16 pence, so that $\frac{4}{6}$ parts maketh 64 pence, that is $\frac{64}{96}$. And so I finde the summes to agrée with the other before.

Master. *So haue you now the Art to bring two such fractions into one denomination. And if there be more then two, then must you multiply all the Denominators together, and set the totall thereof so many times downe as there bee fractions; and then to get for each one a new Numerator, multiply the Numerator of the* first, *by the Denominator of the* second, *and the totall thereof multiply by the Denominator of the* third, *and so forth, if there be more. Likewise multiply the Numerator of the* second, *by the Denominator of the* first, *and the totall thereof by the Denominator of the* third. *And in the same sort multiply the Numerator of the third into the Denominator of the* first, *and the totall thereof into the Denominator of the* second, *and so forth, if there were moe. So these three fractions* $\frac{2}{5}\ \frac{3}{4}\ \frac{2}{3}$ *do make by Reduction these other three fractions of Denomination* $\frac{24}{60}, \frac{45}{60}, \frac{40}{60}$. *All which you may bring into one fraction by adding the Numerators together, and putting the totall for the totall Numerator, reseruing still that same common Denominator. And those three fractions make one improper fraction, thus:* $\frac{109}{60}$.

Note the reduction of three Fractions or more, into one.

Scholar. All this I perceiue, and also that this last fraction is more then an vnite, and therefore you did call it an improper fraction.

Master.

Master. There be certaine other formes of working in this Reduction, which I will briefly touch also, to giue you an occasion to exercise your wit therein.

The first variety of Reduction

The first variety is this: When you haue made and written downe your common Denominator (as I haue taught before) then to get a Numerator for the first, do thus: Diuide the common denominator by the denominator of the first fraction, and the quotient multiplied by the Numerator of the same, yeeldeth a new Numerator for the first new fraction, So likewise do with the second and the third, and with all the residue, if there be more.

Scholar. That will I proue in your last example of these three fractions $\frac{2}{5}\frac{3}{4}\frac{2}{3}$. When the denominators be multiplied, they make 60: for 5 into 4 maketh 20, & 20 by three yeeldeth 60, that I set down three times thus: $\frac{}{60}, \frac{}{60}, \frac{}{60}$, then to haue a Numerator, for the first, I must diuide 60 by 5 (the denominator of the first) and the quotient is 12, which I must multiply by 2 (the Numerator first) and that maketh 24, and so haue I for the first fraction $\frac{24}{60}$.

Likewise for the second fraction: I diuide 60 by 4, and there commeth 15, which I multiply by 3, and so haue I 45, for the second fraction $\frac{45}{60}$. Then for the third in like sort will come $\frac{40}{60}$.

The second variety.

Master. *Another way is this: If it happen so, that the lesser denominator can by any multiplication make the greater, then note the multiplier, and by it multiply the Numerator ouer that lesser denomina-*

tor,

tor, and for the lesser denominator put the greater, as thus in these two fractions $\frac{3}{12}$ and $\frac{2}{3}$, three being the lesser denominator multiplied by 4, will make 12, which is the greater denominator: therefore by the same 4 I do multiply 2 which is the Numerator over 3, and that maketh 8: vnder which I do put 12, being the greater denominator, which is also made by multiplication of 4 into 3, and so haue I these two fractions $\frac{3}{12}$, $\frac{8}{12}$, thus shortly reduced, without altering the one fraction.

Scholar. **This I vnderstand.**

Master. *Then marke this third way: If the denominators do not happen so, that one by multiplication may make the other, then looke whether they both may be parts of any other one number, as in $\frac{5}{12}$, and $\frac{7}{18}$, although the lesser taken but twice, be too much to make 18, yet they both may be parts vnto 36: therefore looke how many times twelue is in 36, and that quotient being multiplied by the Numerator ouer 12, the totall shall be put in stead of the Numerator ouer 12, and for 5 put 15, thus $\frac{15}{36}$. So likewise looke how often is 18 in 36, because it is twice, therefore by 2 multiply 7, which is ouer 18, and it will be 14: set that for the Numerator, and in stead of 18 put 36; and then your Fractions reduced stand thus, $\frac{15}{36}$, $\frac{14}{36}$, in stead of $\frac{5}{12}$ and $\frac{7}{18}$.* The third variety.

And if you will proue whether you haue wrought well or no, that may be proued by Reduction **of them againe to their former** denominations, **which** Art, **shall be taught in the** fourth kinde of Reduction, **where greater** termes **of** Fractions **be reduced into smaller in** Proofe.

number

☞ number, but no smaller in proportion. And if in such Reduction the same termes or numbers come againe that were before, then is the worke good, else not.

Scholar. Sir, I heare your words, but I do not understand many of them: which if it please you, declare.

Master. With a good will, when convenient place serueth, but that must be in the said fourth kinde of Reduction; which teacheth how to reduce fractions of fractions into one fraction, and so to one Denomination.

The second form of Reduction of fractions of Fractions into one fraction and Denomination.

When Fractions of Fractions be proponed, you shall multiply the Numerators of each into other, and set the totall for the new Numerator, and then multiply all the denominators likewise, and take their totall for the new Denominator, and so are they speedily reduced.

Scholar. If that be all, then I understand it already, as by this example I will declare. These be the fractions, $\frac{3}{4}$ of $\frac{2}{3}$, of $\frac{6}{7}$ of $\frac{7}{9}$ which I would reduce to one Denomonation, and proper simple fraction.

Therefore begin I with the Numerators, and multiply them together, saying, 3 into 2, maketh 6: and 6 by 6, maketh 36, which multipied by 7, yeeldeth 252: that I set ouer a line for the Numerator, thus:

$$\frac{252}{\quad}$$

Then I multiply the denominator, 4 by 3, maketh 12, and that by 7 bringeth 84, which multiplied by 9, yeeldeth 756, the new

new Denominator. And so the whole reduced fraction is this, which is too hard a fraction for me to vnderstand yet. $\frac{252}{756}$

Master. You thinke so, and no maruell, but anon you shal learne to iudge it easily, for this Fraction is no more indeed then $\frac{1}{3}$, although it be in greater termes, and therefore more stranger, and more obscure.

And this sufficeth for this Reduction, saue that I will shew you by a figure of measure, the iust rate and reason of this kinde of fractions, and also the due vnderstanding of their Reduction.

The entire measure parted into 9.

1	2	3	4	5	6	7	8	9
1	2	3	4	5	6	7	$\frac{7}{9}$	
1	2	3	$\frac{6}{7}$					
1	2	3	4	$\frac{2}{3}$				
1	2	3	$\frac{3}{4}$					

Here you see the longest measure, (which standeth for the whole and entire quantity) first parted into 9 diuisions, whereof 7 are seuered by the second measure: and thereof againe are parted out 6, and that 6 being distinct into three parts, two of them are parted by the fourth measure, of which fourth measure being diuided into foure parts, the lowest

measure doth containe $\frac{3}{4}$, so that the same $\frac{3}{4}$ must be named, not $\frac{3}{4}$ of the whole measure, but indeed is $\frac{3}{4}$ of $\frac{2}{3}$ of $\frac{6}{7}$ of $\frac{7}{9}$.

Scholar. This Example is so sensible, that I cannot chuse but see it. And furthermore see also, that the same fraction is equall to $\frac{3}{9}$ of the entire measure, as the lines which runne up and downe do expressly set forth. Also I see here that $\frac{2}{3}$ of $\frac{6}{7}$ is equall to $\frac{4}{9}$. And further yet, that $\frac{3}{7}$ of $\frac{7}{9}$ is equall to $\frac{6}{9}$ or $\frac{2}{3}$.

Master. I am glad that you see it so well, not doubting but you wil gather greater light of knowledge hereby.

The third forme of Reduction of improper fracti-ons.

But now it is time that we come to the third forme of Reduction, which teacheth of improper Fractions, that is to say, mixt numbers of vnites & Fractions, although they appeare like Fractions, as this $\frac{26}{5}$, which doth include 5 vnites wholly, and $\frac{1}{5}$ ouer. Wherefore first you shall know them, by that the Numerator is greater then the Denominator.

Scholar. Indeed Sir, that appeareth reasonable, that if the Numerator do expresse more parts to be taken of any vnite, then the Denominator doth signifie that vnite to be diuided into, it must needs follow that such a fraction importeth more then the whole, that is to say, the whole with certaine parts ouer: but what Reduction is there in it?

Two seuerall wayes in this Reduction.

Master. There be two seuerall kindes of Reduction, concerning such Fractions.

Some-

Sometimes it shal be needfull to conuert these fractions into vnites, and the proper fractions that will remaine. And sometimes contrariwise, it shall be meet to reduce mixt numbers, that is, vnites written with fractions, into the forme of one simple fraction, and so be there two wayes.

Scholar. What is the meane of the first way to turne improper fractions into vnites with their proper fractions?

Master. *That is thus. Your Numerator being greater then the Denominator, must be diuided by the same Denominator, and the quotient thereof expresseth the vnites: the Remainer shall be put for the Numerator of the fraction that resteth, and the Denominator must be the same that was before.*

The fifth way.

Reduction of improper fractions into vnites, with their proper fractions.

Scholar. For example, I take $\frac{17}{5}$. And diuiding 17 by 5, the quotient will be 3, and there will remaine 2.

Master. That must you write thus, $3\,\frac{2}{5}$ where (you see) I haue written 3 without any line, as entire numbers ought to be written, and the 2 that remained I haue set ouer the former Denominator with a line as a proper fraction. And this number doth signifie now three vnites, and $\frac{2}{5}$ of one.

Scholar. Then if I would by vnites here vnderstand crownes, so it were 3 Crownes, and $\frac{2}{5}$ that is 2 s.

Master. Euen so, and therefore $\frac{17}{5}$ did signifie the same. But this happeneth sometimes

 that

that when the reduction is so wrought, there remaineth nothing. And then it is not a mixt number, but a simple intire number, represented like a fraction.

The second way.

Scholar. As $\frac{15}{5}$ will make 3 iust, and $\frac{18}{3}$ will make euen 6. This I will remember. But now, what is the second forme of Reduction that you speak of for these sorts of fractions?

Reduction of whole numbers either alone, or ioyned with fractions into improper fractions.

Master. *Whensoeuer you haue any of these two sorts of numbers, that is to say, whole numbers without fractions, or whole numbers with fractions, and you would turne them into the forme of a fraction, you must multiply the whole number by that denominator which you will haue to remaine still, and to the totall thereof adde the Numerator, which you haue already, and all that shall you set for the new Numerator, keeping still the former Denominator: as if you haue* $6\frac{3}{4}$ *which you would conuert into an improper fraction, you must multiply 6 by 4, whereof commeth 24, and thereto adde the numerator which is 3, and so haue you 27 for the numerator, and 4 still for the denominator.*

Scholar. Then is $\frac{27}{4}$ equall to $6\frac{3}{4}$.

Note.

Master. Euen iust, and so backward (as appeareth by the former Reduction) $6\frac{3}{4}$ maketh $\frac{27}{4}$. And thus one of these Reductions may be the proofe of the other worke.

Scholar. This I perceiue: But now if you would turne whole numbers without fractions into any fractions, I see not how that may be done, because there is no Denominator to make the multiplication by.

Master.

Master. That is well marked: but this you know, that no man intendeth to turne any whole number into a fraction, but he hath in his minde that Denominator by which the multiplication must be made: for the proofe whereof I set downe 7, which is a whole number. And if you will haue this number conuerted into any certaine fraction, will me to do it.

Scholar. I pray you reduce 7 into a Fraction.

Master. Then you care not what the Fraction be, so it be some Fraction.

Scholar. No, I passe not for the sort of the Fraction.

Master. Then how can you think that you require me to do any thing certaine, when you leaue me to do as I list? And seeing you stand at that stay, whether thinke you that I must first intend in minde what fraction I will make of it, before I can do it indeed?

Scholar. Else you should do ignorantly.

Master. Then will I limit my selfe (seeing you will not) to turne it into quarters. And therefore I multiply 7 by 4 (which is the Denomination of quarters) and there amounteth 28 to be set for the Numerator, and the 4 must be set for the Denominator, and the fraction will be thus $\frac{28}{4}$.

Scholar. Indeed I perceiue this to be reasonable, for without much triall I understand that $\frac{28}{4}$ of any thing doth make 7. And so then

if I would turne 8 into 5 parts, it will make $\frac{40}{5}$, which is all one with 8: for 8 crownes turned into 5 parts, (that is, into shillings) will make 40 shillings, that is $\frac{40}{5}$ of a Crowne.

The fourth forme of reduction.

Master. *Seeing you understand now these three kindes of Reduction, I will declare unto you the fourth kinde, that is, when fractions be written in greater termes then they need, how they may bee brought to lesser termes.*

Scholar. To write any thing in greater termes then needeth, seemeth to be a fault, and so this Rule seemeth to amend that fault.

Master. It were a fault to do any thing without need, which after must be redressed: but in this case it is not so, neither did I say absolutely (as you do) that it needeth not to expresse those fractions in so great termes, but that the fractions do not need, I meane for their value to be understood: but yet it may be needfull for the ease of these workes whereto they be applyed, as for example: In the first kinde of Reduction this was your owne example: $\frac{3}{16}$ and $\frac{4}{6}$, which when you would reduce, you were faine to turne them first into one denomination, and so appeared they thus: $\frac{18}{96}$, and $\frac{64}{96}$, where the fractions (for their owne understanding) needed not to be turned out of smaller termes into greater, but yet the easinesse of working needed it.

Scholar. Sir, I understand now, not onely the difference of this need (for the fractions might better be understood as fractions

seue-

seuerall, each in his value, when they were in lesser terms, although they could not so well be reduced) but also I understand what you meane by greater termes and lesser termes, whereof before I was in doubt: for I see you call the Numerator and Denominator, the termes of the fraction. Termes of fractions.

Master. *I am glad you vnderstand it so well: Now then when you would value any fractions (because they may best be done when the termes are smallest) you shall reduce them to the smallest that you can, which thing you may do thus: Diuide the greatest of any such two termes by the lesser, and if any thing remaine by that remainer, diuide the last diuisor: and if any thing remaine now, by that diuide the first diuisor (which was before the remainer of the last diuision) and so continue still, till nothing do remaine in the diuision: and then marke your last diuisor, for it is the number that will easily reduce your fractions, if you diuide both the numerator and the denominator by the same number, and put for the numerator the quotient of his diuision, and for the denominator also his quotient, that riseth by his diuision.*

Scholar. I take for example $\frac{18}{96}$, and because 96 is the greatest number, I diuide it by 18, and the quotient is 5, & there resteth 6: what shall I do with this Quotient?

Master. Nothing in this worke, but now seeing there remaineth somewhat, by that remainer must you diuide the last Diuisor.

Scholar. If I shall diuide 18, (which was

the last Diuisor) by 6, that was the remainer, so is the quotient 3. and nothing resteth.

Master. As for the Quotient, I omit him yet: but because there doth remaine nothing therefore is 6 (which was your last Diuisor) that number by which you may reduce the Fraction proponed.

Scholar. Then as you taught me, I must diuide the Numerator 18 by 6, and the quotient is 3, which I must put for the Numerator ouer a line, thus: And then by the said 6 must I diuide also the Denominator 96 and the Quotient will be 16, which I must take for the Denominator, and so is the Fraction $\frac{3}{16}$. And so me thinketh this Rule doth proue the work of the first Reduction.

$$\frac{3}{16}$$

Master. That is true, if the first Reduction were made of fractions into their least termes, and else not, without some help, as the second number in that place will declare.

Scholar. The second number was $\frac{4}{6}$ which was turned into $\frac{64}{96}$ by that Rule. Now if I shall by this Rule reduce it againe into the least termes, I must diuide 96 by 64 and there remaineth 32, wherefore I must take that 32 for the Diuisor, to reduce the said fractions. Then do you diuide 64 by 32, and the quotient is 2, which I set for my numerator. Againe, I diuide 96 by 32, and the quotient will be 3, and so I haue but $\frac{2}{3}$.

Master. Muse not at the matter, for you

haue

haue done well enough: but you thinke you haue not the fraction that you looked for, that is, $\frac{4}{8}$, yet haue you one equall to it, as by the parts of a shilling you may proue.

Scholar. Truth it is, for each of them will bring forth 8 pence, so that $\frac{8}{12}$ and $\frac{4}{8}$, and $\frac{2}{3}$, be all three equall. And now I perceiue that because $\frac{4}{6}$ was not written in the least terms, that it might be therefore this Reduction brought forth not it, but that other which is written in the least termes. Now vnderstand I this rule well. But is there any other way to worke this Reduction?

Master. *Yes, but first note this, that if you finde no such Diuisor, to reduce the fraction till you come to* 1, *because one doth make no Diuision, therefore that Fraction is already in his least termes, as by* $\frac{71}{100}$ *you may proue, and so of* $\frac{85}{98}$, *and many other like.*

Another way to Work this Reductiō.

But now for your better aid to finde the due proportion in least termes, with more ease for a young learner, you shall mediate or take the halfe of the Numerator, and also of the Denominator as long as you may vpon a line, alwayes parting them with a right downe dash of your pen as you worke, which may easily be done, if the numbers be euen: as 2. 468, *or* 10, *but if they be odde (though it be but one of them) then must you abbreuiate them by* 3, 5, 7, *or* 9, *&c.*

Note that to mediate any number is to diuide by two.

And because examples do most instruct, I haue here set downe the manner of two or three, whose last number at the end of the line

shew-

sheweth the least terme or valuation of that fraction.

As for example, I would reduce $\frac{288}{576}$ into his least terme or value, whereupon I set forth $\frac{288}{576}$ with a long line drawne from it thus:

288	144	72	36	18	9	3	1
576	288	144	72	36	18	6	2

And because both the Numerator, and the Denominator end in euen numbers, I see this may be abbreuiated by 2, or 4, or 6, &c. Therefore on the other side of the right downe dash toward the right hand, I first take the halfe of the Numerator: saying, the halfe of 2 is 1, the halfe of 8 is 4: and againe, the halfe of 8 is 4: which 144 is now a new Numerator, and therefore I part it with a right downe dash as before.

Then do I also take the halfe of 576, in saying, the halfe of 5 is 2, and the halfe of 17 is 8, and the halfe of 16 is 8, and so haue I 288 for a new Denominator.

Then beginning againe: saying the halfe of 144 is 72, and the halfe of 288 is 144: thus continuing the mediation or diuision by 2 untill you come to the last worke, as appeareth here in the example, where the same is reduced to $\frac{1}{2}$ which is equall to $\frac{288}{576}$.

So this seccond example $\frac{28}{112}$ first abbreuiated by 2, and againe by 2, and last by 7, is reduced to $\frac{1}{4}$ which is equall to $\frac{28}{112}$.

28	14	7	1
112	56	28	4

Againe,

Againe, $\frac{1465}{4395}$ abbreuiated first by 5, then by 293.

$$\frac{1465}{4395} \Big| \frac{293}{879} \Big| \frac{1}{3}$$

Scholar. Sir I thanke you much, this is very easie and good for a yong learner.

Master. So it is, but yet notwithstanding, if you can without that diuision by memory espie the greatest number that may diuide exactly both termes of your fraction proponed, then néed you not to vse that diuision, as in this fraction $\frac{60}{96}$, I sée that 12 is the greatest number that can diuide them both: and therefore without any work, by memory onely, I turne that into $\frac{5}{8}$, but this abilitie in knowledge is got by exercise.

Yet one other way of easie reduction in this kinde there is, when your fraction hath any cyphers in the first places of both termes, then may you by casting away the cyphers, make a briefe reduction as thus: $\frac{300}{400}$ here take away the cyphers, and it will be $\frac{3}{4}$, which is the same in value with $\frac{300}{400}$.

Scholar. And so if I haue $\frac{400}{650}$, it will be $\frac{4}{65}$.

Master. You are deceiued, for you take away more cyphers from the Numerator then you do from the Denominator, which you may not do.

Scholar. I confesse my fault, which came of too much haste, I was more gladder of the

Rule,

Rule then wise in vsing it: but now I vnderstand it I trust.

Master. Then may I go in hand with the fifth or last kinde of Reduction, which teacheth how to turne any fraction proponed into any other Denomination that you list, or into any parts of common coynes, weights or measures, or such like.

The fifth kinde of Reductiō.

For declaration whereof, first you shall marke whether your fraction be a simple Fraction, either else a Fraction of sundry parts, I meane of more termes then two. And if your Fraction be a Fraction of Fractions, or otherwise compound, you must reduce it to one simple fraction. And then marke well the Denomination of that other fraction, into which you would turne this: For by that Denominator you must multiply the Numerator of your first Fraction, and the totall product thereof shall you diuide by the Denominator of your first fraction, and that quotient shall be the Numerator of the Denominator proponed: as for example, I haue this Fraction $\frac{3}{5}$, *which I would turne into ten parts: therefore I multiply this* 10 *by* 3, *that is the Numerator of my Fraction, and there ariseth* 30, *which I diuide by* 5, *and the quotient is* 6, *which must be the Numerator to* 10, *and so* $\frac{3}{5}$ *will be* $\frac{6}{10}$.

To reduce fractions to a denomination appointed

Scholar. This is easie enough to do.

Master. Then shall you see another example of the same fraction that is not so easie: as if I would turne $\frac{3}{5}$ into 8 parts, proue you that worke.

Scholar. I must multiply 8 by 3, and there amount

amounteth 24, which I diuide by 5, and the quotient is 4, then is the new fraction $\frac{4}{8}$.

Master. And see you nothing doubtfull in this worke?

Scholar. I see that when 24 was diuided by 5, there remained 4, which I did not passe of, because ye spake nothing of any remainer, but onely of the quotient.

Master. By likelihood you remember what I said to you in Diuision of whole numbers, that you should not passe of the remainer there, but onely note it as a summe that could not be diuided without knowledge of Fractions. Wherefore now marke this, that in all diuisions of whole numbers, when there is any remainer, you shall set it ouer a line as a Numerator, and set the Diuisor for the Denominator, and that fraction doth make the Diuision compleat, and is part of the quotient: as if I would diuide 48 by 5, the quotient will be 9 $\frac{3}{5}$: so in your former worke when 24 was diuided by 5, the quotient should be 4 $\frac{4}{5}$, and so the new fraction should be thus, $\frac{4}{8}$ and $\frac{4}{5}$ of $\frac{1}{8}$, that is, $\frac{4}{8}$ of the entire number, and $\frac{4}{5}$ of $\frac{1}{8}$ part of any thing, which you may proue by example of some coyne.

Scholar. Then I take a crowne, whose $\frac{3}{5}$ is 3 s. Now if I would proue whether the 3 s be $\frac{4}{8}$ and $\frac{4}{5}$ of $\frac{1}{8}$, I shall haue a cumbrous worke to do.

Master. Indeed for whole pence, your example is a little troublesome: yet turning the crowne

crowne into halfe pence, it is easie enough.

Scholar. That will I try.

¶ First, I see that $\frac{3}{5}$ of a Crowne is 3 shillings, which is 36 pence, or 72 halfe pence. Now if I can finde that this fraction $\frac{4}{8}$ and $\frac{4}{5}$ of $\frac{1}{8}$ be equall unto 3 shillings, then am I fully answered.

Because I cannot take $\frac{4}{8}$ of a crowne, I turne the crowne into halfe pence, as you willed me, which makes 120, which I divide by 8, my quotient is 15, which taken foure times, make 60 ob. Now resteth me to have $\frac{4}{5}$ of the $\frac{1}{8}$ part of a crowne, whereof $\frac{1}{8}$ part is 15 ob. the 15 being parted in 5 parts, the quotient is 3, which taken foure times maketh 12 ob. which with my 60 before amounteth to 72 which are then equall to $\frac{3}{5}$, my desire.

Master. I commend you for your diligence, you might have wrought it thus: either $\frac{4}{8}$ being abbreviated as before I taught, is $\frac{1}{2}$. Now halfe a crowne is 2 shillings 6 pence. Now $\frac{4}{5}$ of $\frac{1}{8}$ is a fraction of fractions, which if you do reduce into one entire fraction, as before you have learned, in saying, five times 8 is 40, for a new Denominator, and once 4 is 4, for a new Numerator: it maketh $\frac{4}{40}$, and abbreviated also make $\frac{1}{10}$. Now the tenth part of a crowne is 6 pence, which put to 2 shillings six pence, make also 3 shillings, your desire.

But now one example more for this rule, and then we shal end it. If I have $\frac{7}{15}$ of a Soueraigne (accounting the Soueraigne 20 shillings)

lings) how many shillings is that $\frac{7}{15}$?

Scholar. I must multiply 7 by 20, and that maketh 140, which I shall diuide by 15, and the quotient will be 9 $\frac{5}{15}$, or else in lesser terms $\frac{1}{3}$.

Master. That is 9 shillings, and one third part of a shilling, that is 4 pence, as by the same Rule you may proue. And this for this time shall suffice for Reduction. And now I will proceed to Addition.

Addition.

When soeuer you haue any Fractions to be added, you must consider whether they be of one denomination or not, and if they be of one Denomination, then adde the Numerator's together, and set that that amounteth for the Numerator ouer the common Denominator, and so haue you done: The reason is because that such differ little in Addition *or* Subtraction *from the worke of vulgar denominations, where the denominators be of the number, as* 3 pence *and* 5 pence, *make* 8 pence, *where the* Denomination *is not altered. But if the fractions be not of one* Denomination, *or any of them be mixt of whole numbers and fractions, then must you first reduce them to one* Denomination, *and after adde them. And if they be many, then adde first two of them, and so the summe that doth amount of the Addition, and the third, and then the fourth, &c. if you haue so many.*

Addition of fractions of one denomination.

Scholar.

Scholar. This séemeth easie enough, now ỹ I haue already learned to reduce, without which I could neuer haue wrought this. And therefore now I sée good reason; why you did place Reduction before Addition.

Master. It is well considered, but yet refuse not to expresse your understanding of it by an example.

Scholar. Then would I adde first $\frac{7}{18}$ with $\frac{5}{18}$, and because the Denominators are like (and so néedeth no reduction) I adde 7 to 5, which maketh 12, and then is my summe $\frac{12}{18}$, that is in smaller numbers, being abbreuiated $\frac{2}{3}$.

To adde fractions of diuers denominations.

And if I haue many numbers to be added as here $\frac{3}{8}$, $\frac{4}{5}$, $\frac{9}{10}$, first I must reduce them (because they haue diuers denominators) into one Denomination, and then they will be thus: $\frac{150}{400}$, $\frac{320}{400}$, $\frac{360}{400}$, or in lesser termes, $\frac{15}{40}$, $\frac{32}{40}$, $\frac{36}{40}$, which by Addition do make $\frac{83}{40}$, that is 2 & $\frac{2}{40}$.

Master. Now may we go to Subtraction.

Sub-

Subtraction.

Subtraction hath the same precepts that Addition had, for if the Denominators be like, then must you subtract the one Numerator from the other, and the rest is to be set ouer the common Denominator, and so your Subtraction is ended: but and if you haue many fractions to be subtracted out of many, then must you reduce them to one Denomination, and into two seuerall fractions, that is, all that must be subtracted into one fraction, and the residue into another fraction, and then worke as I said before.

Subtraction of fractions.

Scholar. For the first example, I take $\frac{15}{12}$ to be subtracted out of $\frac{17}{12}$, and the rest will be $\frac{2}{12}$ or $\frac{1}{6}$.

For another example, I take $\frac{3}{4}$ to be subtracted out of $\frac{7}{8}$, which I must reduce, and it will be thus: $\frac{24}{32}$ and $\frac{28}{32}$.

Then do I subtract 24 out of 28, and there resteth 4, which I set ouer the common Denominator for a Remainer, thus: $\frac{4}{32}$ that is $\frac{1}{8}$.

Now for the third example, I take $\frac{3}{4}$ and $\frac{5}{6}$ to be subtracted from $\frac{7}{8}$ and $\frac{9}{10}$: and because their Denominators be diuers, I do reduce them into one denomination thus: $\frac{1440}{1920}$ $\frac{1600}{1920}$ [illegible] $\frac{1728}{1920}$.

Then do I adde the two first, & they make $\frac{3040}{1920}$. Also I adde the two last, and they yeeld $\frac{3408}{1920}$. Then do I subtract 3040 out of 3408, and there resteth 368, so is the remainer $\frac{368}{1920}$ that is in smaller termes $\frac{203}{120}$. And thus haue I done with Subtraction, except you haue any more to teach me.

Master. Proue one example or more of two fractions of diuers denominations.

Scholar. I take the two fractions, $\frac{7}{8}$ to be subtracted from $\frac{9}{24}$, which being reduced, will stand thus: $\frac{168}{192}$ and $\frac{72}{192}$. Now would I subtract 168 out of 72, but I cannot.

168	72
$\frac{7}{8}$	$\frac{9}{24}$
192	

Master. Then may you perceiue that you mistooke the fractions: for you can neuer subtract the greater out of the lesser, although you may adde, multiply, or diuide the greater with the lesser. And albeit that $\frac{7}{8}$ hath both his terms lesser then $\frac{9}{24}$ yet is $\frac{9}{24}$ the lesser fraction: for generally if you multiply the Numerator and the Denominators of two fractions crosse-wayes, that Fraction is the greatest of whose Numerator commeth the greatest summe, as in this example: 7 multiplyed by 24, maketh 168: and 9 being multiplied by 8, yeeldeth but 72, therefore is the first fraction 7, the greatest of these two, so can you not subtract it out of a lesser fraction.

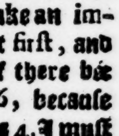
The greatest of two fractions.

But if you should subtract a fraction out of a whole number, what should you do?

Scholar.

Scholar. Marry I would reduce the whole number into a fraction of the same Denomination that my fraction is, and then work by Subtraction.

Master. So may you do, but it is much easier, if your fraction be a proper fraction, that is to say, lesse then an vnite, to take an vnite from the whole number, and then turne it into an improper fraction, and so worke your Subtraction. As if I would subtract $3\frac{2}{5}$ from 4, I may take 1 from 4, and turne it into $\frac{5}{5}$, from which I abate $3\frac{2}{5}$, there will remaine $\frac{3}{5}$. And if the first fraction be an improper fraction, then may I take so many vnites from the whole number, that they may make an improper fraction greater then that first, and then worke by Subtraction. As if there bee proponed $\frac{10}{3}$ to be subtracted from 6, because $\frac{10}{3}$ is more then 3, and not so much as 4. I must take 4 from 6, and turne them into thirds thus $\frac{12}{3}$, then abate $\frac{10}{3}$ and from $\frac{12}{3}$ there resteth $\frac{2}{3}$: so the whole remainer is $2\frac{2}{3}$. Or else you may at your pleasure take $3\frac{1}{3}$, which is $\frac{10}{3}$: from 6 whole: then set 1 under 6, as thus: $\frac{6}{1}$: And then to reduce those two fractions into one Denomination, as here appeareth $\frac{10}{3}$ from $\frac{6}{1}$. Then $\frac{10}{3}$ from $\frac{18}{3}$ resteth $\frac{8}{3}$ which maketh $2\frac{2}{3}$ your desire. And thus will I make an end of the worke of subtraction of fractions, and procéed to Multiplication.



Multiplication.

Multiplication of fractions.

Therefore when any two fractions be proponed to be multiplied together, the Numerator of the one must be multiplied by the Numerator of the other: and the summe that amounteth thereof must be set for a new Numerator: likewise the Denominator of the one must be multiplied by the Denominator of the other, and that that amounteth, shall be set for the Denominator, and this new third Fraction expresseth the product of the multiplication of the two first Fractions proponed, whereof take this example, $\frac{3}{5}$ *multiplied by* $\frac{5}{12}$*, doth make* $\frac{15}{60}$.

$$\frac{3}{5} \qquad \frac{5}{12} \qquad \frac{15}{60}$$

Scholar. I perceiue then that 3, being the Numerator of the first fraction, is multiplied by 5, being the numerator of the second fraction, whereof amounteth 15, the numerator of the third Fraction. And so likewise 5, being the denominator of the first fraction, is multiplied by 12 the denominator of the second fraction, whereof amounteth 60 the new denominator, so that I perceiue how the worke is done, but I do not perceiue how $\frac{15}{60}$ is greater then $\frac{3}{5}$, for if I shall vse my former manner of examination by the parts of some coyne, I see that $\frac{3}{5}$ of a Crowne is 36 pence, and $\frac{5}{12}$ of a crowne is 25 pence, whereof the one multiplied by the other, doth make

900

900 pence, which is 15 Crowns, but by your multiplication there amounteth $\frac{15}{60}$, which is but 15 pence, and that is much lesse then any other of both the first fractions.

Master. That difference is betwéene multiplication in whole numbers, and multiplication in broken numbers, that in whole numbers, the sum that amounteth is greater then both the other whereof it came: but in fractions it is contrariwise; for the summe that amounteth is lesser then any of the other two fractions whereof it is produced.

Scholar. I desire much to vnderstand the reason thereof.

Master. Although I purposed to reserue the reasons of workes Arithmeticall for the perfect Boke of Arithmeticke, yet I will shew you this, because of the strangenesse of the worke. You sée in whole numbers, that of two numbers being multiplied together, is made the third number, which third number, doth beare the same proportion to the nūber multiplied, that the multiplier doth beare to an vnite. And so in fractions, ẏ third nūber which amounteth of multiplication, beareth the same proportion to each of the two first fractions, that the other of those two fractions doth beare to an vnite.

Scholar. Sir, I vnderstand your words thus: when 40 is multiplied by 12, there doth amount 420, which 480 doth containe 40, so many times in it, as 12 doth containe Vnites, that is to say, twelue times. And so it

appeareth that 480 doth containe twelue ſo many times alſo as 40 doth containe vnites, that is 40 times. But now I ſee not how the third number in this example of Fractions can containe any of the two former (as it happened in whole numbers) ſeeing it is leſſer then either of them.

Maſter. No maruell if you cannot ſee that thing which is not poſſible to be ſeene of any man, how the third number in Multiplication of Fractions ſhould be greater then any of the two former fractions: but yet this may you ſee (which I ſaid) that the third number in Fractions ſo multiplied, doth beare the ſame proportion to any of the two former fractions that the other of thoſe two fractions doth beare to an vnite, as in your example, $\frac{3}{5}$ being multiplied by $\frac{5}{12}$, doth make $\frac{15}{60}$. Now ſay I that $\frac{15}{60}$ doth beare the ſame proportion to $\frac{3}{5}$ that $\frac{5}{12}$ doth beare to an vnite, as you may in your owne forme of examination by Coine, try it: for in an old Angel (which in times paſt was currant for 7 ſhillings ſixpence, are 180 halfe pence) which I ſet for the intire vnite, whoſe parts (according to the fractions aforeſaid) are theſe, for $\frac{15}{60}$ ſet 45 halfe pence, for $\frac{3}{5}$ take 108 halfe pence, and for $\frac{5}{12}$ put 75 halfe pence. Now doth 45 beare the ſame proportion to 108, that 75 doth beare to 180, for 45 is $\frac{5}{12}$ of 108, and ſo is 75 alſo $\frac{5}{12}$ of 180.

But theſe reaſons may be better reſerued till another time, when the knowledge of pro-

proportions in due order shall be taught: yet in the meane season I will shew you how it commeth to passe, that in fraction the third summe must needs be lesser then any of the other two.

Consider this, that when a fraction is proponed, as in the former example $\frac{3}{5}$ if it be multiplied by more then 1, it will make more then one entire number. As if I multiply $\frac{3}{5}$ by 5: that is to say, if I take it 5 times, it will make three entire vnites: Example in a Crown, $\frac{3}{5}$ of it maketh 3 shillings, which if I take fiue times, it will amount to 15 shillings, that is, three entire Crownes: so if I take the same $\frac{3}{5}$ but twice, it will yeeld 6 shillings, that is, one entire Crowne, and $\frac{1}{5}$. Now if I take it but once, it cannot be more then it was before, that is 5 shillings. And if I take it lesse then once, it cannot be so much as it was before. Then seeing that a Fraction is lesse then one, if I multiply a fraction by another fraction; it followeth that I do take the first fraction lesse then once, and therefore the summe that amounteth, must needs be lesse then the first Fraction. Note.

Scholar. Sir, I thanke you much for this reason. And I trust I do perceiue the thing, as by example of this same fraction $\frac{3}{5}$ I wil expresse. If I take $\frac{3}{5}$ of a Crowne once, that is to say, if I multiply $\frac{3}{5}$ by 1, it will be as it was before, but 3 shillings: so if I do multiply it by $\frac{1}{2}$, that is, if I take but halfe one time, then

will it be but halfe so much: likewise if I multiply it by $\frac{1}{3}$ that is, if I take but the third part of one, it will yeeld but 12 pence, that is, the third part of the first fraction.

And so to make an end: if I take it but the twelfth part of once, that is, If I do multiply it by $\frac{1}{12}$, it will yeeld but the twelfth part of the first fraction, which is but 3 pence. And it followeth, that if $\frac{1}{12}$ make three pence, then $\frac{5}{12}$ must needs make five times so much, that is 15 pence, which was the summe that hath giuen the occasion of all this doubt.

Master. Then I perceiue you haue sufficient understanding in this sort of multiplication for this time, wherefore I will procéed to the rest.

In multiplication it happeneth sometime, that there be whole numbers to be multiplied with Fractions; and may be in two sorts: for either the whole number is seuerall from the Fraction, and is the Multiplier, or else the whole number is ioyned with one, or both of the Fractions, and so maketh a mixt number thereof. If it be in the first sort, then needeth there no reduction, but onely multiply the Numerator of the Fraction by that whole number, and the totall thereof set for the new Numerator.

To multiply a whole number into a fraction.

Scholar. I understand you thus. If I haue $\frac{6}{23}$ to be multiplied by 16, then must I multiply that 16 with 6, which is the Numerator, whereof commeth 96, and that must I set for the new Numerator, kéeping still 23

for

for the Denominator, and ſo the fraction will be $\frac{96}{23}$, that is $4\frac{4}{23}$.

Maſter. And in this ſort of worke you may abridge the labour, thus. If it happen the Denominator to be ſuch a number, as may euenly be diuided by the ſaid whole number propoſed, then diuide it thereby, and ſet the Quotient of that diuiſion for the former Denominator, but reſerue ſtill the Numerator, and ſo is the multiplication ended.

Scholar. Then I ſaine this example $\frac{7}{20}$ to be multiplyed by 5: and becauſe 5 will iuſtly diuide 20, therefore I take the Quotient of that diuiſion, which is 4, and ſet inſtead of 20, and ſo the fraction will be $\frac{7}{4}$, that is $1\frac{3}{4}$.

Maſter. Which is all one with $\frac{35}{20}$ that would haue followed of the other ſort of worke.

Scholar. I perceiue it very well.

Maſter. *Now then for the other ſort, where the number is mixt, take this way: firſt to reduce the ſaid whole number and fraction into one improper fraction, (as I ſhewed you in* Reduction) *and then multiply them together, as if they were proper fractions.*

How to multiply mixt numbers.

Scholar. $13\frac{3}{5}$ being ſet to be multiplyed by $\frac{5}{8}$ firſt I muſt reduce the mixt number, as in this example appeareth by multiplying 13 by 5, and that maketh 65, whereto I muſt adde the Numerator 3,

$13\frac{3}{5}$	340
68 by	5
5 40	8

and

and so the fraction will be $\frac{68}{3}$, which two Fractions now I shall multiply after the accustomed forme, and it will be $\frac{340}{40}$, or $\frac{34}{4}$.

Master. You haue done well: and so may you see, that although most part of the formes of Multiplication may be wrought without Reduction, yet some cannot, as namely, mixed numbers.

Duplation *And yet one note more will I tell you of Multiplication, before we leaue it: That is, whensoeuer you would multiply any Fraction by 2, which commonly is called* Duplation, *you may do it not onely by doubling the Numerator, but also by parting the Denominator into halfe, if it be euen.*

Scholar. Then if I would double $\frac{5}{12}$, I may chuse whether I will make it $\frac{10}{12}$, or else $\frac{5}{6}$. And indeed I see that is all one, but that the diuiding of the Denominator seemeth the better way to make smaller termes of the Fraction, and so they shall need the lesse Reduction.

Master. It is so: and now I shall not need to tell you that multiplication is proued by Diuision, and Diuision likewise by multiplication: but the like worke that I shewed you in multiplication, will I shew you in Diuision also.

Diuisi-

Diuision of Fractions.

WHensoeuer two Fractions be proponed, that one should be diuided by the other, I must set downe first the fraction that shall be diuided (which is called the Diuidend) and then after it the other which is the Diuisor. Then shall I multiply the Numerator of the Diuidend by the Denominator of the Diuisor, and that which amounteth, I must put for a new Numerator. Againe, I shall multiply the Denominator of the Diuidend by the Numerator of the Diuisor, and the number that amounteth thereof, I must put for the new Denominator. And this third Fraction is the Quotient of the said Diuision. Diuision of Fractions.

Scholar. This seemeth easie in forme, as by example thus: If I would diuide $\frac{5}{8}$ by $\frac{2}{6}$, first I multiply 5 (being the Numerator of the Diuidend) by 6, which is the Denominator of the Diuisor, and thereof riseth 30: then I multiply 8 (being the Denominator of the Diuidend) by 2, being the Numerator in the Diuisor: and so riseth 16, the which I must make in a third Fraction, thus $\frac{30}{16}$.

$$\begin{array}{ccc} & 30 & \\ 5 & & 2 \\ 8 & & 6 \\ & 16 & \end{array}$$

Master. Me seemeth you are quicker in vnderstanding now, then you were when I taught

taught you the Art of whole numbers but that is no marvell: for the more knowledge that any man getteth, the readier shall he finde his wit, and quicker in understanding: but yet of two things I will admonish you, which you might haue obserued here for the ease of work, and lightnesse of understanding, the nature of the Quotient.

Whensoeuer you diuide one fraction by another, either they be both equall together, or else the one is greater then the other: if they be equall, their Quotient shall be such, that the Numerator and the Denominator of it shall be equall also. And if the two first fractions be inequall, their Quotient shall declare the same by the unequality of the Numerator and Denominator, as in these examples following shall appeare.

First, if equall fractions $\frac{4}{9}$ and $\frac{12}{27}$ be equall together, and if the one be diuided by the other the Quotient will be $\frac{108}{108}$, as you may perceiue by that Rule aforesaid.

Now in the unequall fractions, as $\frac{4}{9}$ and $\frac{3}{10}$, the Quotient will be $\frac{40}{27}$, where the Numerator is greater then the Denominator.

Scholar. I see it is so: but I see not the reason why it should be so.

Note how to know the proportion betweene two numbers.

Master. The reason is this: When any Fraction is diuided by another, the quotient declareth what proportion the Diuidend beareth to the Diuisor. So $\frac{1}{2}$ diuided by $\frac{1}{4}$ maketh 2, which must be sounded, not 2 but twice,

twice, declaring that $\frac{1}{4}$ is contained twice in $\frac{1}{2}$.

And note this, that the Numerator in the Quotient representeth the Diuidend, and the Denominator representeth the Diuisor. And this is alwayes true, whether the greater fraction be diuided by the lesser, or the lesser by the greater. But this proportion will not be exactly knowne, till you haue learned the Art of proportions: notwithstanding somewhat of it I haue declared in the Rule of Reduction. But now for the easie remembrance of the Quotient in Diuision: as soone as you haue set down your two fractions the one against the other, then make a straight line for the Quotient: and as soone as you haue multiplyed the Numerator of the Diuidend, by the Denominator of the Diuisor, set the number that amounteth ouer the said line, and then multiply the other two numbers, and set their totall vnder the same line.

Scholar. I perceiue you would not haue me trust to memory till I were better expert, lest oftentimes I happen by misse-remembrance to be abused. This example I take for that declaration.

If I would diuide $\frac{2}{3}$ by $\frac{3}{4}$, I must set the numbers one against the other, (as here doth appeare) and then make another line for the quotient in some good

$\frac{2}{3}$ by $\frac{3}{4}$

distance,

distance, where I may set the numbers of the quotient, as soone as any of them is multiplied. So then as soone as I haue multiplyed 2 by 4, which maketh 8, I shall set that 8 ouer that line, thus: And then multiply 3 by 3, which yeeldeth 9: and that 9 must be set vnder the same line, and then will the whole quotient appeare thus $\frac{8}{9}$; whereby it appeareth (as I remember your words) that $\frac{2}{3}$ is in proportion to $\frac{3}{4}$, as 8 is to 9: but how may I perceiue that?

$$\frac{8}{9}$$

Master. Although you might better perceiue it by the Rule of Reduction, yet this example may be declared in common coines as in a common shilling of 12 pence, of which $\frac{2}{3}$ maketh 8 pence, and $\frac{3}{4}$ doth make 9 pence, and so you may easily see that their proportions do agree. And if you had taken this example before when you tooke the example of $\frac{5}{8}$ and $\frac{2}{4}$ your quotient would appeare (as this doth) more easie to vnderstand; whereas that Quotient being $\frac{20}{16}$, is not an easie proportion for you to perceiue, being yet little acquainted with proportions.

Scholar. If there be whole numbers to be diuided by a Fraction, how shall I performe it?

To diuide a whole number by a fraction.

Master. *When any whole number shall be diuided by a Fraction, you must multiply the said whole number with the Denominator of the Fraction, and set the totall thereof for the new* Numerator, *and for the* Denominator *set*

set the Numerator *of the fraction.*

$\frac{80}{3}$

Scholar. Then 20 diuided $\frac{20}{1}$ by $\frac{3}{4}$ by $\frac{3}{4}$ will make $\frac{80}{3}$, as here appeareth $\frac{80}{3}$.

Master. *Euen so: but if you would diuide the fraction by the whole number, then multiply the Denominator by the same whole number, and set the totall for the Denominator, without changing the Numerator.*

To diuide the fraction by the whole number.

Master. Then to diuide $\frac{20}{23}$ by 4, it will be $\frac{20}{92}$, as here appeareth $\frac{20}{23}$ by $\frac{4}{1}$ in this example $\frac{20}{92}$.

$$\frac{20}{23} \text{ by } \frac{4}{1} \quad \frac{20}{92}$$

Master. You say well. And by the same example you giue me occasion to remember another briefe way to do the same: for if you had diuided the said numerator, by 4, and set the quotient for the numerator, keeping still the old denominator, it would haue béene not onely as well done, but also in a fraction of lesser termes.

Another briefe way

Scholar. I guesse it to be euen so, by a like worke that you taught me in multiplication: And for proofe thereof $\frac{20}{23}$ being the diuidend, and 4, the diuisor, I diuide the Numerator 20 by 4, and the quotient is 5. which I set for 20 ouer 23, thus, $\frac{5}{23}$: And I sée that it is all one with $\frac{20}{92}$, as by diuiding or abbreuiating both these termes by 4, and so reducing them

to

to their least denomination, I may easily proue: as appeareth by this example, $\frac{20}{92}$ $\frac{5}{23}$.

Master. You conceiue it well. And if there be mixt numbers (either one or both) you must first reduce that mixt number into an improper fraction & then work as you haue learned.

Scholar. That was sufficiently taught in Multiplication. Therefore I pray you go forward to some other thing.

Master. Then take this note yet for Diuision: if the Denominators be like, then diuide the numerators as it were in whole numbers, and the quotient whether it be fraction, whole number, or mixt, is a good quotient for that diuision. And generally, if one of the Numerators may iustly diuide the other, by that quotient, multiply the Denominator of the lesser Numerator, and set it that doth amount in the roome of the same denominator, and then for a numerator to it, set the Denominator of the other fraction.

Scholar. Then if I would diuide $\frac{3}{4}$ by $\frac{12}{17}$, I see that 3 will diuide 12, and the Quotient will be 4. by which I must multiply the other 4, that is the Denominator vnder 3, and then it is 16, which is set for the denominator 4, and ouer it instead of 3, I must set 17 the other Denominator, and so it is thus, $\frac{17}{16}$.

Master. And so is $\frac{17}{16}$ instead of $\frac{51}{48}$, which would haue risen by the common worke, as here appeareth.

$$\begin{array}{ccc} & 51 & \\ \frac{3}{4} & \text{by} & \frac{12}{17} \\ & 48 & \end{array}$$

And

And now for mediation (which is to diuide by 2) marke this, if the Numerator be an euen number, set the halfe of it in his place without the Diuisor, and so haue you done: and if the Numerator be not euen, then double the Denominator.

Scholar. That is, if I would mediate $\frac{6}{17}$, I may make the quotient $\frac{3}{17}$, and if I would mediate $\frac{7}{11}$ I must make it $\frac{7}{22}$.

Master. And thus will I make an end of the works of common fractions for this time, not doubting but you can apply them both to the Rule of Progression, and also to the Golden Rule, without any other teaching then you haue learned before, which might seeme tedious to repeat, in regard you haue sufficient knowledge in Reduction, Addition, Subtraction, Multiplication, and Diuision: And therefore will I go in hand with the Rule of Proportion, or Golden Rule, which now will appeare easie en[...]gh.

The Golden rule direct, in Fractions.

Master. Master:

Therefore as touching the Golden Rule, for the placing of the 3 numbers proponed in the question whereby to finde the fourth, and for the forme of their worke, with other like notes, I

The rule of proportion, in fractions.

P referre

refer you to that which you haue already learned.

But this eaſie forme of working by fracti-ons ſhall you note, that if your three numbers be fractions, for an apt worke and certaine, multiply the numerator of the firſt number in the queſtion, by the denominator of the ſe-cond: And all that againe multiply by the Denominator of the third number, and the totall thereof ſhall you keepe for to be the Di-uiſor. Then mulitply the Denominator of the firſt number by the numerator of the ſecond, and the whole thereof by the Numerator of the third, and the totall thereof ſhall be your Diuidend.

Note this for a general rule.

Now diuide this Diuidend by the diuiſor which you found out before, and that num-ber ſhall be the fourth number of the queſtion which you ſeke for, as in this example.

A queſtion of velvet.

If $\frac{3}{4}$ *of a yard of veluet coſt* $\frac{2}{3}$ *of a Soueraigne, eſteemed at* 20 ſhillings, *what ſhall* $\frac{5}{6}$ *coſt?*

Scholar. If it pleaſe you to let mée make the anſwer, I would firſt place theſe three numbers as I learned in whole numbers, thus,

$$\frac{3}{4} \text{ — } \frac{2}{3}$$
$$\frac{5}{6}$$

And then according to your new rule, I muſt multiply 3, being Nnmerator, in the firſt number, by 3 the Denominator of the ſecond: and thereof commeth 9, which I multiply a-gain by 6, the denominator of the third num-ber, and ſo haue I 54, which I kéepe for the diuiſor. Then multiply I 4 the denominator of the firſt, by 2 the numerator of the ſecond,

and

and there ariseth 8. which againe I multiply by 5, the numerator of the third, and it maketh 40. Then must I diuide 40 by 54, and it will be $\frac{40}{54}$ that is $\frac{10}{27}$, in lesser termes, and then the Figure will stand thus.

$$\begin{array}{ccc} \frac{3}{4} & & \frac{2}{3} \\ \frac{5}{6} & & \frac{20}{27} \end{array}$$

But what that is in money I cannot tell, except I shall worke it by Reduction, as you taught me.

Master. It forceth not now, you may reduce it when you list, but it were disorderly done here to mingle diuers works together, where we do not seek the value of the thing in common money, but in an apt number, which ye haue well done. And therefore will I yet shew you another like way of easinesse in work, how you may change your three fractions into three whole numbers, by which you shall worke, as if the question were proponed in whole numbers. The first number you shall find as I taught you: now to finde the diuisor of the second number, take the Numerator for the second fraction: and for the third number take that that ariseth of the multiplication of the denominator of the first, by the numerator of the third, and then work your question.

Scholar. *For example hereof, I put this question. If $\frac{11}{12}$ of* 1 pound *weight of silver be worth $\frac{12}{4}$ of a Soueraigne, what is $\frac{1}{3}$ of* 1 pound *weight worth?* A question of Siluer.

For the answer, first I place the fractions in order thus:

$$\begin{array}{ccc} \frac{11}{12} & & \frac{12}{4} \\ \frac{1}{3} & & \end{array}$$

Then to turne theſe fractions into whole numbers, I multiply 11, which is the numerator of the firſt by 4 (the denominator of the ſecond) and there commeth 44, which I multiply by 2 the denominator of the third, and ſo amounteth 88, which I ſet for the diuiſor in the firſt place. Then in the ſecond place I ſet 12 which is the numerator of the ſecond fraction, & in the third place I ſet the ſum that amounteth of 12, being the denominator of the firſt number, multiplied by one, being numerator in the third number, and ſo the figure will ſtand as here you ſee.

88 —— 12
12 ——

Then to worke it forth, I multiply 12 by 12, and there amounteth 144, which I diuide by 88, and the quotient will be 1 $\frac{56}{88}$, or in leſſer termes, 1 $\frac{7}{11}$, and then the figures will ſtand thus:

$\frac{11}{12}$ —— $\frac{12}{4}$
$\frac{1}{2}$ —— 1 $\frac{7}{11}$

Maſter. Theſe two formes now you underſtand well enough, and as for any other at this time I will not repeat, onely this ſhall you marke for the proofe of this Rule whether your worke be well wrought or no. Multiply the firſt number by the fourth, and note what amounteth: then multiply the ſecond by the third, and marke what amounteth alſo. Now if thoſe two numbers ſo amounting be equall, then is your worke well done, elſe you haue erred. And this ſhall ſuffice for the former rule.

The proof of the golden Rule.

The

The Backer Rule, or Reuerſe Rule in Fractions.

BVt in the Backer Rule, this ſhall you note for eaſe of worke, that you multiply the numerator of the firſt by the numerator of the ſecond, and the whole thereof by the denominator of the third, and that amounteth thereof, ſhall be the diuidend. Then multiply the denominator of the firſt, by the denominator of the ſecond, and that whole by the numerator of the third, and that that ariſeth thereof, ſhall bee the diuiſor. Example of this.

The backer Rule in Fractions.

Note this alſo for a generall Rule.

I did lend my friend $\frac{3}{4}$ of a Porteguiſe ſeuen moneths, upon promiſe that he ſhould do as much for me againe, and when I ſhould borrow of him, he could lend me but $\frac{5}{13}$ of a Portegniſe: now I demand how long time muſt I keep his mony in iuſt recompence of my loane, accounting 13 *moneths in the yeare?*

A queſtion of Loane.

Scholar. The firſt number muſt be the firſt mony borrowed, that is $\frac{3}{4}$ of the Porteguiſe: the ſecond number the 7 moneths, that is $\frac{7}{13}$ of a yeare: and the third number the money that was lent in recompence, that is $\frac{5}{13}$ of a Porteguiſe: then I ſet the numbers thus:

$\frac{3}{4}$ $\frac{7}{13}$

$\frac{5}{13}$

Then (as you taught mee) I multiply 3 (being numerator in the first number by 7, the numerator of the second number, and it maketh 21, which I multiply by 12 the denominator of the third, and so haue I 252 for the diuidend: then I multiply 4 the denominator of the first, by 13 the denominator of the second, & it yeeldeth 52, which I multiply againe by 5 the numerator of the third, and it will make 260, that is the diuisor. Then must I diuide 252 by 260, so it will be in the small fraction $\frac{63}{65}$ of a yeare.

Master. And thus do you see some ease in working, better then to multiply and diuide tediously so many Fractions.

Statute of Assise of Bread and Ale.

Another question yet will I propose, to the intent you may see thereby the reason of the Statute of assise of bread and ale, which in all statute Books, in Latine, French, and English is much corrupted for want of knowledge in this Art, for the right vnderstanding whereof, I propone this question.

Question of Bread.

When the price of a quarter of Wheat is 2 shillings, *the farthing white loafe shall weigh* 68 shillings, *then I demand what shall such a loafe weigh when a quarter of Wheat is sold for* 3 shillings?

Scholar. This question must be wrought as it is proponed in whole numbers, and not in Fractions.

Master. You seeme to say reasonably, howbeit in the Statute of assise, the rate is made

by

by the proportion of parts in a pound weight Troy, elſe could it not be a Statute of any long continuance, ſeeing the ſhillings do change often, as all other monies do: but this Statute being well underſtood, is a continuall rule for euer, as I will anon declare by a New Table of Aſſiſe, conuerting the ſhillings into ounces, and parts of ounces.

Wherefore here by a ſhilling you muſt underſtand $\frac{1}{20}$ of a pound weight, and ſo by a penny $\frac{1}{20}$ of an ounce: wherefore although you might worke this queſtion proponed by whole numbers well enough, for that time when the Statute was made, yet to apply it to your time, and to make it ſerue for all times generally, it is beſt to worke it by fractions, ſetting for 2 ſhillings $\frac{2}{20}$, and for 68 ſhillings, $\frac{68}{20}$, and ſo for thrée ſhillings $\frac{3}{20}$, and then wil the Figure of the queſtion ſtand thus.

$$\begin{array}{cc} \frac{2}{20} & \frac{2}{20} \\ \frac{3}{20} & \end{array}$$

In which queſtion becauſe all the denominators be like, you ſhall worke onely with the numerators.

Scholar. Then ſhall I multiply 68 by 2, whereof commeth 136, which if I diuide by 3, the quotient will be $45\frac{1}{3}$: but how ſhall I make a fraction of that, to ſtand with the other?

Maſter. Haue you ſo ſoone forgotten what was taught you ſo lately? this is his forme.

$$\frac{45\frac{1}{3}}{20}$$

P 4 Scholar.

Scholar. I remember it now, and then it signifieth 45 twenty parts, and the third deale of one twenty part.

Note what a shilling is.

Master. So is it that maketh in shillings 45 shillings 4 pence, whereby you may note one great error in the Statute Books; which haue constantly 48 shillings in that Assise. And by this Rule if you examine the Statute, you shall find many summes false. Wherefore for the true understanding of that Statute, and such like as I haue made mention of it, and somewhat recognised it, so do I wish that all Gentlemen & other Students of the Laws would not neglect this Art of Arithmeticke, as unneedfull to their studies. Wherefore to encourage them thereto, and to gratifie both them and all other in generall, I will exhibite a Table of that part of the Statutes in two columnes, and in a third columne, I will adde the correction of those errors which haue crept into it.

Here followeth the Table.

The

The price of a quarter of Wheat.		The weight of a farthing white loafe by the Statute Books.			The Correction by iust Assise.		
s	d	li	s	d	li	s	d
1	0	6	16	0	6	16	0
1	6	4	10	8	4	10	8
2	0	2	8	0	3	8	0
2	6	2	14	$4\frac{1}{5}$	2	14	$4\frac{4}{5}$
3	0	2	8	0	2	5	4
3	6	2	2	0	1	18	$10\frac{2}{7}$
4	0	1	16	0	1	14	0
4	6	1	10	0	1	10	$2\frac{2}{3}$
5	0	1	8	$2\frac{1}{2}$	1	7	$2\frac{6}{17}$
5	6	1	4	$8\frac{1}{4}$	1	4	$8\frac{3}{32}$
6	0	0	2	8	1	2	8
6	6	0	19	11	1	0	$11\frac{1}{13}$
7	0	0	19	1	0	19	$5\frac{1}{7}$
7	6	0	18	$1\frac{1}{2}$	0	18	$1\frac{2}{3}$
8	0	0	17	0	0	17	0
8	6	0	16	0	0	16	0
9	0	0	15	$0\frac{1}{4}$	0	15	$1\frac{1}{5}$
9	6	0	14	$0\frac{3}{4}$	0	14	$3\frac{15}{19}$
10	0	0	13	$7\frac{1}{5}$	0	13	$7\frac{3}{9}$
10	6	0	12	$11\frac{1}{4}$	0	12	$11\frac{3}{8}$
11	0	0	12	$4\frac{1}{4}$	0	12	$4\frac{4}{11}$
11	6	0	11	10	0	11	$9\frac{21}{23}$
12	0	0	11	4	0	11	4

In the common Books there is no further rate of Assise made then into 12 s the quarter of Wheat, but in an ancient Copy of 30 yeares old (which I haue) there is added the rate of Assise vnto 20 s the quarter, but yet was that assise also either wrong cast at the first penning, or else corrupt since that time, for lacke of iust knowledge in the Rule of Proportion, which I will adde here also to gratifie such as be desirous to vnderstand truth exactly.

The price of a quarter of Wheat.		The weight of a farthing white loafe by the Statute Books.			The Correction by iust Assise.		
s	d	li	s	d	li	s	d
12	6	0	11	0	0	10	$10\frac{14}{25}$
13	0	0	11	$0\frac{1}{2}$	0	10	$5\frac{7}{13}$
13	6	0	10	$1\frac{1}{2}$	0	10	$0\frac{8}{9}$
14	0	0	9	7	0	9	$8\frac{4}{7}$
14	6	0	9	$2\frac{1}{2}$	0	9	$4\frac{16}{29}$
15	0	0	9	$1\frac{1}{2}$	0	9	$0\frac{4}{7}$
15	6	0	9	$1\frac{1}{4}$	0	8	$9\frac{9}{31}$
16	0	0	9	0	0	8	6
16	6	0	8	6	0	8	$2\frac{10}{11}$
17	0	0	8	3	0	8	0
17	6	0	7	10	0	7	$10\frac{9}{35}$
18	0	0	7	6	0	7	$6\frac{2}{3}$
18	6	0	7	3	0	7	$4\frac{3}{37}$
19	0	0	7	2	0	7	$1\frac{17}{19}$
19	6	0	5	10	0	6	$11\frac{9}{13}$
20	0	0	5	6	0	6	$3\frac{3}{5}$

These two Tables I haue set seuerall, because no man should thinke that I would either adde or take away from any Law those parts which might of right seem either superfluous, either diminute: but yet I may not bee so curious as to neglect manifest errours, which is not onely my part, but euery good Subiects duty with sobriety to correct. And for auoiding of offence, I haue rather done it in this Priuate Booke, then in any Booke of the Statutes it selfe, trusting that all Men will take it in good part,

Scholar. I would wish so, but I dare not so hope, sith neuer good man that would reforme errour, could escape the venemous tongues of enuious detracters, which because they either cannot or list not to do any good themselues, do delight to barke at the doings of other, but I beseech you to stay nothing for their peruerse behauiour.

Master. I consider many things that some may obiect, whereunto I am not vnprouided of iust answers, but I will not seeme so hasty to make the answers, before I heare their obiections, but as I trust that men are of a better nature, and more gratefull now then some haue beene in times past, as I haue done in the Statute of Assise for bread in rate of shillings, so will I set forth the like Table in pounds and ounces, and the parts thereof, that it may be easily applied to all times: but I meane not by this to alter any word of the Statute, Concerning the following Tables.

Statute, being so good an ordinance, and of so great continuance) but onely to make it as a kinde of exposition and declaration of the said Statute, trusting that thereby the Statute may be better vnderstood, and consequently better put in execution. And here you shall note, that I haue accounted the shillings after the rate of 60 shillings to the pound weight, because I esteeme it the most apt for our time. Wherefore in the first Columne you finde the price of Wheat directly against it; in the second Columne, you may finde the weight of a Farthing white Loafe in this our time: and if you double the number (as I haue done in the third Columne) then haue you the weight of the Halfe-penny white Loafe; and so in the fourth Columne is set the weight of a penny white-Loafe. It needeth not to tell that the sight doth testifie, how that euery Columne is parted into three smaller pillars, whereof the first Columne hath these three titles; pounds ounces, and penny weights. And as in the first columne 12 pence make a shilling, and 20 shillings make a pound, so in the other three Columnes 20 pence weight maketh an ounce and 12 ounces do make a Pound.

A pound weight.

Gentle

GEntle *Reader*, touching the vnderstanding of the *Table* following, wherein according to our time, *Master Record* alloweth 60 *pence* to the *ounce*, and 3 *pound*, or 60 *shillings* to the *pound*, and thereupon after the rate of 60 *shillings* to the *pound* Troy, doth he frame or produce this his T*able*, beginning at 3 *shillings the quarter*, till he comes to 40 *shillings* 6 *pence the quarter*. And this his *proportion* (for that he hath not set downe any one *Example* to continue the worke) hath beene hard for many to conceiue or comprehend, and therefore the onely chiefe cause why I haue written this *digression* for the better vnderstanding of him therein.

The first thing therefore that is sought for in this T*able*, as in the other aforesaid, is a *Maxime*, grounded vpon the S*tatute*, which is this. When the *quarter of Wheat* is sold for *two shillings*, then the *farthing white-loafe* shall weigh 68 *shillings*, where by *a shilling* is meant $\frac{1}{20}$ of a *pound*, and by a *penny* $\frac{1}{20}$ of an *ounce*. Now therefore for a *generall Rule*, to finde what weight the *Farthing white-loafe* shall weigh at 3 *shillings the quarter*, till you come to 40 *shillings* 6 *pence the quarter*, is thus to be wrought. Comming to the *first ground*, and working by the *Backer Rule*, *say*; if *two shillings the quarter*, giue, or allow the *Farthing white-loafe* to wey 68 *shillings*, what weight ought the *Farthing-white-loafe* to weigh at 3 *shillings the quarter?* Worke, and you shall find 45 *shillings*, 4 *pence*,

as before in the correction of the *first Table* is noted. Then for the *second worke*, say by the *Rule of 3 direct*, if 20 *pence* giue one *ounce*, what giueth 45 *shillings*, 4 *pence*? Multiply and diuide and you shall finde 544 *ounces*, which 544 *ounces* being multiplied by 3, for 3 *pounds*, or 60 *shillings*, yeeldeth 1632 *ounces*, which diuided by 20, produceth 81 *ounces*, and $\frac{12}{20}$, or rather $\frac{3}{5}$ of an *ounce*, equall vnto 12 *penny weight*, which is *halfe an ounce*, and 2 *penny weight*, and so maketh in all 6 *pounds*, 9$\frac{1}{2}$ *ounces*, and 2 *penny weight*. Now the next way to continue this *Table*, to know the weight of the *Halfe-penny White-loafe*, is thus. Multiply 1632 *ounces* by 2, and it bringeth forth 3264 *ounces*, and diuided by 20, it yeeldeth 163 *ounces* and $\frac{1}{5}$, which is equall to 13 *pounds*, 7 *ounces*, and 4 *penny weight*, as *M. Record his Table noteth*.

Thirdly, for the weight of the Penny White-loafe, multiply 1632 *ounces* by 4, and diuide by 20, and after by 12, as before, and you shall find 27 *pounds*, 2 *ounces*, and 8 *penny weight*, &c. This *Method*, or else by doubling the *farthing white-loafe*; for the *weight of the Halfe-penny white-loafe*, and so doubling the *halfe-penny White loafe*, for the *weight* of the *Penny White-loafe*, is the order to continue the *Table* to the end thereof.

The

The price of a quarter of Wheate.			The weight of a farthing white loafe.		
po.	s.	d.	po.	ounces.	penny waight.
0	3	0	6	9	12
0	4	6	4	6	8
0	6	0	3	4	16
0	7	6	2	8	$12\frac{4}{7}$
0	9	0	2	3	4
0	10	6	1	11	$6\frac{2}{7}$
0	12	0	1	8	8
0	13	6	1	6	$2\frac{2}{3}$
0	15	0	1	4	$6\frac{6}{9}$
0	16	6	1	2	$16\frac{8}{11}$
0	18	0	1	1	12
0	19	6	1	0	$11\frac{1}{13}$
1	1	0	1	0	$4\frac{4}{5}$
1	2	6	0	10	$17\frac{3}{7}$
1	4	0	—	10	4
1	5	6	—	9	12
1	7	0	—	9	$1\frac{1}{2}$
1	8	6	—	8	$11\frac{13}{19}$
1	10	0	—	8	$3\frac{1}{5}$
1	11	6	—	7	$15\frac{3}{7}$
1	13	0	—	7	$8\frac{4}{11}$
1	14	6	—	7	$1\frac{21}{23}$
1	16	0	—	6	16
1	17	6	—	6	$10\frac{14}{23}$
1	19	0	—	6	$5\frac{7}{13}$
2	0	6	—	6	$0\frac{2}{47}$

The price of a quarter of Wheate.			The weight of a Halfe-penny white loafe.			The waight of a Penny white loafe.		
l	s	d	po.	oũ-ces.	penny waight	po.	oũ-ces	penny waight.
0	3	0	13	7	4	27	2	8
0	4	6	9	0	16	18	1	12
0	6	0	6	9	12	13	7	4
0	7	6	5	5	$5\frac{3}{5}$	10	10	$11\frac{1}{3}$
0	9	0	4	6	8	9	0	16
0	10	6	3	10	$12\frac{4}{7}$	7	9	$5\frac{1}{7}$
0	12	0	3	4	16	6	9	12
0	13	6	3	0	$1\frac{1}{3}$	6	0	$10\frac{2}{5}$
0	15	0	2	8	$12\frac{4}{5}$	4	5	$5\frac{3}{5}$
0	16	6	2	5	$13\frac{5}{11}$	4	11	$6\frac{10}{11}$
0	18	0	2	3	4	4	6	8
0	19	6	2	1	$2\frac{2}{31}$	4	2	$4\frac{4}{13}$
1	1	0	2	0	$9\frac{1}{3}$	3	0	$19\frac{1}{3}$
1	2	6	1	9	$15\frac{1}{3}$	3	7	$10\frac{2}{3}$
1	4	0	1	8	8	3	4	16
1	5	6	1	7	4	3	2	8
1	7	0	1	6	$2\frac{2}{5}$	2	0	$5\frac{1}{5}$
1	8	6	1	5	$3\frac{11}{19}$	2	10	$7\frac{3}{19}$
1	10	0	1	4	$6\frac{2}{5}$	2	8	$12\frac{4}{5}$
1	11	6	1	3	$10\frac{6}{7}$	2	7	$1\frac{5}{7}$
1	13	0	1	2	$16\frac{8}{11}$	2	5	$13\frac{5}{11}$
1	14	6	1	2	$3\frac{19}{23}$	2	4	$7\frac{15}{23}$
1	16	0	1	1	12	2	3	4
1	17	6	1	1	$1\frac{3}{35}$	2	2	$2\frac{6}{35}$
1	19	0	1	0	$11\frac{1}{13}$	2	1	$2\frac{2}{13}$
2	0	6	1	0	$0\frac{4}{45}$	2	0	$0\frac{8}{45}$

H*auing spoken before for the vnderstanding of the* Table *placed by* M. Record, *a man indued with rare knowledge in* Arithmeticall and Geometricall proportions, *touching the* Statute *of* Coynage, and the Standard thereof, *as appeareth in his Epistle of this* Book, dedicated to K. Edw. the sixt, *insinuating vnto his Highnesse that the* Standard of Coyne *is much altred from the* 14 *yeere of* K. Ed. the third (*when this* Statute & Assise *was confirmed*) *to the* Standard of this our time. *For it appeareth that in* K. Ed. the thirds *time, when the* Assise of Bread and Drink *was established, that a* sterling peny, round without clipping, *did then weigh* 32 corns of wheat dry, *and taken out of the midle of the* eare, *and* 20 *of these* pence *made an* ounce, *and* 12 ounces *made a* pound Troy. *And so from the* weight of a peny *to* 20 shillings sterling, *which then weighed* 12 ounces, *took* Bread his weight *and proportion* : *And now finding* 60 pence *is an* ounce. *That onely cause* (*I perceiue, for the zeale of a Common-wealth*) *moued him to set downe the same* Table in this priuate book; *meaning not thereby to alter any word of the* statute *being so good an ordinance and of so long continuance, but as a kinde of exposition by the way, that thereby the* stat. *may be better vnderstood, & so consequently better put in execution. Which* Assise of his, *is three times greater then the* statute *now alloweth. Therefore also* (*to gratifie such as are desirous of knowledge, according to these* prices of a Quarter of wheat) *I haue added to this Author* these three other new Tables following, *and reduced their* prices *into their iust proportions of* sterling money, *and also reduced the* Money *into known* weight Troy, *according to the* statute. *And thereafter according to proportion in my other three* Tables, *haue I noted the* iust weight, *that a* Farthing, Halfe peny, and peny white loafe *ought to weigh by the* statute.

Z The

The price of a quarter of Wheate.			The weight of a farthing white loafe in Sterling money by assise.			The waight of a Farthing white loafe in Troy weight by Assise.		
L	s	d	L	s	D	ounces	po.	penny waight.
0	3	0	2	5	4	2	3	4
0	4	6	1	10	2 2/3	1	6	2 2/3
0	6	0	1	2	8	1	1	12
0	7	6	0	18	1 3/5	0	10	17 3/5
0	9	0	0	15	1 1/5		9	1 1/5
0	10	6	0	12	15 3/7		7	15 3/7
0	12	0	0	11	4		6	16
0	13	6	0	10	0 8/9		6	0 8/9
0	15	0	0	9	0 4/7		5	8 4/7
0	16	6	0	8	2 10/11		4	18 10/11
0	18	0	0	7	6 2/3		4	10 2/3
0	19	6	0	6	11 9/13		4	3 7/13
1	1	0	0	6	9 3/5		4	1 3/5
1	2	6	0	6	0 8/13		3	12 8/13
1	4	0	0	5	8		3	8
1	5	6	0	5	4		3	4
1	7	0	0	5	0 4/9		3	0 4/9
1	8	6	0	4	9 5/19		3	17 5/19
1	10	0	0	4	6 2/5		2	14 2/5
1	11	6	0	4	3 17/21		2	11 17/21
1	13	0	0	4	1 1/11		2	9 1/11
1	14	6	0	3	11 1/23		2	7 7/23
1	16	0	0	3	9 2/3		2	5 2/3
1	17	6	0	3	7 13/25		2	3 13/25
1	19	0	0	3	5 11/13		2	1 11/13
2	0	6	0	3	4 8/27		2	0 8/27

The price of a quarter of Wheat. l.	s.	d.	The weight of the Halfe-penny white loafe in Troy weight by Assise. po.	ou.	penny weight.	The weight of a penny white loafe in Troy weight by Assise. po.	ou.	penny weight.
0	3	0	4	6	8	9	0	16
0	4	6	3	0	$5\frac{1}{3}$	6	0	$10\frac{2}{3}$
0	6	0	2	3	4	4	6	8
0	7	6	1	9	$15\frac{1}{5}$	3	7	$10\frac{2}{5}$
0	9	0	1	6	$2\frac{2}{3}$	3	0	$5\frac{1}{3}$
0	10	6	1	3	$10\frac{6}{7}$	2	7	$1\frac{5}{7}$
0	12	0	1	1	12	2	3	4
0	13	6	1	0	$1\frac{7}{9}$	2	0	$3\frac{5}{9}$
0	15	0	0	10	$17\frac{3}{5}$	1	9	$15\frac{1}{5}$
0	16	6	0	9	$17\frac{9}{11}$	1	7	$15\frac{7}{11}$
0	18	0	0	9	$1\frac{1}{3}$	1	6	$3\frac{2}{3}$
0	19	6	0	8	$7\frac{1}{13}$	1	4	$14\frac{2}{13}$
1	1	0	0	8	$3\frac{1}{3}$	1	4	$0\frac{2}{5}$
1	2	6	0	7	$5\frac{1}{15}$	1	2	$10\frac{2}{15}$
1	4	0	0	6	16	1	1	12
1	5	6	0	6	8	1	0	16
1	7	0	0	6	$0\frac{8}{9}$	1	0	$1\frac{7}{9}$
1	8	6	0	5	$14\frac{10}{19}$	0	11	$9\frac{1}{19}$
1	10	0	0	5	$8\frac{4}{5}$	0	10	$17\frac{3}{5}$
1	11	6	0	5	$3\frac{14}{21}$	0	10	$7\frac{5}{21}$
1	13	0	0	4	19	0	9	18
1	14	6	0	4	$14\frac{14}{23}$	0	9	$9\frac{5}{23}$
1	16	0	0	4	$11\frac{1}{3}$	0	9	$2\frac{2}{3}$
1	17	6	0	4	$7\frac{1}{25}$	0	8	$14\frac{2}{25}$
1	19	0	0	4	$3\frac{9}{27}$	0	8	$7\frac{5}{23}$
1	0	6	0	4	$0\frac{16}{27}$	0	8	$1\frac{5}{29}$

 Scholar.

Scholar. Sir, I do thanke you most heartily for this, not onely in mine owne name, and in the name of all Students, but also in the name of the whole Commons, to whom the restitution of this Assise (I trust) shall bring restitution of the weight in Bread, which long time hath béene abused. And if you know any like things more, wherein you would vouchsafe to declare the errours, and set forth the truth, you cannot but obtain great thanks of all good hearted men that loue the Commonwealth.

Master. I haue sundry things to declare, but I haue reserued them for a priuate Booke by it selfe, yet notwithstanding because the Statute of the rate of measuring of ground is so common, that it toucheth all men, and yet no more common then néedfull, but so much corrupt, that is, too farre out of all good rate, not onely in the English Books of Statutes, commonly printed, but also in the Latine Books, and in the French also (for I haue read of each sort, and conferred them diligently) I will giue you a Table for the restitution of those errours, as may suffice for this present time. And first I will propose one question to you touching the vse of that Statute, whereby you may perceiue the order how to examine the whole Statute, and euery parcell thereof, and the question is this.

A question of measuring of ground.

When the Acre of ground doth containe foure Perches in breadth, then must it containe 40 Perches

ches in length. Then do I demand of you, how much shall the length of an Acre be, when there is in the breadth of it 13 Perches. But before you shall answer to this question, I will declare vnto you another Statute, which is the ground of the former Statute. And that Statute is this.

It is ordained that thrée Barly-cornes dry and round, shall make vp the measure of an inch: 12 inches shall make a foot, and 3 foot shall make a yard, (the common English books haue an Elne) fiue yards and an halfe shall make a Perch, and 40 Perches in length, and 4 in breadth, shall make an Acre. This is that Statute, whereby you may perceiue, that the intent of the Statute is, that one Acre should containe 160 square Perches. Now let me heare you answer to the question.

Statute measure.

An Acre.

Scholar. As I perceiue by the words of the Statute, a Perch to be the $\frac{1}{160}$ part of an Acre, so could I make those numbers all in fractions, and so worke the question: but séing I may do it also in whole numbers, I take that forme for the most ease, therefore thus I set the question in forme. Then do I multiply 40 by 4, and it maketh 160, which I diuide by 13, and the quotient is $12\frac{4}{13}$.

4 Z 40

13 Z $12\frac{4}{13}$

Master. Now turne that $\frac{4}{13}$ into the common parts of a Perch, as they be named in the former Statute: howbeit it shall be best to take one of the least parts in Denomination

for auoiding of much labour, as Feet, whereof the Perch containeth $16\frac{1}{2}$.

Scholar. Then to returne $\frac{4}{13}$ into Feet, I multiply $16\frac{1}{2}$ by 4, and it maketh 66, which I must diuide by 13, and the quotient is $5\frac{1}{13}$.

Master. So I finde, that if the Acre hold in breadth 13 Perches, it shall containe in length 12 Perches, 5 Foot, and $\frac{1}{13}$ of a Foot, which is not fully an Inch, for the Inch is $\frac{1}{12}$ of a Foot. But here all the Statute Books in Latine and English (that I haue seene) do note it to be 13 Perches, 5 Foot, and one Inch, which maketh aboue 13 Perches too many in the Acre: so that I would haue thought the errour to haue crept into the Printed Bookes, by the great negligence that Printers in our time do vse, saue that in written Copies of great antiquity, I do finde the same: yet haue I one French copy which hath 12 Perches $\frac{1}{4}$ and one Foot, and that misseth very little of the truth.

Note this errour.

Scholar. Then I see it is true that I haue often heard say, that the truest Copies of the Statutes, be the French Copies.

Master. That is often true, but not generally, as I haue by conference tried diuersly: but in this Statute the French Booke is most corrupt: in all other places lightly.

But now to performe my promise, I will set forth the Table for measuring of an Acre of ground, onely by such parts as the statute doth mention, because at this time I do of

purpose

purpose write it for the better understanding of that Statute, and hereafter with other things intend to set forth this same more at large.

In this Table following, I haue not done as in the other Statute before compared by restitution with the faults crept into the Statute, but onely haue written that true measure, which the equity of the Statute doth pretend. For it were vile to iudge of so noble Princes and worthy Counsellors, as haue authorised and set forth this Statute, that they would make one Acre in any form greater then another, but euery one to be iust and equall with each other, which is the ground also of my worke: and hereby may all men perceiue how needfull Arithmeticke is to the Students of Law. But now I thinke best to make an end of these matters for this present time, sith the Table hath in it none obscurity that I should need to declare.

The breadth of the Acre.	The length of the Acre.			
Perches.	Perches.	Feet.	Inches.	Parts of an Inch.
10	16	0	0	0
11	14	9	0	0
12	13	5	6	0
13	12	5	0	$\frac{12}{13}$
14	11	7	0	$\frac{6}{7}$
15	10	11	0	0
16	10	0	0	0
17	9	6	9	$\frac{9}{17}$
18	8	14	8	0
19	8	6	11	$\frac{7}{19}$
20	8	0	0	0
21	7	10	2	$\frac{4}{7}$
22	7	4	6	0
23	6	15	9	$\frac{9}{23}$
24	6	11	0	0
25	6	6	7	$\frac{1}{5}$
26	6	2	7	$\frac{6}{13}$
27	5	15	3	$\frac{1}{3}$

The

The breadth of the Acre.	The length of the Acre.			
Perches.	Perches.	Feete	Inches.	Parts of an Inch.
28	5	11	9	$0\frac{3}{7}$
29	5	8	5	$0\frac{12}{19}$
30	5	5	6	0
31	5	2	7	$0\frac{29}{31}$
32	5	0	0	0
33	4	14	0	0
34	4	11	7	$0\frac{13}{17}$
35	4	9	5	$0\frac{1}{7}$
36	4	7	4	0
37	4	5	4	$0\frac{8}{37}$
38	4	3	5	$0\frac{13}{19}$
39	4	1	8	$0\frac{4}{13}$
40	4	0	0	0
41	3	14	10	$0\frac{28}{41}$
42	3	13	4	$0\frac{2}{7}$
43	3	11	10	$0\frac{33}{43}$
44	3	10	6	0
45	3	9	2	0

Scholar.

Scholar. Indeed Sir, I understand the Table (as I think) by those other which you set forth before. For in the first columne is set the Perches of the breadth of an Acre, & then in the two columnes following appeareth how many Perches, and how many foot that same Acre must haue for his length.

Master. You take it well: howbeit to speak exactly of breadth and length, and the first columne doth sometime betoken the breadth, & sometime the length: for properly the longest side of any square doth limit his length, and the shorter side doth betoken the breadth, yet it is no great abuse in such Tables, where a man cannot well change the title, to let the name remain, although the proportions of the numbers do change: for still by the first columne is expressed the measure of the one side, and by the two other Pillars in one columne, is set forth the measure of the other side. And this shall be sufficient now for the vse of the Golden Rule.

The Rule of Fellowship.

NOw somewhat will I touch certaine other Rules which for their seuerall names may seeme diuers Rules, and distinct from this, but indeed they are but branches of it: yet because they haue seueral workings in appearance, but

also

also pleasant in vse, I will giue you a taste of each of them. As for the Rule of Fellowship, both single and double, with time and without time, I shall need to say little more then I haue already said in teaching the workes of whole numbers: yet an example or two will we haue to refresh the remembrance of the same; and to declare certaine proper vses and applications of it, as this for one.

The Rule of Fellowship without time.

Foure men got a booty or prize in time of warre, the prize is in value of money 8190 pound, and because the men be not of like degree, therefore their shares may not be equall: but the chiefest person will haue of the booty the third part, and the tenth part ouer: the second will haue a quarter, and the tenth part ouer, the third will haue the sixth part: and so there is left for the fourth man a very small portion, but such is his lot (whether he be pleased or wroth) he must be content with one 20 part of the prey. Now I demand of you what shall euery man haue to his share?

A question of inequal society.

Scholar. You must be faine to answer to your owne question, else it is not like to be answered at this time.

Master. The forme to understand the solution of this question, and all such like, is this: Reduce all the Denominators into one number by Multiplication, except that any of them be parts of some other of them, for all such parts you may ouerpasse, and take for them all those numbers, whose parts they be: As in this example the shares be these $\frac{1}{3}\frac{1}{10}$, $\frac{1}{4}\frac{1}{10}$, $\frac{1}{6}\frac{1}{10}$, if

I

I multiply all the Denominators together, beginning with 3, and so go on unto 20, it will make 144000: but considering that 3 is a part of 6 I will omit that 3, and likewise 10, which is a part of 20, I may ouerpasse also, and then is there but 3 denominators to multiply, that is, 4, 6, & 20 which make 480, which summe I take for my worke, because all the denominators will be found in it. Then I take such parts of it as the question importeth, that is, for the first man $\frac{1}{3}$ and $\frac{1}{10}$, the $\frac{1}{3}$ is 100, the $\frac{1}{10}$ is 48, which I put in one summe for the first mans share, and it maketh 208. Then for the second mans share, I take $\frac{1}{4}$, which is 120, and $\frac{1}{10}$ which is 48, and that maketh in the whole 168. Now for the third man which must haue $\frac{1}{6}$ I take 80. And for the fourth man there remaineth but 24, which is $\frac{1}{20}$ of the whole summe: so that if the whole prey had béene but 480 pound, then were the question answered: but because the summe was of greater value, by this meanes now shall I know the partition of it. I must set my numbers by the order of the Golden Rule, putting in the first place the number of that I found by multiplying the Denominators, and in the second place the summe of the booty. And looke what proportion is betweene the first number and the second, the same proportion shall be betweene the parts of that first number, and the parts of the second, comparing each to his like. Therefore I must put in the third

The reason of this rule.

third

third place, one of the parts or ſhares, and then worke by the former Rule of proportion or Golden Rule. And becauſe I haue foure ſeuerall parts of the firſt number, by which I would finde out foure like parts of the ſecond number, therefore muſt I make foure ſeuerall figures.

Scholar. Now I truſt I can anſwer to your queſtion, as by your fauour I will proue.

And to trie it, I ſet the foure figures thus, marked with A, B, C, D, to ſhew their order:

A

480 — 8190
208

B

480 — 8190
160

C

480 — 8190
80

D

480 — 8190
24

And then in each of them I multiply the ſecond number by the third, and diuide their totall by the firſt, and ſo amounteth the fourth ſumme which I ſeeke for: For if I do multiply 8190 by 208, it maketh 1703520, which being diuided by 480, maketh in the quotient 3549 for the firſt mans portion.

And ſo working with the other three figures, I finde for the ſecond man 2866 $\frac{1}{2}$, and for the third man 1365, and then for the fourth man 409 $\frac{1}{2}$, and ſo euery mans ſhare is ſet forth in the figure here annexed.

A

A		B	
480	8190	480	8190
208	3549	168	$2866\frac{1}{2}$
C		**D**	
480	8190	480	8190
80	1365	24	$409\frac{1}{2}$.

And thus I thinke I haue done well.

The proofe by Addition.

Master. If you misdoubt your working, and list to proue it, adde all the shares together; and if they make the totall, then seemeth it well done.

Scholar. I may set them thus: and then by Addition the iust summe doth amount, that is, 8190, and therefore (as you say) it seemeth to bee well wrought.

$$\begin{array}{r} 3549 \\ 2866\frac{1}{2} \\ 1365 \\ 409\frac{1}{2} \\ \hline 8190 \end{array}$$

But I beseech you, is there any doubt in this triall, that you vse that word, Seemeth?

Master. You may easily coniecture, that if you did assigne the first mans share to the last, and so change all the rest, and one had anothers share, yet would the Addition appeare all one, and therefore is not the proofe exact.

The iust proofe.

But if you will make a iust proofe for the first mans part, take $\frac{1}{3}$ and $\frac{1}{18}$ of the whole summe, and if it agree with the number in the figure, then it is well done. And so do for the second, third, and fourth summes, and this proofe faileth not. Now will I propound certaine other questions, which haue beene set

forth

forth by certaine learned men, albeit not without some ouersight, which questions I protest heartily, I do not repeat to depraue those good men, whose labours and studies I much praise, and greatly delight in. But onely according to my profession, to seeke out truth in all things, and to remoue all occasions of errour as much as in me lyeth: and for that cause I will onely name the questions without hurting the Authors name.

The first question is this.

Foure men did build an house, which cost them 3000 crowns, their shares were such that one man should pay $\frac{1}{2}$ of the summe, and six crowns ouer: the second should pay $\frac{1}{3}$ and 12 crownes ouer: the third man must lay out $\frac{1}{3}$ abating 8 crownes: and the fourth man should pay $\frac{1}{4}$ and 20 crownes more. Can you answer to this question?

A question of building.

Scholar. No, I cannot sir, and that you know best of any man, for I know no more then you haue taught me.

Master. Then I dare say you cannot do it, neither yet the best learned man that euer did propose it: for the question is impossible. For declaration whereof, I will be bold to vse first the representation of the numbers in their aptest forme (although I haue not yet taught that manner of worke) because it may appeare plainly that the question is not possible. For here I haue set the parts, and added them, and they make the whole summe, and $\frac{3}{4}$ and 30 more. Now, how is it possible to

An impossible question.

to diuide truly either gaines, either charges, so that the particulars $1\frac{3}{4}$ shall be more then the totall?

Scholar. It is against the forme of proofe by addition of parts.

$\frac{1}{2}$ +	6
$\frac{1}{3}$ +	12
$\frac{2}{3}$ —	8
$\frac{1}{4}$ +	20
$1\frac{3}{4}$ +	30

Master. You say truth. And (because you shall perceiue it the better) I will trie it after the vulgar forme, as in this figure you see where the $\frac{1}{2}$ with 6 ouer, is 1506, for the totall as you heard before, is 3000, the $\frac{1}{3}$ and the 12 more is 1012: the $\frac{2}{3}$ would be 2000, but then abating 8, it is but 1992, and then last of all, the $\frac{1}{4}$ is 750, and the 20 more maketh 770: which all being added in one summe, do make 5280, where the totall summe should bee but 3000, which sum of 3000 if you diuide by $1\frac{1}{3}$ or $\frac{3}{4}$, you shall haue $\frac{3}{4}$ of it, that is 22[illegible], and thereto adde 30 more, then will those 3 summes make 5280: whereby you may see how this forme (as well as the other) doth declare that the particulars in that question would make more then the whole summe by $\frac{3}{4}$, and 30 more, and therefore can that question not be accepted as a possible thing, but yet do certaine learned men propound such questions, and answer to them: There-

1506
1012
1992
770
5280

2000
2250
5280

Therefore somwhat to say to their excuse (rather of their good meaning, then for their doing) I will anon declare what may bee said for their defence: but in the meane season, I will propound the Question as it may bee wrought by good possibility.

As if foure men build a house together, and it cost them 3000 crownes, and then for the partition they agree thus: that as often as the first man doth pay 6 crownes, so often the second man shall pay 4, the third man 8, and the fourth man 3. Or else thus, that the first man shall pay double so much as the fourth, and the second man shall pay $\frac{2}{3}$ of the first mans charge: the third man shall pay double so much as the second: and these two wayes are to one end) but further for their agreement it is appointed also, that the first shall giue 6 crownes ouerplus, and the second 12, and the fourth shall giue 20: but the third man shall giue no ouerplus, but shall haue 8 crownes abated of his charge.

The former question of building now possible.

Now is the question possible to be assoiled, and this is the way to doe it. Marke the proportion of the seuerall charges, and set out small numbers in that rate, by which you may reduce the worke to the golden rule, as here in the first forme the numbers are already named, 6, 4, 8, 3: and in the second forme (although they be but plainely named, yet they may bee the same numbers: for 6 is double to 3, and 4 is $\frac{2}{3}$ of 6: and againe 8 is double to 4. Now adde these together, and they make 21, which 21 must bee set for the first number in the gol-

den Rule: for if it with the overplus of each mans charge would make the totall summe of the charges, then were those severall summes the charges of each man, besides his overplus: but now it is not so.

The Rule. But yet this is true: (so excellent are conclusions Arithmeticall) that looke what proportion each of their severall summes doth beare to 21, the same proportion doth the iust charges of every man (besides his overplus) beare to the totoll of the charges, the overplus being deducted: wherefore this may you note, that before you doe apply the totall of his charges to the Golden rule, you must deduct the overplus, which is 6, 12, and 20, that is in the whole 38: but then 8 must be restored for the abatenent of the third man, and then remaineth to be deducted 30: take 30 therefore out of 3000, and there will rest 2970, which I must set in the Golden rule, for the second sum: and for the third summe, I must put each of the small numbers before mentioned, which although they bee not severall charges, yet they represent them in proportion. And so making for every mans charge a severall question, the figures will bee 4, which I marke with foure letters, A, B, C, D, thus.

A

21 Z 2970
6 Z $848\frac{4}{7}$

B

21 Z 2970
4 Z $565\frac{5}{7}$

C

21 Z 2970
8 Z $1131\frac{3}{7}$

D

21 Z 2970
3 Z $424\frac{2}{7}$

Where I haue set for briefnesse the summe of every mans charge in the fourth place, presupposing that you can tell how to try out that fourth summe by so many Examples as yée haue had.

Scholar. As I trust that I understand this forme, so I desire much to know what may bée sayd for them that mistoke this Question.

Master. You séeme so desirous to know this errour, that you haue forgotten to examine, whether this worke be without fault.

Scholar. Me séemeth this worke to be well done, because the Addition of the foure seuerall numbers doth make the totall summe of 2970, which was to be diuided into such foure parts

Master. But then haue you forgotten that the first man must pay sixe crownes more besides his share, and the second man 12 Crownes more, the third man 8 Crownes lesse, and the fourth man 20 Crownes more: for without these, your first totall of 3000 Crownes will not be made:

Scholar. Then must I adde to the first mans summe 6 more, and it will bée $854\frac{4}{7}$, and to the seconds summe, I must adde 12, and it will be $577\frac{5}{7}$: from the third s summe I must abate 8, and then will the summe bee $1123\frac{3}{7}$: then adding unto the fourths summe

20, it will bee 444 $\frac{2}{7}$, and these foure summes will make 3000, which is the whole change, as in this example it may appeare, where first I gather the $\frac{14}{7}$, that maketh 2, and so procéed I in the Addition to the end.

854 $\frac{4}{7}$
577 $\frac{5}{7}$
1123 $\frac{3}{7}$
444 $\frac{2}{7}$
3000

Master. Now haue you well done, and this worke in the same summes is brought of other learned men for the true solution of the question, as it was first proponed, which (as I sayd) was impossible: and now examine by these seuerall summes, and sée whether it doth agree with the summes in the question proponed.

The first man must pay $\frac{1}{2}$ and 6 ouer of the totall summe: how thinke you, is 854 $\frac{4}{7}$ the halfe, and 6 more of 3000?

Scholar. No that it is not, for it should bee 1506: and for the second man 1012: and for the third man 1992, & for the fourth man 770: whereof not one summe agreeth to this worke. But I maruell, that so wise men could bee so much ouer-seene.

Master. It is commonly seene, that when men will receiue things from elder Writers, and will not examine the thing, they seeme rather willing to erre with their Ancients for company, then to bee bold to examine their workes or writings. Which scrupulosity hath ingendred infinite errors in all kindes of

knowledge,

knowledge, and in all ciuill administration, and so in euery kinde of Art. But these Learned men did not meane any other thing by this question, then to finde such numbers as should beare the same proportion together, as those numbers in the question proponed did beare one to another: which thing you shall perceiue more plainely by another question of theirs, that is this.

A man lying vpon his death-bed, bequeathed his goods (which were worth 3600 Crownes) in this sort. Because his Wife was great with child, and hee yet vncertaine whether the Childe were male or female, hee made his bequest conditionally, that if the Wife bare a Daughter, then should the Wife haue halfe his goods, and the Daughter $\frac{1}{3}$; but if shee were deliuered of a Sonne, then that Sonne should haue $\frac{1}{2}$ of the goods, and his Wife but $\frac{1}{3}$. Now it chanced her to bring forth both a Sonne and a Daughter; the question is; How shall they part the goods agreeable to the Testatour his Will? A question of a Testament.

Scholar. If some cunning Lawyers had this matter in scanning, they would determine this Testament to be quite voide, and so the Man to dye intestate, because the Testament was made insufficient, sith this condition was not expressed in it, and also it might haue chanced, that she should haue brought foorth neither Sonne nor Daughter, as often hath beene seene: so is the Will insufficient in that point also.

Master. Such Scanners should seeme to cunning, and yet not so cunning as cruell: for the minde of the Testator is to be taken fauorably for the aide of the Lygatories, when there ariseth such doubt. But let vs try this worke, not by force of Law, but by proportion Geometricall, seeing the Testator did minde to prouide for each sort of them.

Scholar. If the Sonne shall haue $\frac{1}{3}$ by force of the Testament, so must the mother haue $\frac{1}{3}$. Againe, because she hath a Daughter also, therfore ought shee to haue $\frac{1}{2}$, and the Daughter $\frac{1}{3}$, that is both wayes $\frac{1}{2}\frac{1}{3}$, and $\frac{1}{3}\frac{1}{2}$, which commeth to the whole goods, and $\frac{2}{3}$ more.

Wherefore it seemeth also impossible.

Master. In this matter, the minde of the Testator is so to be vnderstood, that such proportion should be betweene the portion of the Wife, and the Sonne, as is betweene $\frac{3}{1}$ and $\frac{1}{2}$, that is, the Sonne must haue $\frac{3}{6}$ for $\frac{2}{6}$ to his Mother, so shall hee haue 3 to 2, that is, as much as his Mother, and halfe as much more; and the Mother must haue the like rate in comparison to her Daughter. Then must I finde out three numbers in such proportion, that the first may haue as much as the second, and halfe as much more (that is) in proportion sesquialtera, and the second to the third, in that same proportion: such numbers be 9, 6, 4.

Scholar. I pray you Sir, how shall I finde out these numbers?

Master. That will I gladly tell you.

What

Whatsoeuer the proportion bee of any three numbers, multiply the Termes of that proportion together, and the number that amounteth, shall bee the middle number of the three: then multiply that middle number by the lesser terme, and diuide that totall by the greater, and the least number of the three will amount. So if you multiply that middle number by the greater extreame, and diuide the totall by the lesser extreame, then will the greatest number of that Progression amount. To finde 3 numbers in any proportion.

Scholar. Then in this example to finde the proportion of $\frac{1}{2}$ to $\frac{1}{3}$, I must diuide (as you taught mee in diuision) $\frac{1}{2}$ by $\frac{1}{3}$, and the quotient will be $\frac{3}{2}$, that is, $1\frac{1}{2}$, whereby I perceiue that the proportion in this question, is as 3 to 2. Therefore as you taught mee euen now, I multiply 3 by 2, and the summe is 6, which must bee the middle number: then I multiply the middle number 6 by 2, which is the least tearme, and the summe is 12, that I doe diuide by 3, beeing the greater terme, and the quotient is 4: so is 4 the least number of the three. Then I multiply 6 by 3, whereof commeth 18, and that I diuide by 2, and so haue I 9 which is the greatest number of the three. To finde the proportion between two numbers.

Master. Another way yet may you finde the third number in any Progression, if you haue two of them: for if the middle number be one of them which you haue, then multi-

ply it by it selfe (as in thir example, 6 by 6, maketh 36) and that totall diuide by the other number which you haue, and the third number will be the quotient.

Scholar. Then I diuide 36 (which commeth of 6 multiplyed by it selfe) by 4 the quotient will be 9: and if I diuide 36 by 9, the quotient will be 4. But what if I know the first number and the third, and would haue the middle number?

Master. Multiply the two numbers together, and in their totall you must seek the root of that number, and it shall be the middle number: but because as yet you haue not learned to extract Roots, therefore vse the first forme which I haue taught you, till I teach you to extract Roots. And now goe forwards with the answer to the same question.

Note.

Scholar. I perceiue then, that the Sonne must not haue $\frac{1}{3}$ of the goods, neither the Mother $\frac{1}{3}$, nor yet the Daughter $\frac{1}{3}$, but yet must the goods be diuided into such proportion, that the Sonne shall haue 9 Crownes for 6 to his Mother, and the Mother shall haue 6 Crownes for euery 4 to her Daughter. Then I apply it to the Golden rule in three examples, as followeth.

Where the first number is the Addition of those three numbers 9, 6, 4: and the third is one of them seuerally: the second is the totall of the goods in that Testament: and then

by the worke of the Golden Rule, I finde out the fourth number in euery worke: that is for the Sonne $1705\frac{5}{19}$, for the Mother $1136\frac{16}{19}$, and for the Daughter $757\frac{17}{19}$: the which ſummes added together, doe make the ſumme of the whole goods as may be ſeene by this Example.

19	3600
9	
16	3600
9	
19	3600
4	

$1705\frac{5}{19}$
$1136\frac{16}{19}$
$757\frac{17}{19}$
3600

And this (me thinketh) I doe perceiue, becauſe in this caſe there is a neceſſary remedie deuiſed againſt an urgent inconuenience: therefore theſe learned men thought they might vſe the like liberty in that other queſtion.

Maſter. Your geſſe is good, but they had ſo good reaſon for them in the one, as they haue in the other: As in another example of theirs, it may better appeare, as in this.

A man left vnto his three Sonnes 7851 crownes to be parted in ſuch ſort, that the firſt Sonne ſhould haue $\frac{1}{2}$, the ſecond Sonne $\frac{1}{3}$, and the third Sonne $\frac{1}{4}$, which is not poſſible: for $\frac{1}{2}$, and $\frac{1}{3}$, and $\frac{1}{4}$ doe make $\frac{26}{24}$, or $\frac{13}{12}$, that is, $1\frac{1}{12}$, ſo it is more then the whole, but reduce theſe Fractions into one denomination, the leaſt that they will come to, and they will Another queſtion of a Teſtament.

be

be $\frac{6}{13}$, $\frac{4}{13}$, $\frac{3}{13}$, and ſo may you part the goods into ſuch proportion as theſe three numerators beare together, that is, the firſt to haue 6 for euery 4 to the ſecond, and the ſecond to haue 4 as often as the third hath 3: and ſo their portions will bee for the firſt, 3623 $\frac{7}{13}$, for the ſecond 2415 $\frac{9}{13}$, and for the third 1811 $\frac{10}{13}$; and theſe three ſhares added together, will make the totall ſumme of the whole goods, as you may eaſily ſee in this example.

3623 $\frac{7}{13}$
2415 $\frac{9}{13}$
1811 $\frac{10}{13}$
7851

Another Queſtion **is there propoſed thus.**

Another like queſtion.

There are 450 crownes to bee diuided betweene three men, ſo that the first man muſt haue $\frac{1}{2}$ and $\frac{1}{3}$, the ſecond man $\frac{1}{3}$ and $\frac{1}{4}$; the third man ſhall haue $\frac{1}{4}$ and $\frac{1}{5}$

Scholar. **I maruell that any man ſhould be ſo ouerſeen, to propone that** queſtion **as a thing poſſible, ſith** $\frac{1}{2}\frac{1}{3}$, $\frac{1}{3}\frac{1}{4}$, $\frac{1}{4}\frac{1}{5}$, **doe make** 1 $\frac{12}{13}$ **that is almoſt double the** whole ſumme.

☞ **But I perceiue it might bee thus proponed: that as often as the** firſt man **did receiue** 50 crównes, **ſo often the** ſecond man **ſhould receiue** 35, **and the** third man 27: **for** $\frac{1}{2}$ **and** $\frac{1}{3}$ **is equall to** $\frac{50}{60}$: **and ſo is** $\frac{1}{3}$ **and** $\frac{1}{4}$ **equall to** $\frac{35}{60}$, **and** $\frac{1}{4}$ **and** $\frac{1}{5}$ **is** $\frac{27}{60}$: **and ſo working the** queſtion **the** three figures **wil appeare in this forme: whereby the** firſt mans **portion is found to bée** 200 $\frac{50}{56}$: **the** ſecond mans **part is**

112	450
50	200 $\frac{50}{56}$
112	450
35	140 $\frac{35}{56}$
112	450
27	108 $\frac{17}{58}$

$140\frac{35}{36}$: the third mans share $108\frac{27}{36}$: which in the whole doth make 450 Crownes to bee diuided betweene them.

Master. And thus you are (I thinke sufficiently instructed in the Rule of Fellowship.

The Rule of Alligation.

Now will I goe in hand with the Rule of Alligation; which hath his name, for that by it there are diuers parcells of sundry prices, and sundry quantities alligate, bound or mixed together: whereby also it might bee well called the Rule of Mixture; and it hath great vse in composition of Medicines, and also in mixtures of Metalls, and some vse it hath in mixtures of Wines: but I wish it were lesse vsed therein then it is now adayes. The order of this rule is this. The rule of Mixture.

When any summes are proponed to bee mixed, set them in order one ouer another, and the common number (whereunto you will reduce them) set on the left hand; then marke what summes bee lesser then that common number, and which bee greater, and with a draught of your Penne euermore linke two numbers together, so that one bee lesser then the The reason of this Rule

the common number, and the other greater then hee: (for two greater or two smaller cannot well bee linked together) and the reason is this, that one greater and one smaller, may bee so mixed, that they will make the meane or common number very well: but two lesse can never make so many as the common number, being taken orderly: no more can two summes greater then the meane, ever make the meane in due order, as it shall appeare better to you hereafter. And as it is of necessity to linke every smaller (once at the least) with one greater, and every greater with one smaller, so it is at liberty to linke them oftner then once: and so may there bee to one question, many solutions. When you haue so linked them, then marke how much each of the lesser numbers is smaller then the meane or common number, and that difference set against the greater numbers, which be linked with those smaller, each with his match still on the right hand, and likewise the excesse of the greater numbers aboue the meane, you shall set before the lesser numbers, which bee combined with them. Then shall you (by addition) bring all these ☞ differences into one summe, which shall bee the first number in the golden Rule, and the second number shall be the whole masse that you will haue of all those particulars: the third summe shall be each difference by it selfe, and then by them shall bee found the fourth number, declaring the iust portion of every particular in

that

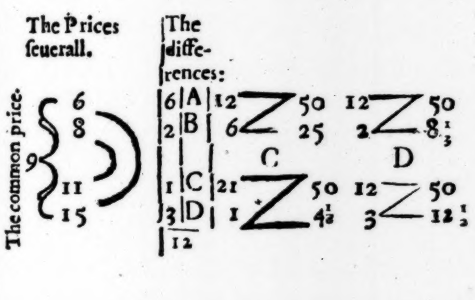
that mixture: As now by these Examples I will make it plaine.

There are foure sorts of Wine, of severall prices, one of 6 pence *a gallon, another of* 8 pence, *the third of* 11 pence, *and the fourth of* 15 pence *the gallon. Of all these Wines would I have a mixture made to the summe of fifty gallons, and so the price of each gallon may be* 9 pence. *Now demand I, how much must bee taken of every sort of Wine?*

A question of mixing of wines.

Scholar. If it shall please you to worke the first example, that I may marke the applying of it to the rule: then I trust I shall bee able, not only to doe the like, but also to see the reason in the order of the worke.

Master. Marke then this forme, and the placing of every kinde of number in it.



Heere (you see) I have set downe the severall

rall prices, which bee 6, 8, 11, 5, and haue linked together 6 with 15, and 8 with 11. The common price 9, I haue set on the left side, and the difference betwéene it, and euery particular price, I haue set on the right hand, not against the summe (whose difference it is) but against the summe that it is linked withall, so the difference of 15 aboue 9, is 6, which I haue set, not against 15 but 6, that is linked, with 15, and the difference betwéene 6 and 9 (that is 3) I haue set against 15. So likewise the difference betwéene 8 & 9, is but 1, that I haue set against 11, and the difference of 11 aboue 9 (which is 2) I haue set against 8. Then adde I all those foure differences, and they make 12, which I set for the first number in the Golden Rule: the second number I make 50, which is the summe of Gallons that I should haue, and the third summe is euery particular difference. Now if you worke by the Golden Rule, you shall finde the number of Gallons that shall be taken of each sort of wine: For the better distinction whereof, I haue set these letters, A, B, C, D, both against the numbers for which the workes doe serue, and ouer the worke also, which seuerally serue for each of them. And now (if you list to examine the truth of these workes) adde these foure summes together, and they will make 50, that is the totall which I

The proofe of this Rule

would

would haue, as by this example you may eaſily perceiue And (for to proue how the prices doe agrée) doe this: multiply the totall ſumme 50, by the common price 9, and it will make 450: then kéepe that ſumme by it ſelfe, and afterward multiply euery ſeuerall ſumme of Gallons, by the price belonging to the ſame Gallons, & if that ſumme doe agrée with this, which you haue kept firſt, then is your worke well done. As here 25 is the number of Gallons of 6 pence price, multiply then 25 by 6, and it maketh 150, which you ſhall ſet downe, then multiply $8\frac{1}{3}$ by 8, which is the price for the number of Gallons, and it will make $66\frac{4}{6}$: ſo againe $4\frac{1}{6}$ multiplied by 11: doth make $45\frac{5}{6}$. And laſt of all, $12\frac{1}{2}$ multiplyed by 15, maketh $187\frac{3}{6}$: and theſe added together doe make 450, as in the Example annexed you may ſée, wherefore ſéeing it doth agrée with the former ſumme of 50, multiplyed by 9, I may iuſtly affirme this worke to be good, and well done.

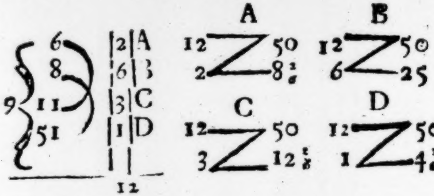
25
$8\frac{1}{3}$
$4\frac{1}{6}$
$12\frac{1}{2}$
50

150
$66\frac{4}{6}$
$45\frac{5}{6}$
$187\frac{3}{6}$
450

And now to prooue how you can doe the like, I propound the ſame Queſtion, onely willing you to vſe ſome other forme of combining or linking the ſummes. The variation of this queſtion.

Scholar. That ſhall I proue with your fauour, and therfore I combine 8 with 15, and 6 with 11, & then the forme wil be as 5 followeth,

6



Whereby amounteth the same summe in totall of the differences as did before: and yet now the differences bee altered as the combination is changed, whereof I understand the reason by your former worke. And therefore here appeareth no strange thing, but that now I have $8\frac{2}{6}$ gallons of 6 pence, and 25 gallons of 8 pence, and 2 gallons and $\frac{3}{6}$ of 11 pence, and so consequently 4 gallons and $\frac{1}{6}$ of 15 pence: so that multiplying $8\frac{2}{6}$ by 6, it maketh 50, and then 25 multiplyed by 8, maketh 200: likewise $12\frac{3}{6}$ multiplyed by 11 yeeld $137\frac{3}{6}$, and $4\frac{1}{6}$ multiplyed by 15, maketh $62\frac{3}{6}$, which 4 summes added into one, will yeeld in the totall 450, which agreeth with the multiplication of 50 (being the totall summe of gallons) be 9 the common or meane price.

50
200
$137\frac{1}{2}$
$92\frac{1}{2}$
450

Master. Seeing you conceive this worke so well, I will propound another example unto you of more variety in the Alligations or combinings, as thus:

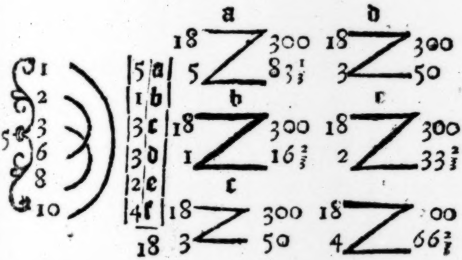
A

A Merchant being minded to make a bargaine for Spices, in a mixt masse (that is to say) of Cloues, Nutmexgs, Saffron, Pepper, Ginger, and Almonds: the Cloues being at 6 shillings, Saffron at 10 shillings, Pepper at 3 shillings, Ginger at 2 shillings, and Almonds at 1 shilling. A question of Spices.

Now would hee haue of each sort some, to to the value of 300 pound in the whole, and each pound one with another; to beare in price 5 shillings: How much shall hee haue of each sort?

Scholar. That will I try thus.

First I set downe those sixe seuerall prices, and at the left hand I set the common price 5 shillings. Then I linke them thus, 1 with 10, 2 with 6, and 3 with 8: as in the example following.



Master. I had minded to haue combined them in more variety: but I am content to see your owne worke first, and then more varieties in combination may follow anon.

Scholar. Then to continue as I beganne, I seeke the difference betweene 1 and 5, (which is 4) and that I set against 10; then against 1 I set 5, which is the excesse of 10 aboue 5, so I gather the difference betweene 2 and 5, which is 3: and that I set against 6, because it is combined with 2: and likewise the difference of 6, aboue 5, (which is 1) I set against 2. Then take I the difference of 3 from 5, which is 2, and that I set against 8: and before that 3, I set the difference of 8 aboue 5, which is 3. Then gather I all these differences by Addition, and they make 18, which I set for my first number in the Golden rule, and so appeareth by those workes, that of Almonds I must take $83\frac{1}{3}$ pound, of Ginger $16\frac{2}{3}$ pound, Pepper 50 pounds, of Cloues 50 pounds, of Nutmegs $33\frac{1}{3}$ pounds, and of Saffron $66\frac{1}{2}$ pounds.

$83\frac{1}{3}$
$33\frac{1}{3}$
150
300
$266\frac{2}{3}$
$666\frac{2}{3}$
1500

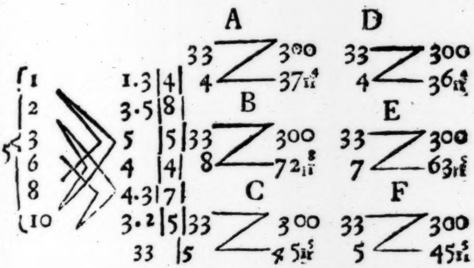
Then for triall hereof, I multiply every parcell by his seuerall price, as $83\frac{1}{3}$, which is the summe of Almonds, I multiply by one, which is their price.

Also $19\frac{2}{3}$ the summe of Ginger, I multiply by 2, which is the price of it: and so each other in his kinde, as this Table annexed doth represent, and then adding them altogether, I finde the totall to be 1500, which also will amount by the multiplication of the grosse masse of 300, by the common price 5, where-

fore

fore it appeareth well wrought.

Maſter. Now I will make the alligation to proue your cunning ſomewhat better: but becauſe you ſhall not thinke your ſelfe preſſed ſo much, I will alſo note the differences, as by this Example you may ſée, where I haue



alligated 1 with 6 and 8, and therefore haue I ſet againſt 1 both their differences, that is 1 and 3: Likewiſe, becauſe 2 is combined with 8 and 10, I ſet before him their differences, 3 and 5. Againſt 3 I haue ſet onely 5, which is the difference of 10, with whom 3 is combined onely. Likewiſe 6 is onely alligate to 1, & therfore is the differences of 1 from 5, which is 4 onely ſet againſt it: 8 is linked with 1 & 2, and therefore hath ſet 4 againſt him, both their differences, 4 and 3: and 10 is ioyned with 2 and 3, therefore hath hee their differences, 3 and 2. And becauſe of eaſe for you, in another columne I haue ſet the differences reduced into one number, for euery ſeuerall ſort,

and haue also added them together, whereby appeareth that they make 33, & so consequently you see the workes of the Golden Rule set forth. For the sixe drugges I haue added the letters, A, B, C, &c. as before.

But I would not wish you to cleaue still to these elementary aids, but accustome Memory to trust her selfe: so shal occasion of negligence best be auoided, And as for the proof, try it at more leisure, because the time now is short, and you sufficiently instructed in that proofe. And there resteth diuers things behinde yet, of which I would gladly giue you some taste, before your departure.

Note.

Scholar. But if it may please you to let me see all the variations of this question, before you goe from it, for mee thinketh I could varie it two or three wayes more yet.

Master. I am content to see you make two or three variations, but I would bee loath to stay to see all the variations: for it may be varied aboue 300 wayes, although many of them would not well serue to this purpose.

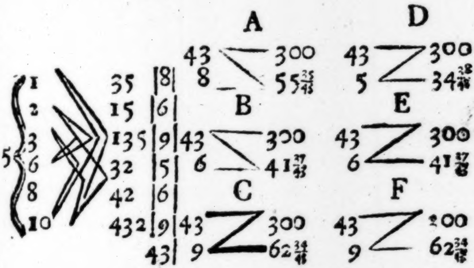
Scholar. I thought it impossible to make so many variations.

Master. Maruell not thereat, for some questions of this Rule may be varied aboue 1000 wayes; but I would haue you forget such fantasies till a time of more leisure. And now goe forward with some variation of this question.

Note.

Scholar. For the first variation, I linke the

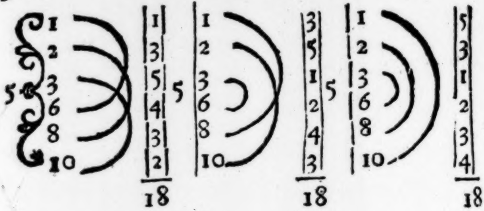
the first number 1 with 8 and 10, and 2 I combine with 9 and 10: then ioyne I 3 with 6, 8, and 10, as in this forme.



And so doth there appeare the proportion of weight for euery kinde of Drugge in this mixture. Now for the triall.

Master. Nay stay there: you shall not néed to make triall in one example so often, or if you list to do it by your selfe, I am content. But now set forth (for declaration that you conceiue the Rule) two or three examples of seuerall combinations, and then will wee passe to some other example, and so end this Rule.

Scholar. As it pleaseth you, so will I doe. And these be the varieties: in which, as the

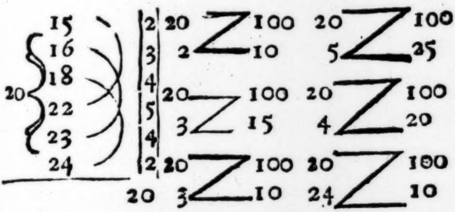


combinations are seuerall, so doth it plainly appeare, that the differences by which the proportion of each seuerall kind is taken, are also seuerall And yet I sée in the three first of these fiue varieties, and in one other before, the totall summe of the differences to be one, that is to say, 18, whereby I perceiue that the variety of their mixture doth depend on the variety of their differences seuerall, and not of the variety of their totall summe.

Master. So is it. And séeing you conceiue it so well, I will make an end of this Rule, onely exhibiting vnto you one Question or two of the mixture of Metalls, that by it you may deuise others like, and exercise your selfe therein also, because the vse of it serueth often in

in businesse of charge, not so much for Goldsmiths, as of coynage in Mints. First, I demand of you this question; *If a Mynt-master have Gold of 22 Karects, and some of 23 Karects, some of 24: Againe some 15 some 16, and some of 18 Karects, and would mixe them, so that he might have 100 ounces of 20 Karects: How much must hee take of each sort?* A question of mixing Gold.

Scholar. To know that, I answer in order, thus:



Mast. You have wrought the question well: but how chanced you made no doubt of that new name Karect?

Scholar. Because I thought it out of time to demaund such questions now, seeing you make so much hast to end: and againe in this case the proportion of the number, is sufficient for my purpose in this worke: trusting that another time you will instruct mee as well of this, as of sundry other things, which as I have heard you talke of, so I have a great desire to them.

Master. Your answer is reasonable, and your request and trust (with Gods helpe)

I intend to satisfie. And now to goe forward with this matter, let me see your examination of this last worke.

Scholar. First for the one part I adde together all the particular summes, as they appeare in the worke, and they make 100, as here by their Addition doth appeare.	10 15 20 25 20 10 — 100

And so it seemeth that the summes are well gathered: but for the further triall of them: I multiply first

120 240 360 550 460 240 — 2000	20 which is the common or meane summe of the Karects by 100, which is the summe of the whole Masse, which I would haue, and it maketh 2000 Then I multiply euery particular summe by the Karects that it doth containe, as 10 by 15, and

that maketh 150.

Likewise I multiply 15 by 16, and it yeeldeth 240: so 20 by 18, maketh 360. And 25 by 22 yeeldeth 550: likewise 20 by 23, bringeth forth 460; and last of all, 10 multiplyed by 24, yeeldeth 240: which summes all ioyned together make 2000, that doth agree with the like summe before, wherefore I may well say, that the worke is good. And now (if it please you) I would set forth some varieties of this question to proue my wit.

Master. Go to let me see.

Scholar. Here be foure varieties,

And

20	15	34	7
	16	3	3
	18	2	2
	22	2	2
	23	54	9
	25	5	5
			28

20	15	23	5
	16	34	7
	8	4	4
	2	5	5
	23	54	9
	24	42	6
			36

20	15	234	9
	16	4	4
	18	3	3
	22	5	5
	23	52	7
	24	54	9
			37

20	15	4	4
	16	4	4
	18	234	9
	22	2	2
	23	2	2
	24	542	11
			32

And more yet could I make, but not like to the number that you speake of in the variation of the other question.

Master. That will I teach you at more leisure, seeing it is a thing rather of pleasure then of any necessity.

But now for your exercise in this Rule, one other question I will propose. A Mint-master hath sixe Ingots of silver of sundry finenesse, some of foure ounces fine, and some of five ounces, some of sixe, and other of eight, some of 11, *and other of* 12, *and his desire is to mixe* 500 *pounds weight, so that in the whole masse every pound weight should beare nine ounces of fine silver: How much shall he take (say you) of every sort of silver?* A question of mixing of silver.

Scholar

Scholar. To finde out that, I set the numbers thus in order.

	4	2	2
	5	2	2
	6	3	3
9	8	3	3
	11	54	9
	12	31	4
			23

And gathering the differences, it will appeare, that, of the first sort there must be $43\frac{11}{23}$ of the second like much: of the third sort $65\frac{5}{23}$: and of the fourth sort as much: of the fift sort $195\frac{15}{23}$ and of the sixt sort $86\frac{22}{23}$, which in the whole will make 500 pound weight, and in ounces after 9 ounces fine 4500, that is of the first sort $173\frac{21}{23}$, and of the second sort $217\frac{9}{23}$: of the third sort $391\frac{7}{23}$: of the fourth sort $521\frac{17}{23}$: of the fift sort $2152\frac{14}{23}$, and of the sixt sort $1045\frac{11}{23}$: which altogether doe make 4500 ounces, agréeable to the multiplication of 9 by 500.

Master. This is well done of you, therfore now make three or foure varieties, and so an end of this Rule.

Scholar. These 4 varieties I set for example.

	4	3		3
	5	3		3
	6	3		3
9	8	2		2
	11	1		1
	21	5	4	2 12
				24

	4	13	5
	5	2	2
	6	2	2
9	8	2	2
	11	5431	13
	12	5	5
			29

Master.

Maſter. And by theſe it appeareth, that you can finde out more, with which I will not now meddle, ſaue onely (for to ſhew you an eaſie helpe in drawing the lines of Combination) I will ſet forth two varieties here.

And this ſhall ſuffiſe now for the Rule of Alligation or mixture: for by theſe examples may you eaſily coniecture ſuch other as doe appertaine to it, as well for the due working, as for variety of drawing the lines of combination.

Scholar. Sir, albeit it pleaſed you erewhile to put me from my muſing at the many varieties that may fall in theſe combinations, and termed them phantaſies, yet my phantaſie

giueth

giueth me, that the consideration of this should in many other examples and cases of importance be very needfull, and the knowledge of it most profitable. Therfore ye may well think, that at another time conuenient I will request you to aid me herein.

Master. Truth it is that this consideration may fall in practice as well politicke as philosophicall, and sundry wayes in them be applied: Therefore when time shall fall fit, for the discussing of this consideration, you shall not want my helping hand.

The Rule of Falshood.

The occasion of the name.

Now will I briefely also teach you somewhat of the Rule of Falsehood, which beareth his name, not for that it teacherh any fraud or Falsehood, but for that by false numbers taken at all adventures, it teacheth how to find those true numbers you seeke for.

Scholar. So might any other Rule be called the Rule of Falshood, for they worke by wrong numbers, and by them finde out the right numbers: so doth the Rule of Alligation, the Rule of Fellowship, and the Golden Rule partly.

Master.

Master. In the Golden Rule, the Rule of Fellowship, and the rule of Alligation, although the numbers that you work by, be not the true numbers that you seeke for, yet are they numbers in iust proportion, and are found by orderly worke, whereas in this Rule the numbers are not taken in any proportion, nor found by orderly worke, but taken at all adventures:

And therefore I sometimes being merry with my friends, and talking of such questions, to call unto them such children or Idiots, as hapned to bee in the place, and to take their answere, declaring that I would make them solve those questions, that seemed so doubtfull.

And indeed I did answer to the question, and worke the triall thereof also by those answers which they happened at all adventures to make: which numbers seeing they be taken as manifest false, therefore is this Rule called the Rule of false positions, and for briefnesse, the Rule of Falsehood: which Rule for readinesse of remembrance, I have comprised in these few verses following, in forme of an obscure Riddle.

Gesse at this worke as hap doth lead,
By chance to truth you may proceed,
And first worke by the question,
Although no truth therein be done.

Such

Such falsehood is so good a ground,
That truth by it will soone be found.
From many bate too many mo,
From too few take too few also;
With too much ioyne too few againe:
To too few adde too many plaine.
In crosse-wise multiply contrary kinde,
All truth by falshood for to finde.

The sense of these verses, and the summe of this Rule is this:

When any *question is proponed* appertaining to this *rule*, first Imagine any *number* that you list, which you shall name the *first position*, and put it in stead of the *true number*, and then worke with it as the *question* importeth: and if you haue missed, then is the *last number* of that work either too *great* or too *little*: that shall you note as hereafter shall be taught you, and you shall call it the *first errour*.

The exposition of the Rule.

Then beginne againe, and take another *number*, which shall bee called the *second position*, and worke by the *question*: if you haue missed againe, note the excesse or default as it is, and call that the *second errour*. Then multiply crosse-wise the *first position* by the *second errour*, and againe the *second position* by the *first errour*, and note their *totalls* seuerally by the names of *totalls*. Then marke whether the two *errours* were *both alike*, that is to say, both *too much* or both *too little*: or whether they bee *vnlike*, that is, the one too *much*, and the other too little:

little: for if they be like, then ſhall you *ſubtract* the one *totall* from the other (I meane the *leſſer* from the *greater*) and the *remainer* ſhall bee your *Diuidend*: ſo muſt you abate the *leſſer errour* out of the *greater*, and the reſidue ſhall bee the *Diuiſor*. Now diuide the *Diuidend* by that *Diuiſor*, and the *quotient* will ſhew you the *true number that you ſeeke for*. But, and if the *errours bee vnlike*, then muſt you adde both thoſe *totalls*, (which you noted) together, and take that *whole number* for the *Diuidend*, ſo ſhall you adde both *errours* together, and that *whole number* ſhall bee the *Diuiſor*, and the *Quotient* of that Diuiſion ſhall giue you the *true number* that the *queſtion* ſeeketh for, and this is the whole Rule.

Scholar. This Rule ſeemeth ſo vnlike any other, that without ſome example, I ſhall not eaſily vnderſtand it.

Maſter. With a good will: propoſe halfe a ſcore ſundry queſtions and examples of variety, for the better vnderſtanding of the worke thereof: and for the firſt, take this example. *A Maſon was bound to build a wall in* 40 *dayes, and it was couenanted ſo with him, that euery day that he wrought, he ſhould haue for his wages* 2 ſhillings 1 penny, *and euery day that hee wrought not, hee ſhould bee amerced* 2 ſhillings *ſixe pence*, *ſo that when the wall was made, and the reckoning taken of the dayes that hee wrought, and of the other that hee wrought not, the Maſon had clearely but fiue* ſhillings *fiue* pence *for the worke.* *Now* A queſtion of Maſonry the firſt example.

doe

Such falsehood is so good a ground,
That truth by it will soone be found.
From many bate too many mo,
From too few take too few also;
With too much ioyne too few againe:
To too few adde too many plaine.
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The sense of these verses, **and the** summe **of this** Rule **is this:**

When any *question is proponed* appertaining to this *rule*, first Imagine any *number* that you list, which you shall name the *first position*, and put it in stead of the *true number*, and then worke with it as the *question* importeth: and if you haue missed, then is the *last number* of that work either too *great* or too *little*: that shall you note as hereafter shall be taught you, and you shall call it the *first errour*.

The exposition of the Rule.

Then beginne againe, and take another *number*, which shall bee called the *second position*, and worke by the *question*: if you haue missed againe, note the excesse or default as it is, and call that the *second errour*. Then multiply crosse-wise the *first position* by the *second errour*, and againe the *second position* by the *first errour*, and note their *totalls* seuerally by the names of *totalls*. Then marke whether the two *errours* were *both alike*, that is to say, both *too much* or both *too little*: or whether they bee *vnlike*, that is, the one too *much*, and the other too little:

little: for if they be like, then shall you *subtract* the one *totall* from the other (I meane the *lesser* from the *greater*) and the *remainer* shall bee your *Diuidend*: so must you abate the *lesser errour* out of the *greater*, and the residue shall bee the *Diuisor*. Now diuide the *Diuidend* by that *Diuisor*, and the *quotient* will shew you the *true number that you seeke for*. But, and if the *errours bee vnlike*, , then must you adde both those *totalls*, (which you noted) together, and take that *whole number* for the *Diuidend*, so shall you adde both *errours* together, and that *whole number* shall bee the *Diuisor*, and the *Quotient* of that Diuision shall giue you the *true number* that the *question* seeketh for, and this is the whole Rule.

Scholar. This Rule seemeth so vnlike any other, that without some example, I shall not easily vnderstand it.

Master. With a good will: propose halfe a score sundry questions and examples of variety, for the better vnderstanding of the worke thereof: and for the first, take this example. *A Mason was bound to build a wall in 40 dayes, and it was couenanted so with him, that euery day that he wrought, he should haue for his wages* 2 shillings 1 penny, *and euery day that hee wrought not, hee should bee amerced* 2 shillings *sixe pence*, *so that when the wall was made, and the reckoning taken of the dayes that hee wrought, and of the other that hee wrought not, the Mason had clearely but fiue* shillings *fiue* pence *for the worke. Now*

A question of Masonry the first example.

doe

doe I demand how many dayes did he worke of those 40, and how many did hee not worke ?

Scholar. I pray you expresse the order of the worke, that I may partly by imitation, and partly by comparing it with the Rule, be able againe to doe the like.

Master. This order shall you keepe in the worke of this rule: first take some number (as you list) at adventure, as for example, I say he plaied 12 daies, and wrought 28 daies. Now cast you the wages of every day, and see whether it will agree with the summe of 5 shillings 5 pence.

Scholar. The 28 dayes that he wrought after 25 pence the day yeeldeth 700 pence: Then 12 dayes that he wrought not, at 30 pence each day, doth amount to 360 pence, which if I abate out of 700 pence, there resteth 340: but you say hee had not so much.

Master. Hee had but 65 pence, and by this supposition he should have had 340: therefore is this summe too much by 275, which summe I must set downe after this sort as you see here, where first I have made a crosse (commonly called S. Andrewes crosse) and at the over corner on the left hand I have set the first position 12: and at the other corner under it I have set 275, which is the first errour, with this figure + which betokeneth too much, as this line — plaine without a crosse line betokeneth too little.

12

×

275 +

On

On the right hand of the crosse I haue left two like roomes for the second position, and his error. Therfore to prosecute the worke, I suppose he played 16 dayes, and wrought 24.

Scholar. I was a while in doubt why you named the dayes of his working, seeing they bee not set in the figure: and I doubted how you knew them, or else whether that you did suppose them at all aduentures, as you did the dayes that hee played: but now I gather, that seeing 40 dayes is the whole time limited, then the dayes that hee played beeing supposed, the rest of 40 must needs be the dayes that hee wrought, and therefore 28 followed 12 of necessity, and 24 followeth 16 also of necessity, but yet I scarce perceiue why you set not in the figures as well 28 as 12.

Master. It forceth not which of them I take, so that in the second position I take the numbers of the same nature that is here both of working dayes, or both of idle, but now examine you this second position.

Scholar. If hee played 16 dayes, then abating 16 times 30 pence, the summe will be 480 pence, and for 24 daies that he wrought, euery day yeelding 25 pence, the totall is 600 pence: so that abating 480 out of 600, there resteth 120, and as you say, it should be but 65: therfore it is too much by 55: that must bee set on the right hand of the figure at the neather part and ouer it on the same side 16, which is the second position, thus.

12 16

275+ 55+

And as I gather by your wordes, it were all one if I did ſet 28 in ſtead of 12, and 24 in ſtead of 16.

Maſter. So were it. But this ſhall you marke, that, of what nature ſoeuer the two poſitions be, of the ſame nature is the quotient. Therefore when the poſitions in this queſtion are 12 and 16, which both being numbers of the playing dayes, the quotient ſhall declare the true number of playing dayes: whereas if the poſitions had béene 28 and 24, which are ſuppoſed to be the working dayes, then would the Quotient declare the true number of the working dayes, and not of playing dayes, as it will doe now. And therefore to continue the worke of this queſtion, and to finde the true number of playing dayes, I muſt multiply croſſe-wiſe the firſt poſition by 55, that is the ſecond errour, and the totall will bee 660. Then I multiply 275 and 16, and it yéeldeth 4400. Now becauſe the errors are like, that is to ſay, both too much, I muſt ſubtract 660 out of 4400, and ſo remaineth 3740, which is the diuidend. Againe, I muſt ſubtract the leſſer errour 55 out of 275, that is the greater errour, and there will remaine 220, which will bée the diuiſor: then diuiding 3740 by 220, the quotient will bee 17. Wherefore I ſay now conſtantly, that 17 is the true number of dayes that the Maſon played: and then it followeth that hee wrought 23 dayes, and ſo is the queſtion anſwered.

Now

Now for the order of triall of this worke, there needeth none other triall but onely this, to worke with this number according to the question, and if it agree, then appeareth the number to be it that you would haue. The proofe of this rule

And here now seeing he wrought 23 dayes, and must haue for euery day 25 pence, the whole summe commeth to 575. Then againe seeing he played 17 dayes, and must abate 30 pence for euery day, the whole summe of the abatement will be 510: Therefore I subtract 510 out of 575, and there will remaine 65, which maketh 5 shillings, 5 pence, the cleare wages of the Mason, for his worke, according to the question.

Scholar. Now I trust I understand the worke and the Rule so well (and the better by this proofe) that I can be able to doe the like: And for a proofe, I take the same question all saue the last number, where I will suppose that he had 10 shillings, for his wages cleare. And now to gesse at the number of the dayes that he wrought, I suppose first that hee wrought 20 dayes: then say I, if he wrought 20 dayes, his wages must be 500 d. then did he play other 20 daies, for which must be abated 600 d. and then he loseth 100 d. And so am I at a stay, for it is not like to your former worke.

Master. You should haue required of mee some question, and not haue taken a question of your owne phantasying, untill you were more expert in this Art. for so might you as well

happen to an impossible question, as on a possible: but now to goe forward, consider that this number is too little by 220 seeing he should gaine by your supposition 120 pence, and in this position hee loseth 100, those both make 220, which you shall set downe for the first errour with this signe —, betokening too little, as here in this forme following doth appeare.

20
220-

And now for the rest goe forward your selfe once againe.

Scholar. As my errour hath uttered my folly, so it hath procured mee better understanding.

Now therefore considering this position not to solue the question, I take another, supposing that hee wrought 30 dayes. Then for his wages hee must be allowed 750 pence, and for the 10 dayes which hee wrought not, hee must abate 300 pence, and so remaineth cleare 450 pence, but it should bee onely 120 pence, therefore it is too much by 330, which I set downe in the figure with the former position and his errour; and the figure appeareth thus:

20	30
220-	330+

Now first I multiply in crosse wayes 20 by 330, and it will be 6600.

Then againe I multiply 30 by 220, and it will be also 6600. Wherefore if I shall subtract the one out

out of the other, there will remaine nothing to be the Diuidend.

Master. In this you forget your selfe againe; for in as much as the signes in the errours bee vnlike, therefore must you worke by Addition, adding together those two totalls to make the Diuidend, and also adding the two errours to make the Diuisor. And because you shall no more forget this part of that Rule, take this briefe remembrance.

Vnlike require Addition,
And like desire Subtraction.

Scholar. You meane, that if the errors haue like signes, then must the Diuidend and the Diuisor be made by Subtraction, as is taught before: And if those signes bee vnlike (as in this last example they be) then must I by Addition gather the Diuidend and the Diuisor. Therefore must I adde 6600 to 6600, and it will bee 13200, which will be the Diuidend. Then againe I adde 220 to 330, and it will bee 550, which must be the Diuisor: wherefore diuiding 13200, by 550, the quotient will be 24, whereby I know that the Mason wrought 24 dayes, and then it followeth that he played 16 dayes.

Master, Examine your worke, whether it be agreable to the question or no.

Scholar. For 24 dayes worke, the wages must bee 600 pence, and for 16 dayes which the Mason wrought not; there must bee abated 480 pence, & then remaineth cleare to the

Mason 120, as the question importeth: where-fore it is euident that 24 is the true number of dayes that he wrought.

Master. Although you seeme now to vnder-stand this worke, yet to acquaint your minde the better with the new Trade of this Rule, I thinke it good to propone to you 5 or 6 exam-ples more before I make an end of it.

Scholar. Sir, I thanke you that you doe so consider my commodity & profit in knowledge, for vndoubtedly it is practice and exercise that maketh men prompt and expert in euery kinde of knowledge.

Master. You say well, so that they follow some certaine precepts to gouerne and rule their practice by, else may practice procure custome of errour, and a repugnance to exact-nesse of knowledge: namely, as long as the error is not plainly knowne to the vulgar sort. But to returne to your worke.

A question of wares, the second example.

There is a seruant that hath bought of Veluet and Damaske for his Master 40 yards, the Veluet at 20 shillings a yard, and the Damaske at 12 shillings, and when hee commeth home, his Master demandeth of him how much hee hath bought of each sort: I cannot tell (saith he) exactly: but this I know, that I payd for Damaske 48 shillings, more then I payd for Veluet: now must you gesse how ma-ny yards there is of each sort.

Scholar. Although the gesse seemeth dif-ficult, yet I will proue what I can doe: for I remember your saying, that it forceth not

not how fond or false the gesse be, so it be some-what to the question, and not an answer of a contrary matter.

Therfore first I imagine that he bought 20 yards of damaske, for which hee should pay after the former price 240 shillings: then must he needs haue of veluet other 20 yards (to make vp the 40 yards) and that would cost 400 shillings. So that the totall of the price of the Damaske is lesse then the summe paid for Veluet 160 shillings, and should bee more by 48. Therfore the first error is 208 too litle: Then beginne I againe, and suppose hee bought of Damaske 30 yards, that cost 360 shillings, then had he but 10 yards of Veluet, which cost 200 shillings: and now the price of the Damaske is greater then the price of the Veluet by 160 shillings, and should bee but 48, therefore is the second errour 112 too much, which I set in forme of figure as here doth appeare. Then doe I multiply in crosse wayes 208 by 30, and the summe will be 6240. Also I multiply 112 by 20, and there will amount 2240.

20 30

208- 112+

And in as much as the signes of the errours bee vnlike, I know I must worke by Addition, therefore adde I these two totalls together, & they make 8480, which is the Diuidend: then adde I also the two errours together, 208 and 212, and they make 320, which is the Diuisor: wherefore

diuiding 8480 by 320, the quotient will bee 26½, which is the true summe of yards of Damaske that he bought, and in veluet 13 yards ½, and that appeareth by examination, thus: 26½ yards of Damaske at 12 shillings the yard, maketh 318 shillings: then in Veluet hee had but 13 yards, and ½, and cost 270 shillings, at 20 shillings the yard. Now subtract 270 out of 318, and there will remaine 48, which is the number of shillings that the Damaske did cost more then the Veluet.

Master. Now shall you haue a question of another kinde.

A question of debt, the third example.

There are three men that doe owe money to me, and I haue forgotten what the totall summe is, and what the particulars be.

Scholar. Why, then it is impossible to know the debt.

Master. Peace: you are too hastie, there is more helpe in it then yet you see, I haue three seuerall notes, whereby it appeareth that I did conferre their debts together, and found the debt of the first and the second to amount to 47 pound, the debt of the first man and the third man did make 71 pound, and the second man his debt with the third, did rise to 88 pound. Now can you tell what euery man did owe, and what was the whole summe?

Scholar. Nay in good faith: but as I perceiue that it must bee found by coniecture, so will I gesse at it, supposing that the first man did owe 20 pound, and the second man 30, and the third. 6- 1

Master.

Master. Nay stay there, you are too farre gone already: you may not suppose a severall summe for every man, for it is enough to suppose one summe for the first man, and let the other rise as the question importeth. Therefore seeing you set the first man his debt to bee 20 pound, the second man cannot owe 20 pound, for the declaration is, that their debts added together did make 47 pound, so must the second man his debt bee but 27 pound. Now the second debt with the third, must make 88: therefore subtract 27 out of 88, and there will remaine 61, as the third man his debt. Then saith the declaration, that the first and third mans debts doe make 71: but by this supposition they make 81, that is 10 too much, which I must set for the first errour. Now worke you the second position.

Scholar. I suppose the first mans debt to bee 24 pound: then must the second mans debt (by your declaration) be but 23 pound, seeing both they make but 47 pound. And the second man his debt with the third, doe make 88 pound, and the second man oweth but 23: therefore the third man must owe 65 pound. Now the third mans debt with the first, should make by the declaration 71 pound, and they doe make 89 pound, that is 18 pound too much, and that is the second errour, which I set downe with the first, and their positions in this forme, and then I doe multiply in crosse wayes 20 by 18, & it is 360.

And

And 10 by 24 maketh 240. Also because the signes of the errours be like, I must worke by subtraction: therefore I subtract 240 out of 360, and there resteth 120, which is the Diuidend: then doe I subtract 10 out of 18 by the same reason, and so is the Diuisor 8, which is found 15 times in 120: therefore I say that the first man did owe 15 l. and then the second man must owe 32 l. for those 2 doe make 47 l. and the third mans debt is 56: for so much remaineth if I abate 15 out of 71, or if I take 32 out of 88.

10	24
10 +	18 +

The fourth example.

Master. *For the fourth Example, take this easie question for the variety in worke. Two men hauing severall summes, which I know not, doe thus talke together: the first saith to the second, if you giue me* 2 shillings *of your money, then shall I haue three times so much money as you. The second man answereth; It were more reason that our summes were made equall, and so will it bee if you giue mee* 3 shillings *of your money. Now gesse what each of them had.*

Scholar. I imagine that the first had *9 s.*

Note.

Master. Consider evermore in your imagination that you take a likely summe, as in this question, take such a summe, that hauing 2 added unto it, may bee diuided into 3 parts even.

Scholar. Why? I remember you said before, it forceth not how fondly soeuer I gessed.

Master.

Master. As for the possibility of the solution, it is truth: but for easinesse in worke, the aptest numbers are most conuenient.

Scholar. I thought no lesse, and therefore I tooke 9 as an apt number to be parted into thrée: but I perceiue I should haue considered the aptnesse of that partition after the addition of two vnto it, and then 7 had béene more méet.

Master. That is truth, and then should the second man his summe be 5: for although hée haue now but the third part of 9, that is 3, yet you must remember that hée lent the first man 2, and so had he 5.

Scholar. Then to goe forward: if the second man had thrée of the first man, then should hée haue 8, and the first man but 4: so hath hée double to the first man: yet hee said in the question they should haue equall: wherefore it appeareth that hée hath 4 too much.

Therfore I note that error with his supposition, and gesse againe that he hath 10 shillings: whereunto I adde 2 shillings borrowed of the second man, and then he hath 12 shillings: so the second man hath remaining but foure, whereunto if I adde the 2 that he lent to the first man, so had he but 6 shillings at the beginning.

Then take 3 s from the first man, and giue to the second, then hath the first man but 7, and the second hath 9, which are not equall, but there are 2 to many,

7 10

4+ 2+

many, wherefore I set downe both the positions with their errours as before you see, and multiply a crosse, so commeth there 40 and 14: and because the signes bee like, I take 14 out of 40, and so resteth 26 to bee diuided: then likewise I take 2 out of 4, and there resteth 2, by which I diuide 26, and the quotient will be 13, which is the summe that the first man had. And so appeareth that 2 being added thereto, the summe will bee 15, so hath the second man but 5, and before hee had 7: then take 3 from the first, and put to his 7, and so haue each of them 10, and that is equall as the question would.

The fifth example: A question of Lambes.

Master. *For the fifth example, take this question. One man said to another, I thinke you had this yeare two housand Lambes: so had I, said the other; but what with paying the tythe of them, and then three seuerall losses, they are much abated: for at one time I lost halfe as many as I haue now left, and at another time the third part of so many, and the third time ¼ so many. Now gesse you how many are left.*

Scholar. Because heere is mention made of certaine parts, I must take a number that may haue all these parts, that is to say, ½ ⅓ and ¼ which will be 24, howbeit 12 hath the same parts. Therefore I take first 12 to bee the number that doth remaine, so hath he lost 6, 4, and 3, that is 13, and the whole 23, but it should be 2000.

Master. Ye are deceiued yet still, you haue

for-

forgotten the 10 part, which muſt be defalked, that is 200, ſo there remaineth but 1800, and now goe on againe.

Scholar. Then to finde the errour, I take 25 out of 1800, and there remaineth 1775 too few, which I ſet for the errour. Then for the ſecond poſition I take 24, whoſe halfe is 12, the third part 8, and the quarter 6, whereby riſeth 50, which is too little by 1750, therfore I ſet downe both the poſitions with their errours thus:

12	24
1775-	1750-

And multiply in croſſe wayes 1775 by 24, whereof cometh 42600. Alſo I multiply 1750 by 12, and there ariſeth 21000. And becauſe the ſignes are like, I doe ſubtract the one from the other, and ſo remaineth the Diuidend 21600. Then doe I ſubtract 1750 out of 1775, and there reſteth 25, by which I diuide 21600, and the quotient is 864, whereof the halfe is 432, and the third part is 288 the quarter is 216, which all being added together, will make 1800. And if you adde thereto the tenth which was abated before, then will the whole ſumme be 2000.

864
432
288
216
1800

And now doth there come a queſtion to my memory which was demanded of me, but I was not able to anſwer to it: and now me thinketh I could ſolue it.

Maſter. Propone your queſtion.

Scholar.

A question of sheepe & tillage, the sixth example.

Scholar. *There is supposed a Law made, that (for furthering of tillage) euery man that doth keep sheepe, shall for euery ten sheepe eare and sow one Acre of ground: and for his allowance in sheepe pasture, there is appointed for euery foure sheepe one Acre of pasture. Now is there a rich Sheepe-master which hath 7000 Acres of ground, and would gladly keepe as many sheepe as hee might: by that Statute I demand how many sheepe shall hee keepe?*

Master. Answer to the question your selfe.

Scholar. First I suppose hee may keepe 500 sheepe, and for them hee shall haue in pasture after the rate of foure sheepe to an Acre, 125 Acres, and in Arable ground 50 Acres, that is, 175 in all: but this errour is too little by 6825. Therefore I gesse againe that hee may keepe 1000 sheepe, that is in pasture 250 Acres, and in tillage, 100 Acres, which make 350, that is too little by 6650. Both these errours with their positions, I set downe as you see, and multiply them crosse 6825 by 1000, and it maketh 6825000, also I multiply 6650 by 500, and there commeth 3325000, which summe

500	1000
6825-	6650-

I subtract out of the former, and there remaineth 3500000 for the Diuidend: likewise I subtract the lesser errour out of the greater, and there resteth 175, by which I diuide 3500000 (the Diuidend aforesaid) and the

the quotient will bee 20000, so that by this rate, hee that hath 7000 Acres of ground may keepe 20000 sheepe.

Master. You haue done well, notwithstanding both this last question, and the next before might be wrought without the second position by the Rule of proportion, as this: When in this question you found in the first errour that for 500 sheepe there must be 157 Acres, then might you reduce it to the Golden Rule, thus:

Another way of working.

175 ⟍⟋ 500
7000 ⟋⟍ 2000

If 175 Acres will admit in allowance 500 sheep, then 7000 will haue 2000. And so by one position, with the helpe of the Golden Rule may you answer that question.

Likewise for the question of Lambes, when you had found that 12 came of 25, you might haue set the figure as followeth, and haue said:

25 ⟍⟋ 12
1800 ⟋⟍ 864

If 25 doe leaue but 12 what shall 1800 leaue? and it would appeare to be 864.

Scholar. Sir, I thanke you for this aide, for it doth much shorten the worke of this Rule.

Master. Yet againe, I will shew you another way to answer to this last question without the Rule of false position, and that by the Rule of Fellowship, for it appeareth in the propounding of the question, that 10 sheepe

Another way yet.

must

must haue in pasture two Acres and $\frac{1}{2}$, and for them must there bee eared but one Acre; so it followeth, that for 2 Acres eared, there must be 5 set to pasture: and if you put them both into one summe, they will make 7. Therefore looke what proportion 7 being this totall, doth beare to 5 and to 2, such proportion shall any totall in this question beare to the pasture ground, and the eared ground.

Scholar. This serueth wondrous aptly. Therefore to proue it, I demand this by the former supposition: if a man haue 300 Acres, how much shall hee leaue in pasture, and how much shall he turne to tillage? You say that as 7 is to 5, so shall 300 be to the Acres of pasture: and as 7 is to 2, so is 300 to the Acres of tillage, whereof for both I haue set examples here following, whereby appeareth that of Pasture, there shall bee 214 $\frac{2}{7}$ Acres, and of Tillage 85 $\frac{5}{7}$, which both summes added together, doe make 300.

7 — 5
300 — 214 $\frac{2}{7}$

7 — 2
300 — 85 $\frac{5}{7}$

Another question, the seuenth example.

Master. *Now take another Example: A man hath three siluer Cups with one Couer, the Couer weigheth 18 ounces, the second Cup weigheth euen halfe the weight of the first and the third. Now if the Couer be put to the first Cup, they weigh iust as much as all the three Cups doe weigh: and if the Couer be ioyned with the second Cup, they weigh as much as the second twice, and the third: and if the Couer*

Couer bee put to the Cup, they will make twice as much as the first and the second Cup. Now try you what was the iust weight of euery Cup.

Scholar. I doe set the weight of the first Cuppe to bee 9 ounces, then in as much as these two (that is to say, the couer and the first Cup) doe weigh the weight of the three Cuppes, I see that the three Cuppes must weigh 27 ounces, for so much is 18 and 9. Also because the first and the third doe weigh double so much as the second, therefore it is the third part of that weight, that is 9, and then would it follow, that the third Cuppe also should weigh 9 ounces; but then the question saith, that the Couer being ioyned to the second Cuppe, they weigh as much as the second twice, and the third once, that should bee 27, and so it doth; that being ioyned with the third Cup, they should weigh twice as much as the first and the second, that should bee 36, and they weigh but 27, so is that errour 9 too little. Then beginne I againe, and say, that the first Cup doth weigh 12 ounces, which I ioyne with the Couer, and they make thirty ounces: then seeing the second is of that weight, it must needs weigh 10 ounces, and the third must weigh 8 ounces, seeing the first and the third must weigh 20 ounces. Now put I the Couer to the second Cuppe, and they weigh 28 ounces, which should be euen so: then ioyne I the couer with the third Cup, and so should it wey twice the first & the second,

Pra?

that is 44 ounces, and they wey but 26, that is 18 too little: those errours with their positions I set down, and multiply in crosse wayes 9 by 12, whereof commeth 108: Also 9 by 18, and that yéeldeth 162: and in as much as the signes bee like, I abate the lesser out of the greater, and there doth remaine 54. Then doe I also abate the lesser error from the greater, and so remaineth 9, by which I diuide 54, and the quotient is 6, which I take for the true weight of the first Cuppe, which being ioyned with the Couer, must weigh as much as the three Cups, so doe they weigh but 24 ounces. Then seeing the second Cup is the third part of that weight, for the other two Cups (you say) must weigh double his weight, the weight of the second Cup is 8 ounces, and so the weight of the third Cup must be 10 ounces. Now put the Couer to the second Cup, and it will make 26 ounces: that must be the weight of the second twice, and the third once, that is, twice 8, and once 10, and so is it. Againe put the Couer to the third Cup of 10 ounces, and they must wey twice as much as the first and the second, that is, 28, and so is all agréeable.

9	12
9-	18-

Master. Then answer to this Question.

A question of water: the eighth example.

There is a Cisterne with foure Cockes, containing 72 barrells of water: and if the greatest Cocke be opened, the water will auoid cleane in sixe houres; at the second Cocke it will aske eight houres: at the

third

third Cocke it will auoyd in no lesse then nine hours : and at the smallest it will require twelue houres. Now I demand in what space will it auoid, all the Cockes being set open?

Scholar. First, I imagine it will auoid in two houres.

Master. Then must there auoid by the first Cocke $\frac{1}{3}$ of the water, that is 24 barrells, and by the second Cocke $\frac{1}{4}$, that is 18, and by the third Cocke $\frac{2}{9}$, that is 16 barrells, and by the smallest Cocke $\frac{1}{6}$, that is 12 Barrells, all which summes put together, doe make 70, as by their Addition it doth appeare, but it should be 72; therefore the errour is 2 too few.

Scholar. Then will I beginne againe by your fauour, because I thinke I vnderstand the worke and put three houres for the due time: so shall there runne out at the greatest Cocke $\frac{1}{2}$, that is, 36 Barrells, and at the second hole $\frac{3}{8}$, that is 27, and at the third Cocke $\frac{1}{3}$, that is 24, and at the smallest hole $\frac{1}{4}$, that is 18 Barrells, which altogether doe make 105, and should bee but 72, so is it too much by 33; therefore doe I set the errours in order of the figure with their positions, and worke by Multiplication, in crosse, saying, two times 3 is 6, and two times 33 maketh 66, and because the signes are vnlike,

$$\begin{array}{r} 24 \\ 18 \\ 16 \\ 12 \\ \hline 70 \end{array}$$

$$\begin{array}{ccc} 2 & & 3 \\ & \times & \\ 2- & & 33+ \end{array}$$

I must adde those two totalls together, which make 72: also I adde the two errors, and they make 35 by which I diuide 72, and the Quotient riseth $\frac{2}{35}$, whereby I see that all the Cocks being set open, the water will auoid in two houres, and $\frac{2}{35}$ of an houre.

Master. This exercise maketh you to grow expert in the Rule. Therefore I will inure you somewhat more with a question or two.

A question of partners

The ninth example.

There were two men that had beene partners, & had in account between them 300 Duckets; whereof the one should haue for his part 180, and the other 120: but in the parting of them, they fell at variance; so that each of them catched as many as hee could: yet afterward being reconciled, they agreed that he which had gotten most part of them, should lay downe $\frac{3}{4}$ of them againe, and he that had gotten least, should lay down $\frac{1}{3}$ of those which he had taken, and then parting them into two equall parts, each man to haue halfe thereof, and so had they their iust portions, as they ought: now I demand of you what each of them had gotten by the scambling?

Scholar. I suppose hee that had least, got 108 duckets, then the other had 192: wherefore in laying downe againe of the 192, there was put downe $\frac{3}{4}$, that is 144, and so had hee left but 48. Also of the 108, there was laid downe 36, that is $\frac{1}{3}$, and so hee had left 72. Then I put together 144 and 36, and it maketh 180, which I part into two parts euen, and so commeth 90 to bee giuen to each of them:

them: which summe put to 72 maketh 12 and ioyned to 148, it maketh 238: and now I doubt how I shall goe forward.

Master. You need not to take but one of them, which you list, the greater or the smaller, for all commeth to one purpose: and so may you compare it that you take to any of the other summes, remembring that you make comparison to the same in the second worke: as for example of the first part. If you compare 138 with the lesser summe due, that is, 120, so is it 18 too much, and if you compare it with the greater summe, then is it 42 too little. Againe, if you compare 162 to the greater summe, the errour will be 18, as it was in the other: but it will haue a contrary signe: and if you compare it with the lesser summe, it will be 42 too much: so that the errour both wayes is either 18, or 42: and as for the signes it little forceth, for in them is nothing considered here, but likenesse and vnlikenesse, which in this case doth neither further or hinder: But now goe on with the worke. Note.

Scholar. If it bee so, then am I out of my greatest doubt. Then I ioyne that 90 (which I found as the halfe of the latter partition) vnto 48, which is left with the one man, and so hath hee 138, which (I may say) is 18 too many, for the least should bee but 120, that error doe I note, and then make a new position, supposing the one man to haue 204, and the other to haue 96: wherefore of the 204, there

 must

muſt bee layd downe 153, and ſo remaineth with him 51. Alſo of the 96, there muſt be layd down $\frac{1}{3}$, that is 32, and ſo reſteth with that man 64: Now of the 153 and 32, I make one ſumme, as 185, which I muſt diuide into two equall parts, and ſo each man ſhall haue $92\frac{1}{2}$, whereunto if I adde their former portions reſerued, then the one ſhall haue $156\frac{1}{2}$, and the other hath $143\frac{1}{2}$. Wherefore take the leſſer ſumme now againe, as I did before, that is, $143\frac{1}{2}$, and finde that hée hath too many by $23\frac{1}{2}$, for he ſhould haue but 120, & ſo haue I for my two poſitions two errors, which I ſet downe as here may be ſéene, each error vnder his poſition, & then by the Rule I doe multiply in croſſe wayes 108 by $23\frac{1}{2}$, and there riſeth 2538, which I note, then againe I multiply 96 by 18, and thereof amounteth 1728.

$$\begin{array}{ccc} 108 & & 96 \\ & \times & \\ 18+ & & 23\frac{1}{2}+ \end{array}$$

Now becauſe the ſignes are both like, that is, both too many, I muſt worke by Subtraction, and ſo abating 1728 out of 2538, there will reſt for the Diuidend 810: then for the Diuiſor I ſubtract 18 out of $23\frac{1}{2}$, and there remaineth $5\frac{1}{2}$, by which I diuide 810, and the quotient will be $147\frac{3}{11}$, which is the iuſt portion of him that had the leaſt ſumme. And if I doe ſubtract it out of 300, being the totall ſumme, then will there remaine $152\frac{8}{11}$, as the portion that the other did get.

Master. For the proofe of this worke, you may chuse whether you will examine those numbers according to the forme of the questions, or else worke by other two positions for to finde the second number: and if those positions bring the same numbers that did amount by the first two positions, then doth each worke confirme other. Note.

Scholar. By your patience, I will prooue both waies, not only to seeke their agreement, but also to accustome my minde to those workes, for I perceiue it is exercise that must bee the chiefe engrauer of these Rules in my memory.

Master. You consider it well: then go to.

Scholar. First, I will by two other positions try to finde the portion of him which had most.

Master. Although you may doe it with any positions, yet to see the agreement of your worke the better, take the same positions that you did before comparing them now to the greater, as you did before vnto the lesser.

Scholar. Then I suppose that hee that had most, had 192, so had the other 108. Now if I take ¾ out of 192, that will be 144, and there will rest to that man but 48. And from the second which had 108, if I take ⅓, that is 36, there will remaine to him 72: then ioyning 144 with 36, it will make 180, the halfe whereof being 90, If I adde to each of those

two mens portions remaining with them, the one shall haue 138, and the other 162, of which two I take the greater (that is 162) and see it to be 18 too few; for it should be 180, that errour I note vnder this position. Then for the second position I take (as I did before) 204 for the one, and so resteth 96 for the other: then (take I $\frac{3}{4}$ of 204, and it will bee 153, and there resteth to him 51. Also of the 96 I take $\frac{1}{3}$ that is, 32, and there remaineth to him 64; now put I that 32 to 153, and it yeeldeth 185: which being parted in equall values, maketh $92\frac{1}{2}$ to bee added to each mans remainder, and so the one hath $143\frac{1}{2}$, and the other $156\frac{1}{2}$: wherefore I take the greatest summe, and it is $23\frac{1}{2}$ too little, that doe I note also, and set both these errors vnder their positions, as in this Example following doth appeare.

And then multiplying 102 by $23\frac{1}{2}$, there doth arise, 4512.

Againe, I multiply 204 by 18, and it maketh 3672, which I doe subtract out of 4512, because the signes bee like, and there resteth

102	204
18	$23\frac{1}{2}$

840 for the diuidend, then subtracting 18 out of $28\frac{1}{2}$ there will remaine $5\frac{1}{2}$, which I must take for the Diuisor. And so diuiding 840 by $3\frac{1}{2}$ the quotient will be $152\frac{8}{11}$, whereby I haue found an agreeable summe to that which I found by the former positions, for him that had

most

[m]oſt, which I doe ſubtract out of 300, that is the totall, there will reſt $147\frac{3}{11}$, which was the portion of him that had the leaſt part.

Maſter. So by diuers poſitions, you ſee, that one doth confirme the worke of the other. Now examine thoſe two numbers by the forme of the queſtion, and ſo ſhall you proue your worke good alſo.

Scholar. If that hee which gate moſt, had $152\frac{8}{11}$, then muſt hee lay downe $\frac{3}{4}$ of this ſumme. That is $114\frac{6}{11}$, and ſo ſhall remaine with him but onely $38\frac{2}{11}$. The other which had leaſt, that is, $147\frac{3}{11}$, muſt put downe of his ſumme $\frac{1}{3}$, that is $49\frac{1}{11}$, and ſo doth there remaine with him yet $98\frac{2}{11}$. Then doe I adde together $114\frac{6}{11}$, and $49\frac{1}{11}$, and it will make $163\frac{7}{11}$, which I muſt part into two equall parts, and that will bee $81\frac{9}{11}$ to be giuen to each of them: putting $81\frac{9}{11}$ vnto $38\frac{2}{11}$, there doth amount 120 iuſt, which is the true portion of him that ſhould haue the leſſer ſumme: and adding $81\frac{9}{11}$ $98\frac{2}{11}$, the totall will bee 180, the true portion of the other. And ſo is the worke by this proofe alſo tryed to be good, And this I marke by the way, that in their ſcambling, hee got moſt (as it chanceth often) that ought to haue had leaſt by iuſt partition.

Maſter. Let your ſtudy bee to learne truth and iuſt Art of proportion, & to diſtribute and part according thereunto, as often as occaſion ſhall be miniſtred. And here would I make an end of this Rule, ſaue that I remember

one

one pleasant question which I cannot ouer-passe, which I will declare somewhat largely, because you shal as well understand some reason in the pleasant invention, as apt proceeding in the witty working thereof.

The tenth example of mixture of gold and silver.

Hiero *king of the Syracusans in Sicilia had caused to bee made a Crowne of Gold of a wonderfull weight, to be offered for his good successe in wars: in making whereof, the Goldsmith fraudulently took out a certaine portion of gold, and put in silver for it, so that there was nothing abated of the full weight, although there was much of the value diminished.*

Which thing at length being uttered (as no evill can alwayes ly hid) the King was sore moued: and being desirous to trie the truth without breaking of the Crowne, propsned the doubt to Archimedes, unto whose wit nothing seemed unpossible, which although presently hee could not answer unto, yet he had good hope to devise some policie for that invention, and so musing thereon, as he chanced to enter into a baine full of water to wash him, hee obserued, that as his body entred into the baine, the water did runne over the Tub, whereby his ready wit of such small effects coniecturing greater works, conceiued by and by a reason of solution to the Kings question, and therefore reioycing exceedingly, more then if he had gotten the Crowne it selfe, forgat that hee was naked, and so ranne home, crying as hee ranne, ἕυρηκα, ἕυρηκα, I haue found, I haue found. And thereupon caused two massy

peeces,

peces, one of gold, and another of Siluer, to be prepared, of the same weight that the said Crowne was of: and considering that Gold is heauier of nature then Siluer, and therefore Gold of like weight with Siluer, must needs occupy lesse roome by reason it is more compact and sound in substance, hee was assured that putting the masse of Gold into a vessell brim full of water, there would not so much water runne out, as when he should put in the siluer masse of the like weight. Wherefore hee tryed both, and noted not onely the quantitie of the water at each time, but also the difference or excesse of the one aboue the other, whereby hee learned what proportion in quantity is betweene Gold and Siluer of equall weight. And then putting the Crowne it selfe into the vessel of water brim full (as before) marked, how much water did runne out then, and comparing it with the water that ranne out when the Gold was put in, noted how much it did exceed that: and likewise comparing it to the water that ranne out of the Siluer, marking how much it was lesse then that, and by those proportions found out the iust quantity of Gold that was taken out of the Crowne, and how much Siluer was put in stead of it: but seeing Vitruu us which writeth this History doth not declare the particular worke of this triall, it shall bee no inconuenience to suppose an example for declarations sake, wherein although the true and iust

pro-

proportion be not expressed, yet the form of triall shall be truely set forth. And for an example, I suppose the weight of the crowne to be 8 pound, and so of each the other two Masses. And when the Masse of Gold was put into the water, I imagine that there ranne out two pound of water: and when the masse of siluer was put in, I suppose there ranne out three pound $\frac{1}{2}$. Againe, when the Crowne was put in, there ranne out two pound $\frac{1}{4}$: Now to know what quantity of siluer was in the Crowne, worke by the Rule of false position, and imagine that there was two pound of siluer, then must there bee sixe pound of gold, then say thus by the Rule of proportion. If eight pound of gold doe expell two pound of water, what shall sixe pound expell? and it will be 1 pound $\frac{1}{2}$. Againe, for the siluer: if eight pound of siluer expell three pound $\frac{1}{2}$ of water, what shall two pound of siluer put out? it will be $\frac{7}{8}$, now adde these two weights of water together, and they will make two pound $\frac{3}{8}$, and it should be by the supposition two pound $\frac{1}{4}$ so is it too much by $\frac{1}{8}$.

Scholar. Now doe I vnderstand the worke as I thinke, therefore I pray you let mee worke the rest of the question. And because this first supposition did erre, I note that position and his errour, and take a new position, esteeming the siluer to be but one pound, so must there be in Gold seuen pound. Then say I: if eight pound of Gold doe yeeld two

pound

pound of water, what shall seuen pound yéeld? and it will be 1 pound $\frac{3}{4}$. Againe, if 8 pound of siluer expell 3 pound $\frac{1}{2}$ of water, what shall 1 pound expell? and it will be $\frac{1}{16}$. Now must I adde those two summes together, and they make 2 pound $\frac{5}{16}$, and they should make 2 pound $\frac{1}{4}$, so is it too little, by $\frac{1}{16}$ Therefore I set the positions, with their errors, in order as heere followeth: And then I multiply in crosse wayes 2 by $\frac{1}{16}$, and it maketh $\frac{1}{8}$: Likewise 1 multiplyed by $\frac{1}{8}$, maketh $\frac{1}{8}$. And because the signes be vnlike, I must adde these two summes, which make $\frac{1}{4}$: and that is the diuidend.

$$\begin{array}{ccc} 2 & & 1 \\ & \times & \\ \frac{1}{8}+ & & \frac{1}{16} \end{array}$$

Againe, I must adde $\frac{1}{8}$ to $\frac{1}{16}$, and it will bée $\frac{3}{16}$, that is the Diuisor. Now I shall diuide $\frac{1}{4}$, by $\frac{3}{16}$, and the quotient will be $\frac{16}{12}$, that is $1\frac{1}{3}$: whereby I know that there was put 1 pound and $\frac{1}{3}$ of siluer into the Crowne, and so much Gold taken out for it.

Master. Proue it now by examination, according to the question.

Scholar. If there were 1 pound $\frac{1}{3}$ of Siluer, then was there of Gold 6 pound $\frac{2}{3}$. Now say I by the Rule of proportion: If 8 pound of Gold expell two pound of water, what shall 6 pound $\frac{2}{3}$ expell?

$$\begin{array}{cc} 8 & 2 \\ 6\frac{2}{3} & 1\frac{2}{3} \end{array}$$

It

$8 \qquad 3\frac{1}{2}$

$1\frac{1}{3} \qquad \frac{7}{12}$

It will bee 1 pound $\frac{2}{3}$. Again, If 8 pound of siluer expell 3 pound $\frac{1}{2}$ of water, what shall 1 $\frac{1}{3}$ expell? It will bee $\frac{7}{12}$. Now must I adde together 1 pound $\frac{2}{3}$ and $\frac{7}{12}$, & they will make 2 pound $\frac{9}{36}$, that is 2 pound $\frac{1}{4}$, according to the supposition of the question: whereby I perceiue the worke to be wel done. And I cannot but much reioyce of this excellent inuention, so my desire is kindled behemently to be perfectly instructed in euery part thereof, and namely in this point, whether the proportion betwéene water and gold be such, that for 8 pound of gold put into a vessell full of water, there shall runne out two pound of water, and for as much siluer, whether 3 pound $\frac{2}{3}$ of water would aucid.

Master. I perceiue your meaning, and coniecture your imagination to be thus, that if you knew the exact proportion betwéene gold and siluer, and water, both in their weight and quantities, then could you easily finde out the mixtures of them, which thing I haue reserued for another worke that intreateth of such matters especially. And at this time you must consider that you learne Arithmeticke, which intreateth of the manner to solue doubtfull questions touching number, without regard what matter is signified by that number: else were it necessary in Arithmeticke to teach all Arts, séeing in it may be moued questions of all Arts.

Note.

But

But seeing you are so desirous to know these things, I will tell you in such a sort, that you shall practise your Art in finding it, and propound it in forme of a question. Gold beareth a greater proportion to water, then siluer doth, and their two proportions be in proportion together, as forty eight to twenty fiue. But to helpe you somewhat in this riddle, you shall note that the proportion of quick-siluer vnto water, is the iust middle number proportionall in progression Geometricall betweene the proportion of gold and siluer vnto water.

A question of the proportion of gold, siluer, and quick-siluer, vnto water.

And this proportion is $\frac{290}{21}$. Now if you will know the iust numbers of these three proportions, then must you finde out thrée numbers in Progression Geometricall, whereof the middlemost must bée $\frac{290}{21}$, and the first must be vnto the last, as 25 to 48. And thus I will leaue you to finde those numbers when you bée at leisure.

Scholar. Yet Sir, I thanke you heartily for thus much, for now I sée the possibility to finde them out. Howbeit, because this question seemeth strange, if it might please you to instruct me somwhat in the order of working for it, I should the more easily finde the true working.

Master. You desire too much if you will study for nothing: Therefore to occasion you to study the better, I will leaue this doubt wholly to your owne search: But as touching the generality of the Rule, Archimedes néeded not to take two Masses of Gold and

DEFECTIVE ORIGINAL

and Siluer equall in weight with the Crowne, for the proportion might as well be found in any other weight, yea, although the Masse of Gold were of one weight, and the Masse of Siluer of another. As for example: If the Crowne were of 18 pound weight as I did suppose, and I haue not so much other fine Gold, but onely one pound, and trying that by water, and finding that it doth expell but $\frac{3}{4}$ of an ounce of water, yet then by it I may inferre, that 8 pound of Gold would expell 6 ounces of water. And likewise of siluer, whereof if I had but two pound, and find that it doth expell three ounces of water, then might I affirme that 8 pound would expel 12 ounces, that is, one pound weight: and so is it as good as if the three Masses were all of one weight. And thus for this time I will make an end of this other part of Arithmeticke.

Sch. Although I cannot sufficiently thanke you for this, yet your promise made me to looke for the Art of Extraction of roots, whereof hitherto I haue learned nothing.

Master. I will not breake my promise, but intend (God willing) to performe it within this three or foure moneths, if I perceiue this my paines to bee well taken in the meane season. And you shall not repent the tarrying for it: for it shall be increased by the tarrying. And in the meane time, you shall take this Addition, not for the second Part of Arithmeticke which I promised, but for an

mentation of the first part, unto which I would haue annexed the extraction of Rootes square and cubicke, namely for examples of the Statute of Assise of wood, but that in the second part I must write of diuers other Roots, and thought it best to reserue those Rules also with their examples unto the same second part.

Scholar. Sir, although I cannot recompence your goodnesse, yet I shall alwayes doe mine endeauour to occasion you not to repent your benefit on me thus imployed.

Master. That recompence is sufficient for your part.

FINIS.

The third part

OR

Addition to this Booke

Entreateth

Of briefe Rules, called Rules of Practice, of Rare, pleasant, and commodious effect, abridged into a briefer Method then hitherto hath beene published.

With diuers other necessary Rules, Tables, and Questions, not only profitable for *Merchants*, but also for *Gentlemen*, and all other *Occupiers* whatsoeuer, as by the Contents of this Booke may appeare.

Set forth by IOHN MELLIS *Schoole-master.*

Ee 2

Item { 4 3 2 1 } frome

3

1 3

The First Chapter of this Addition, entreateth of briefe Rules, called Rules of *Practice*, with diuers necessary questions, profitable *not onely for Merchants, but also for all other occupyers whatsoeuer.*

THe working of Multiplication in practice is no other thing then a certaine manner of multiplying of one kinde by another: whereupon is brought forth the product of the proponed number, which is accomplished by the meanes of Diuision in taking the *halfe*, the *third*, the *fourth*, the *fifth*, or such other parts of the summe which is to be multiplyed.

And for the better vnderstanding of such conuersions, you shall vnderstand that in the manner and vse of these rules of practice, you ought first to know the euen or aliquot parts of a shilling, which in this Table following doth appeare.

Item	6	*pence is the*	$\frac{1}{2}$	*of a shilling.*
	4		$\frac{1}{3}$	
	3		$\frac{1}{4}$	
	2		$\frac{1}{6}$	
	1		$\frac{1}{12}$	

Wherein as you see according to the order

of these rules of Practice: at 6 pence the yard of any thing; you must take ½ of your number which is to be multiplyed, and the product that commeth thereof shall bee shillings, if any vnite doe remaine, it is 6 pence.

For 4d. take the ⅓ of the number that is to be multiplyed, and the product also produceth shillings, if any vnites doe remaine, each one shall bee worth in value 4 pence. The like is to bee vnderstood of the other 3, &c.

I Example.

At 6 d. the yard, what —— 379 yards?
189 s ——6 d.

II

At 4 d. the yard, what —— 104 yards?
34 —— 8 d.

III

At 3 d. the yard, what —— 5014 yards?
1253 s ——6 d.

IV

At 2 d. the yard, what —— 532 yards?
88 s —— 8 d.

V

At 1 d. the yard, what —— 409?
34 s —— 1 d.

Heere you may see in the first example, that 379 yards at 6 d. the yard, are worth 189 s. 6 d. in taking the ½ of 379. And in the second example the 104 yards at 4 pence the yard, are worth 34 s. 8 d. in taking the ⅓ of 104: Likewise in the third example, 5014 yards at

3 d.

3 d. the yard, bringeth forth 1253 s. 6 d. in taking the ¼ of 5014. Also in the fourth example at 2 d. the yard, maketh 88 s. 8 d.

And lastly in the fifth example: 409 yards at 1 d. the yard, amounteth to 34 s. 1 d. in taking the $\frac{1}{12}$ of 409: and so is to be done also of all other questions the like, when the number of the pence is any of the euen or aliquot parts of 12 d.

Item, to bring the products of these shillings, and all other the like into pounds is very easy in diuiding of it in your minde by 20, for it is to be understood that as often as 20 is found in that product, so many pounds doth it containe: which with facility to performe, alwayes strike off the figure towards your right hand, with a right downe dash of your pen, for the 0 that appertaineth to the 20. And then beginne at the left hand, in taking the halfe of the rest. And if that at the last any vnite doe remaine, the same shall bee ioyned with the figure that is cut off, which shall represent the odde shillings, contained in that worke.

As for example, in your third question at 3 d. the yard, which amounteth to 1253 s. 6 d. the product whereof maketh 62 li. 13 s. 6 d. as here you may see, is easily performed by this example.

```
   1
 125|3
 -----------
 62 -- 13 -- 6
```

Also for the working of one penny the yard, it is something harsh and hard to take the $\frac{1}{12}$ of some products: therefore to ease that hard worke, you shall first bring your deliuered

 summe

summe into groats, by taking ⅓ part of the product, and if any vnites remaine of that ⅓ part, as sometimes there may, they are pence: and must be signified with a line from the *groats* with their title of *pence*; and because that 60 *groats* maketh a pound or twenty shillings, *strike* off the first figure toward your right hand, for the 0 that appertaineth to 60 (as you did euen now for the 0 that belongeth to 20:) Then in taking the ⅙ of that product, if there doe remaine any vnites, the same shal you ioyne with the figure that you cut off, esteeming them as *groats*, which keepe in your minde, and by taking the ⅓ part of them, you shall turne them into *shillings*, and so haue you done: *As for example*, by a question or two hereafter proponed, shall more plainely by the worke appeare.

At 1 d. the yard, what 54368 yards?

	1359	2 *groats.*
℔li	226	10 s. 8 d.

Heere in taking the ⅙ part of 1359, in comming to the last worke, the ⅙ part of 39 being taken, the *remainder* is 3, which ioyned with the two that was cut off, maketh 32 *groats*, which conuerted into *shillings* by taking the ⅓ part, maketh as appeareth 10 *shillings* 8 d. Many other wayes there are, but none more apt for a *young learner* to vnderstand then this: wherefore this one way well impressed in memory, is better then 20 wayes doubtfully vnderstood.

At

At 1 penny the yard, what 4533 yards?

113|3 groat—1 d

li 18—17—9 d

At 1 penny the yard, what 64768 yard?

1619|2 groats

li 269—17—4 d.

Now followeth also to bee understood that if the number of pence be not an aliquot part of 12, you must reduce them into some aliquot part of 12: and after the aforesaid manner, you shall make of them two or three products, as need shal require, and adde them together into one summe. And here for thy furtherance appeareth a note of the order of their parts, as they are to be taken. 2 Rule.

for pence	take	and	or		
5	3	2	4	&	1
7	4	3	6		1
8	4	4	6		2
9	6	3	4·4		1
10	6	4	4·4	and	2
11	6	4 & 1	4·4		3

Heere in the first note of this *Table* at 5 d you shall first take for 3 d. the $\frac{1}{4}$ of the number that is to be multiplyed: and likewise for 2 d. the $\frac{1}{6}$ of the same number, adding together both the products; But if you will worke by 4 and 1, you must for 4 d, first take the $\frac{1}{3}$ of the number that is to bee multiplyed: and for 1 d, take the $\frac{1}{12}$ of the whole summe, or rather, which is more better, for 1 penny you may take the $\frac{1}{4}$ of the product which did come of the 4 pence:

pence: because that 1 d. is the $\frac{1}{4}$ of 4 pence. The totall summes of these two numbers shall bee the solution to the *question*. And in like manner is to be done of all others, as by these examples following shall appeare.

I

At 5 d: the yard, what	758 yards?	
3 d	189	6 d
2 d	126	4 d
shillings,	315	10 d

Otherwise.

At 5 d. the yard, what	758 yards?	
4 d	252	8 d
1 d	63	2 d
shillings	315	10 d

II

At 7 d. the Ell, what	563 Ells?	
4 d	187	8 d
3 d	140	9 d
shillings	328	5 d

III

At 8 d. the pound, what	112 pound?	
4 d	37	4 d
4 d	37	4 d
shillings	74	8 d

Otherwise.

At 8 pence the pound, what	112 pounds?	
6 d	56	0
2 d	18	8
shillings	74	8 d

IV

IV

At 9 pence the Ell, what	356 Ells?
6 d	178—0
3 d	89—0
shillings	267—0 d

V

At 10 pence the peece, what	795 peeces?
6 d	397—6
4 d	265—0
shillings	20)662—6
	li 33. 2. 6

VI

At 11 pence the pound, what	7576 pound?
6d	2788—0
4 d	2525—4
1 d	631—4
	6941—8 d
Pounds	347—4 s—8 d

1 Here in this first example, where it is demanded (at 5 d. the yard) what will 758 cost? First, for 3 d. I take the $\frac{1}{4}$ of 758; and thereof commeth 189 s. 6 d. Then for 2 d. I take the $\frac{1}{6}$ of the same 758, which amounteth to 126 s. 4 d. these two summes added together, doe make 315 *shillings* 10 *pence*: and so much are the 758 *yards* worth at 5 d. the *yard*.

1 Item, for the same againe: First for 4 d. I take the $\frac{1}{3}$ of 758, and thereof commeth 252 s 8 d. then for 1 peny I take the $\frac{1}{4}$ of the same 758, that is to say, of 252 s. 8 d. and it yeeldeth me 63 s. 2 d. which both added together maketh 315 s.—10 d. as before.

2 *Item, for 7 d. there is taken the* $\frac{1}{3}$ *and the* $\frac{1}{4}$ of

the whole summe which is to bee multiplyed, and adde them together, that is to say, first, for 4 pence there is taken ⅓ of 563: which comes to 187 s——8 d. as appeareth by the worke, and for 3 d. there is taken the ¼ of the whole summe, which amounteth to 140 s——9 d. Both which products added together, doe make 328 s—5 d. and so much comes 563 *ells* to at 7 d. the *ell*.

3 *Item, for the first* 8 d. *there is taken for* 4 d. the ⅓ of the whole summe, and another ⅓ for the other 4 d. which added together, as in the example doth euidently appeare, amounteth to 74 s—8 d.

Againe, for the second worke of 112 ll. there is taken first the ½ of the whole summe for 6 d. which comes to 56 s. then for that 2 d. you haue to take ⅙ of the whole summe, or if you will, the ⅓ of the product that came of 6 d. either of which maketh 18 s—8 d. These two summes being added together, doe make 74 s. 8 d. as in the third example appeareth.

4 *Item, for* 9 d. *there is taken for* 6 *pence the* ½ of the whole summe: and the ¼ of the whole summe for 3 d. or otherwise for the 3 d. you may take the ½ of the product that came of 6 d. because 3 d. is the ½ of 6 d. which added together, as plainly appeareth in the *fourth example*, amounteth to 267. s. 0 d.

Item, for 10 d. *first there is taken for* 6 d. the ½ of the whole summe, which amounteth to 397 s—6 d. Then for 4 d. there is found 265 s.

both

both which added together, make 662 shillings, 6 d. as appeareth in the *fifth example*, it may also be wrought, *as appeareth by the second note in the Table*, by 4 d. twice taken, and the $\frac{1}{2}$ of the product of 4 d. or else by the $\frac{1}{6}$ of the whole summe, &c.

Item, for 11 d. *there is first taken the* $\frac{1}{2}$ *for* 6 d. then the $\frac{1}{3}$ of the whole summe for 4 d. Lastly, the $\frac{1}{4}$ of the last product for 1 d. All which 3 summes added together,, maketh in shillings 6944 s—8 d. and in pounds 347—4—s—8 d:

Item, likewise by the same reason, when you will multiply (by shillings) any number that is under 20 s. *you shall haue in the product pounds, if you know the euen or aliquot parts of* 20, *which are here in this little Table set downe to sight.*

Item	10 s.	is the	$\frac{1}{2}$	of one pound.
	5		$\frac{1}{4}$	
	4		$\frac{1}{5}$	
	2		$\frac{1}{10}$	
	1		$\frac{1}{20}$	

So that for 10 s. *which is the* $\frac{1}{2}$ *of a pound,* you may take the $\frac{1}{2}$ of the number which is to bee multiplyed, and you shall haue in your product pounds: if an vnite do remaine, it shall be worth 10 shillings.

Likewise for 5 *shillings you must take the* $\frac{1}{4}$ of the number which is to be multiplyed, and if there doe remaine any *vnites*, they shall be fourth parts of a *pound*, euery vnite being in value fiue *shillings*.

For 4 *shillings take the* $\frac{1}{5}$ *of the number* which is

is to bee multiplyed: and if there doe remaine any *vnites*, they shall bee fift parts of a *pound*, each vnite being in value 4 *shillings*.

For 2 shillings you must take the $\frac{1}{10}$ *of the* number to be multiplyed, wherefore to take the $\frac{1}{10}$ of any number, you must cut off the last figure of the same number (which is nearest your right hand) from all the other figures with a small right down line or dash with a pen, and so haue you done: for all the other figures which doe remaine toward your left hand from the same figure that you doe separate, shall be the said $\frac{1}{10}$ of a *pound*: and that figure so separated towards your right hand, shall bee so many peeces of 2 s. the peece, the which figure you must double to make thereof the true number of shillings, as by the example shall appeare.

Finally for 1 *shilling needeth small worke*, for it is so many shillings as bee proponed in the summe, which to bring into pounds, hath been already taught in the *first rule*.

Example.

At 10 s. the peece, What		6543 peeces?
$\frac{1}{2}$	li	3271 10 s.
At 5 s. the Ell, What		4373 Elis?
$\frac{1}{4}$	li	1093. 5 s.
At 4 s. the yard, What		7839 yards?
$\frac{1}{5}$	li	1567. 16 s.
At 2 s. the pound weight, What		752|7 pound
$\frac{1}{10}$	li	752. 14 s.

At

At 1 s. the peece, what 757|3 peeces?
$\frac{1}{20}$ li. 378.13 s.

NExt followeth in order to be understood, that if the number of shillings be not some euen or aliquot part of 20, you must then conuert the same number of shillings into the aliquot parts of 20, and thereof make two or three products as need shall require: which done, adde them together, and bring them into pounds. And here for thy furtherance I haue set downe a note of the order of their parts, as they are to be taken. 4 Rule.

3		2&1			13		10.2&1
6		4&2		5&1	14		10.&4
7		5&2			15		10.&5
8	of	4&4	or	5.2.1	16	of	10.5.1
9		5&4		4.4.1	17		10.5.2
11		10&1			18		10.4.4
12		10&2			19		10.5.4

For 3 s. according to the tenour that you see is expressed in the Table, you must first take for 2 *shillings* the $\frac{1}{10}$ of the number that is to be multiplyed. Then for one *shilling* you must take the $\frac{1}{2}$ of the product which did come of the same $\frac{1}{10}$ part; which two summes added together produceth the effect desired.

Item, for 6 shillings according to the note set forth in the table first, for 4 s. I take the $\frac{1}{5}$ of the number

number that is to bee multiplyed: Then for 2 s. the $\frac{1}{2}$ of the product that came of 4 s. and adde them together.

Or else as appeareth also in the Table, for 5 shillings you may take the $\frac{1}{4}$ and the $\frac{1}{5}$ part of the product that came of 5 *shillings*, and adde them together.

Item, for 7 s. *first for* 5 s. *take* $\frac{1}{4}$ *of the product* that is to bee multiplyed, then for 2 s. take the $\frac{1}{10}$ of the number that is to bee multiplyed, and adde them together, &c.

Item, for 8 s. *according to reason, and the* intent of the *Table*, for the first 4 shillings take the $\frac{1}{5}$ of the product, and the same number again for the other 4 shillings, and adde them together.

Item, for 9 *shillings: first for* 5 *shillings*, take *the* $\frac{1}{4}$; then for foure shillings take the $\frac{1}{5}$: and adde them together.

Otherwise as you see by the intent of the Table, worke twice for 4 shillings, as was taught euen now for 8; and then take the $\frac{1}{4}$ of the last product for the 1 shilling: but 5 and 4 is the shorter.

Item, for 11 s. *first dispatch* 10 *shillings*, for which you must take the $\frac{1}{2}$ of the product, then lastly, for 1 shilling take the $\frac{1}{10}$ part of the summe produced of the $\frac{1}{2}$ of the product, and adde them together.

Item, for 12 *shillings, where I will end with the first part of my Table.* First take the $\frac{1}{2}$ for 10 *shillings*. And then for 2 *shillings*, take the $\frac{1}{5}$ of the summe that came of 10 *shillings*, take and adde

adde them together, or elſe if you pleaſe for 2 ſhillings, you may take the $\frac{1}{10}$ of the whole giuen number.

To write more of the manner of taking the true parts, I omit. The deſirous practitioners will no doubt conceiue it. Alſo the *Table* is ſome aide to helpe the vnperfect, whereupon by and by I will ſet downe three or foure of theſe notes in *Examples*, and the reſt I will leaue to thine owne induſtry and practice, to labour vpon.

This is the order moſt commonly vſed in practice, when the number of ſhillings is not an *aliquot* part of a pound. But (*louing Reader*) after I haue touched the euen or aliquot parts of a pound that falleth out in pence and ſhillings, I will deliuer two new rules that ſhall drowne this common order quite and cleane: wherein ſhall bee comprehended in one line, or working both of euen and odde parts of ſhillings vnder 20, without regard whether it bee an *aliquot*, or not an *aliquot* part; which two Rules (when they come in place) I commit to thy friendly iudgement in working.

Now follow the examples vpon the notes beforeſaid.

At 6 ſhillings the yard, what	3215	yards?
4 ſhillings	643	
2 ſhillings	321 —	10
li	964 —	10 s.

Otherwise by Multiplication of 6.

	3215
6 *shillings*	1929|0
li	964 — 10 *shillings.*

At 7 shillings the Ell, what 4563 Ells?

5 shillings	1140	15
2 shillings	456	6
li	1597	1 shilling.

Otherwise by Multiplication of 7.

	4563
7s	3194|1
	1597 — 1

At 8 s. the peece, what 7563 peeces?

4s	1512.	12
4s	1512.	12
pounds	3025.	4s

Otherwise by Multiplication.

	7563
8 s:	6050|4
pounds	3025 — 4 shillings:

At 13 s, the peece, what 401 peeces?

10 s	200	10
2 s	40	2
1 s	20	1
pounds	260	13

Other-

Otherwise by Multiplication.

	401
13 s	1203
	401
	521|3
Pounds	260——13 s.

These and such like questions of compound numbers, which I haue heere in this fourth rule for orders sake set downe, for that it hath beene heretofore a common course of worke, I account but superfluous. For in the eighth and ninth rules of this my simple Addition shall appeare, that the given price of any euen or odde number of shillings, either under or aboue 20. shall bee wrought at one or two workings at the most, how difficult soeuer the question bee.

Item, *there resteth yet a kinde of practice, how to bring pence into pounds at the first working* 5 Rule. *whereupon you must understand that* 240 *pence maketh one pound, or* 2 s. *In consideration whereof I cut off the last figure or* 0, *and there remaineth but* 24 (*of which* 24) 8 d. *is the* $\frac{1}{3}$ *part thereof,* 6 d. *is the* $\frac{1}{4}$ *part,* 4 d. *is the* $\frac{1}{6}$ *part, and* 2 *pence is the* $\frac{1}{12}$ *part thereof.*

To reduce pence into pounds at one operation.

Whereupon if it were demanded what 1486 yards or pounds of any thing commeth to, at 8 pence the yard, in pricking or cutting off the first figure towards your right hand, for the

the o that appertaineth to 240. There is remaining of the said summe 148, whereout I taking the $\frac{1}{3}$ part, and it commeth to 49 li. and there resteth 1, which 1 I put to the 6, that I pricke or cut off, and it maketh 16 peeces of 8 pence, which I double to make into groats, and they make 32, whereof the $\frac{1}{3}$ part maketh 10 s. and there remaineth $\frac{2}{3}$ s. which is 8 d. whereby it followeth, that the 1489 yards at 8 pence the yard, maketh 49 li. 10 s. 8 d. as by the example shall appeare.

Item, for 6 pence, take $\frac{1}{4}$ part of the number from the prickt figure; and if any vnites doe remain, they are so many sixe-pences, whereof taking the $\frac{1}{2}$ they are shillings, if there doe remain yet one, it is in value sixe pence.

Item, for 4d. take the $\frac{1}{2}$ part of the number from the prickt figure; If any vnites remaine, they are so many groats, which to conuert into shillings, take the $\frac{1}{3}$ part. And if any thing yet remaine, they are thirds of shillings, each one in value being worth 4 pence.

Item, for three pence, take the $\frac{1}{8}$ part from the prickt figure, if any vnites remaine, they are so many peeces of 3 pence, whereof in taking the $\frac{1}{4}$ part, maketh shillings: If any thing yet remaine, they are fourth parts of shillings, each one being in value 3 pence.

Item, for 2 pence, as appeareth also by the Table, take the $\frac{1}{10}$ part of the number from the prickt figure: if any thing remaine, they are so many peeces of 2 pence, which by taking the

the $\frac{1}{3}$ part, you shall turne into shillings, and if any vnites remaine, they are so many sixt parts of shillings, or peeces of two pence, whether you will.

If one cost 8 pence, what maketh pounds	148|6? 49—10—8 d.
If one cost 6 pence, what maketh pounds	786|5? 196—12—6 d.
As 4 pence the yard, what maketh pounds	873|6 yards? 145—12—0 d.
If one cost 3 pence, what maketh pounds	987|4 worth? 123—8—6 d.
At 2 d. the Ell, what maketh pounds	789|Ells to? 65—15—8 d.

BVt if your number of pence bee not an aliquot or euen part of 24: then must you bring them into the aliquot parts of 24, and make thereof diuerse products which must bee added together, as by the question heereafter following shall appeare. 6 Rule.

Item, for 5 d. first take for 3 d. then for 2 d. and adde them together, according to the instruction of the *second Rule*: Or else first take for 4 d. then for 1 d.

Item, for 7 d. first take for 4 d. then for 3 d. and adde them together.

Item, for 9 d. first take for 6 d. then for 3 d.

and adde them together.

Item, for 10 d. first take for 6 d, then for 4 d. and adde them together.

Item, for 11 d. first take for 8 d. then for 3 d. and adde them together: as by these examples

Examples.

1. If one yard cost 5 d. what 759|6?

4 pence	126—12
1 penny	31—13
maketh pounds	158—5

Otherwise.

1——5——759|6

3 pence	94—19
2 pence	63—6
maketh pounds	158—5 s.

2. If one cost 7 d. what 98|7 worth

4 pence	16—9
3 pence	12—6—9
maketh pounds	28—15—9d

Otherwise.

1——7——98|7

6 pence	24—13—6
1 penny	4—2—3
maketh pounds	28—15—9 d.

3 If one cost 9 d. what 98|7 worth

6 pence	24—13—6
3 pence	12—6—9
maketh pounds	37—0—3d

Other-

Otherwise.

1 —— 9 ——	98|7
6 pence	24—13—6
3 pence	12—6—9
maketh pounds	37—00—3

4 If one cost 10 pence, what	98|7?
6 pence	24—13—6
4 pence	16—9—0
maketh pounds	41—2—6

5 If one cost 11 pence, what	98|7?
8 pence	32—18—0
3 pence	12—6—9
maketh pounds	45—6—9

But if you haue any shillings and pence to be multiplyed together: Then are you to take for the shillings according to the instruction of the third rule: and for the pence according to the first rule before mentioned: unlesse you can spie the aduantage thereof, and thereby helpe your selfe: as appeareth in this second example, where first I work for 6 d. which is to be rebated out of the giuen number, and I haue 719 li. 11 s. my desire.

At 19 s. 6 d. the yard, what 738 yards?

			Otherwise by Rebating.
	738		738
10 s	369 ——	6 d	18 — 9 s
5 s	184 — 10	li 7	19 — 11 s
4 s	144 — 12		
6 d	18 — 9		
pounds	719 — 11 s		

The like again is done by rebating, as by these two Examples appeareth.

At 18 s. the Ell, what	418 Ells?
2 s	41 ———— 16
pounds	376 ———— 4 s.

At 16 s. the Ell, what	517 Ells?
4 s	103 ———— 8
pounds	413 ——— 12 s.

7 Rule. *And now I will touch a little the euen part of a pound, that falleth out in pence and shillings, whereof for those parts you shall take such like part out of the giuen number that is to be multiplyed, as the price of that giuen number beareth in proportion to a pound, which also for their better aid is heere set downe.*

1 s. 8 d			$\frac{1}{12}$	
2 6	*is the*		$\frac{1}{8}$	*part of a pound.*
3 4			$\frac{1}{6}$	
4 8			$\frac{1}{3}$	

Item, first for 1 shilling 8 pence take the $\frac{1}{12}$ part

part of the giuen number, and if any thing doe remaine, they are twelue parts of a pound, each one being in value 1 shilling 8 pence.

Item, for 2 shillings 6 pence, take the $\frac{1}{8}$ part of the number that is to bee multiplied. And if any thing doe remaine, they are eight parts of a pound, each one being in value 2 shillings 6 pence.

Item, for 3 shillings 4 pence, as appeareth by the Table, you must take the $\frac{1}{6}$ part of the giuen number, and if any thing doe remaine, they are 6 parts of a pound, each one being in value 3 shillings 4 pence.

Item, for 6 shillings 8 pence, take the $\frac{1}{3}$ part of the number that is to bee multiplyed: And if any vnites doe remaine, they are thirds of a pound, euery one being worth 6 shillings 8 pence.

Other infinite numbers there are, that may be reduced by abbreuiation into the proportionate parts of a *pound*, as 16 shillings 8 pence maketh $\frac{5}{6}$: which 16 shillings 8 pence is easily reduced into groats, by multiplying 16 by 3, and thereto adde 2, which maketh 50 groats.

Then set 60 the groats of a pound vnder 50, cutting off the 2 ciphers as is here performed.

16—8
3
5|0
6|0

And then haue you brought 16 *shillings* 8 d. into the known parts of a *pound*, which maketh

But

But yet gentle Reader, for thy further instruction, I haue hereunto annexed in a Table, how pence and shillings beare proportion to a pound, which I commit to thy friendly beneuolence; it will be some aid vnto the ungrounded Practitioner: but I count him the best workman that can presently reduce his giuen price into the knowne and proportionate parts of a pound.

James Laird

November
17-1808 Thursday
After Dinner

I started part one ...
in Churches House
in the Parlour
... Courtney

A Table

A Table of the aliquot parts of a pound, or 20 shillings.

s	d	l	s	d	l
0	2	1/120	8	4	5/12
0	3	1/80	8	9	7/16
0	4	1/60	9	0	9/20
0	6	1/40	10	0	1/2
0	8	1/30	11	0	11/20
1	0	1/20	11	3	9/16
1	3	1/16	12	8	3/5
1	8	1/12	12	0	3/30
2	0	1/10	13	0	13/20
2	6	[illegible]	13	4	2/3
3	0	3/20	13	9	11/16
3	4	1/6	14	0	[illegible]
3	9	3/16	15	0	3/4
4	0	1/5	16	0	4/5
5	0	1/4	16	8	5/6
6	0	3/10	17	0	17/20
6	3	5/16	17	6	7/8
6	8	1/3	18	0	9/10
7	0	7/20	18	4	11/12
7	6	3/8	18	9	15/16
8	0	2/5	19	0	19/20

James Prescott ... November 10-1800

Here follow foure Examples vpon the foure Notes deliuered.

At 1 s. 8 d. the yard, what 3884 yards?
maketh pounds 323 — 13 — 4 d.

At 2 s. 6 d. the yard, what 4563 yards?
maketh pounds 570 — 7 — 6 d.

At 6 s. 8 d, the Ell, what 7562 Ells?
maketh pounds 2520 — 13 — 4 d.

Now by custome you are able to worke by all sorts of summes being deliuered in shillings and pence, as one shilling one penny, two shillings two pence, three shillings three pence, and so of all other: wishing you to haue some consideration of your questions, when they are set downe, for there are many subtill abbreuiations, and great aduantages to bee gotten, and easily to bee perceiued.

As of 3 s — 8 d. of 2 s. and 1 s. 8 d.
Of 4 s — 2 d. of 3 s — 4 d. and 10 d. which 10 d. is $\frac{1}{4}$ of 3 s — 4 d.
Of 5 s — 8 d. of 4 s. 1 s — 8 d.
Of 5 s. 10 d. of 5 s. and 10 d. which 10 d. is $\frac{1}{6}$ of 5 s.

And by this meane when you haue taken one product, you may oftentimes vpon the same take another more briefly then vpon the summe

summe which is to bee multiplyed, &c.

NOw (Gentle Reader) that you haue seene the vertue of the euen or aliquot parts of a pound in shillings alone, and also in the aliquot parts of shillings and pence: according to my promise heereafter followeth a briefe and easier method for any euen number of shillings, either vnder or aboue 20, *then euer yet hath beene published; Notwithstanding* M. Humfrey Baker, *whose trauell is worthy commendation, and whom for knowledge sake I reuerence, hath in some part touched this first part, though not in his method. The worke of the rule both pleasant, ready, and briefe, as by the variety of the examples deliuered thereupon shall appeare. And first I will set forth a question, thereby the better to expresse or teach you the order thereof: which is this.* 8 Rule.

If one cost *6* s. what 8574?

1	6	8574
maketh pounds		2572—4 s.

To the vnderstanding of this example, after you haue set downe your giuen number in forme of the rule of 3, with a line drawne vnder it, you shall presently set a pricke vnder your first figure 4. towards your right hand drawing from the pricke, as heeretofore hath beene practised, a little short line, thereto set downe the Mr. *Iohn Mellis* his first rule.

the ſhillings anon, which done, multiply the firſt figure 4 by 6, the value of your price, (which heere you ſee ſtandeth in ſight aboue the line) it maketh 24, which is one pound foure ſhillings. The one pound keep to carry to the next place, and the foure ſhillings ſet downe at the end of the preſcribed line towards your right hand. Thus haue you done now with 6 aboue the line, and alſo with 4 in the firſt place (for the pricke vnder 4 doth ſignifie that 4 hath done his office) Then ſecondarily for a generall rule take but the $\frac{1}{2}$ of the giuen price, which here is 3, which 3 is the number that ſhal now continue the reſt of the multiplication and end the worke, wherupon I multiply 3 into 7, ſtanding in the ſecond place it maketh 21, and with the one pound I kept in minde 22; ſet downe 2, and keepe 2 in minde, working according to the rule of multiplication, deliuering the tenths in minde in their due place, which done, the product from the prick to your left hand repreſenteth the pounds, and the other at the end of the line the ſhillings, as appeareth by the examples.

Note a generall rule.

If one yard coſt 2 s. what	7536?
1 2	7536
maketh pounds	753 — 12 s
If one yard coſt 4 s. what	8792?
1 4	8792
maketh pounds	1758 — 8 s.

If

If one peece cost 6 s. what 9537?

1	6	9537
maketh pounds		2861——2 s.

If one peece cost 8 s. what 7509?

1	8	7509
maketh pounds		3003——12 s.

If one cost 12 s. what 5794?

1	12	5794
maketh pounds		3476——8 s.

If one cost 14 s. what 3705?

1	14	3705
maketh pounds		2593——10 s.

If one cost 18 s. what 5703?

1	18	5703
maketh pounds		5132——14 s

If one cost 22 s. what 953?

1	22	953
maketh pounds		1048——6 s.

Let these suffice (gentle Reader) for an entrance into euen numbers. And now I will shew the like rule for any odde or vneuen part of a pound.

TO helpe you to the vnderstanding of these other Questions that heereafter follow: where in my first Example the giuen number is 6487 at Rule.

3 s.

3s. the yard: I multiply 3 aboue the line into 7, it maketh 21. The one shilling is set downe, and the 1 pound I keepe. Now am I to take the ½ of three, which because it is an odde number I cannot.

Mr. Iohn Mellis his second rule

Therefore I shall keepe and continue my multiplication by three still, and worke by the ½ of the rest of the giuen figures or number, to wit, 648. And first the ½ of 8 which is 4 multiplyed into 3, maketh 12, there to ioyne the 1 li. in minde, it maketh 13. set downe 3, keepe one. Then againe multiply by two the ½ of foure, it maketh sixe, and with one in minde it maketh 7. Then lastly, take the ½ of sixe, which is 3. saying, 3 times 3 is 9, which 9 set down, and so is the question answered, as appeareth by the practice, and examples following.

At 3 s. the yard, what 6487?

	1	3	6487
maketh pounds			973 — 1 s.

If one yard cost 5 s. what 4269?

	1	5	4269
maketh pounds			1067 — 5 s.

At 7 s. the Ell, what 6489?

	1	7	6489
maketh pounds			2271 — 3 s.

If one Ell cost 9 s. what 2807?

	1	9	2807
maketh pounds			1263 — 3 s.

Rules of Practice
July 20 1700 be thursday
twelve O'Clock

Clock striking

If

At 11 s. the Pistolet, what 8263?

1	11	8263
maketh pounds		4544—13 s.

If one peece cost 13 s, what 4629?

1	13	4629
maketh pounds		3008—17 s,

But now note (gentle Reader) when the giuen price falleth vpon an odde number, as 3, 5, 7. 11, 13, &c. then it is to bee presupposed that the giuen summe to bee multiplyed, must bee a summe made of euen numbers. as 2, 4, 6, 8, 10, &c. else cannot that question bee wrought at one line or working.

Prouiding alwayes that it may beare an odde figure in the first place towards your right hand, as appeareth in these sixe examples, which last were wrought, and such like, &c. which may beare an odde number for the price, and bee done at one line or working very well.

But if the giuen price bee an odde number, and the summe to bee multiplyed, odde numbers also: then can it not bee done at one working, but requireth the aid of two workings, for odde with odde will not agree, which notwithstanding to bring to passe, take this for a generall rule. First worke for the euen number, contained in that question, or giuen price, according as you haue learned, and then A generall rule.

afterwards for the one odde shilling, take the ⅒ of the summe giuen to bee multiplyed, omitting the first prickt place, as was taught for the working of one shilling in my first rule of Practice, and adde those two together, and you shall haue your desire.

Example.

At 3 s. the yard, what	7539 yards?
2 s	753 — — 18
1 s	376 — — 19
maketh pounds	1130 — — 17

At 7 s the Ell, what 7539?	
1 7	7539
6 s	2261 — — 14
1 s	376 — — 19
maketh pounds	2638 — — 13

At 13 s. the yard, what	7534?
12 s	4520 — 8
1 s	376 — 14
maketh pounds	4897 — 2

Note this well.

And thus haue I abridged into these two Rules how to bring any number of shillings, whatsoeuer they be, into pounds, with a briefer Method, than euer yet hath beene published, which I commend vnto thy friendly censure and iudgement in the vse and practice thereof.

If

If one cost 6 s. 5 d. what	1231 ?
6 s	369——6
4 d	20—10—4
1 d	5—2—7
maketh pounds	394—18--11

At 14 s. 2 d. what	2825 ?
14 s	1977——10
2 d	23—10—10
maketh pounds	2001—0—10

At 16 s. 4 d. what	2531 ?
16 s.	2024—16
4 d	42—3--8 d
maketh pounds	2066—19—8

At 3 s. the Pistolet, what	8325 ?
maketh pounds	1248——15 s

At 7 s. the crowne, what	6529 ?
	2285——3 s.

At 9 s. the peece, what	6567 ?
maketh pounds	2955——3 s.

These three last questions may seeme something harder, yet they are easie enough, if you marke them well: if I should explaine them, then are they too easie. Therefore I leaue them to whet the minds of the desirous.

10 Rule. ITem, *when any one of the summes, which is to be multiplyed, is composed of many denominations, and the giuen number but of one figure alone; then shall you multiply all the denominations of the other summe by the same one figure, beginning first with that summe which is least in value toward your right hand, and bring the product of those pence into shillings, and the product of the shillings into pounds, as by this example appeareth.*

At 3 li. 7 s. 4 d. a yard, what are 9 worth?
maketh pounds 30—6 s—0 d.

11 Rule. BVt *if in any of the summes that are to bee multiplied, there bee a broken number, First worke for the whole according to the instructions that you haue learned, and then take such part of the giuen price, as that broken number beareth in proportion to the price, as in the examples following. After you haue wrought for 3 s. and for 6 d. then are you to take the ½ of 3 s. 6 d. for the ½ yard, and adde that to the summe: So adding all the 3 products together, which make 43 li. 2 s. 9 d. the iust price of 245 ½ ells, and thus must you doe of all other.*

At 3 s. 6 d. the Ell, what	246 ½ ?
3 s.	36—18
6 d.	6—3
½	1—9
maketh	43—2—9

At

At 16 s. 4 d. the peece, what	14 $\frac{3}{4}$?
16 s.	11 — 4
4 d.	0 — 4 — 8
$\frac{3}{4}$	12 — 3
maketh pounds	12 — 0 — 11

If one peece cost 4 li. 3 s. 6 $\frac{1}{2}$ d. what 12 peeces?

4 li.	48 —
3 s.	1 — 16 —
6 d.	6 — 6
$\frac{1}{2}$	6
maketh pounds	50 — 2 — 7

The proofe.

If 12 peeces cost 50 li. 2 s. 6 d. what one peece?

maketh pounds 4 — 3 — 6 $\frac{1}{2}$

ITem, *touching the manner how to understand the order of this question, and others the like, first seeke how many times* 12 *is contained in* 50, *which is* 4 *times, and so resteth* 2 *pound, which* 2 *pound converted into shillings, and ioyned with the other* 2 *shillings, maketh* 42 *shillings: wherein is found* 12, *three times, resteth* 6 *shillings: which turned into pence, putting thereto the* 6 *pence in the first place, it maketh* 78, *wherein* 12 *is found* 6 *times, resteth* 6 *pence: which containeth* 12, *but* $\frac{1}{2}$ *a time. put that* $\frac{1}{2}$ *to the* 6 *pence, and then the solution is* 4 li. 3 s, 6 $\frac{1}{2}$ d. *as appeareth by the practice thereof.* 12 Rule

13 Rule. *Item, the like is to bee done of any thing that is bought or sold after fiue score to the hundreth, or the Quintall. As for example.*

If 100 pound cost 27 li. 13 s. 4 d. what one pound?

27 li — 13 s — 4 d
20
s 5 | 53
12
1 | 10
5 | 3
d 6 | 4 | 0
| 10 | 0 or $\frac{2}{3}$
Maketh 5 s 6 $\frac{2}{3}$ d.

I haue wrought this at length for the aid of the yong learner, because he should vnderstand how all the Multiplication is set downe.

But to worke it more neatly, it is by a little vnderstanding ended thus.

27 li — 13 s — 4 d
20
s 5 | 53
| 12
d 6 | 40
| 100
Maketh 5 s. 6 $\frac{2}{3}$ d.

☞ *Item, to the vnderstanding of this and such like questions, the right downe line is all the guide, which is pulled downe close by 20, as you see in the example, where 27 pound 13 shillings is reduced all into shillings, and maketh 553 shillings.*

The 5 towards the left hand beeing separated with the hanging or right downe line, is the iust number

number of shillings, that answereth to the question, Nextly, 53 shillings is multiplyed by 12, to reduce them to pence, putting to the 4 d. it yeeldeth for the multiplication of the first figure two 110: the one beyond the line towards the left hand, is 1 penny towards the rest of the price: then 53 also multiplyed by 1 yeeldeth 53: but the 5 behinde the line toward the left hand, is also 5 pence more, towards the price, which 1 and 5 I adde together under the line, it maketh 6 d. So is there found now, as appeareth by the Titles of shillings and pence, 5 shillings 6 pence.

Finally, I come now on this side the line towards the right hand, and under 12 I finde first 10, and then 3, which added together, maketh 40, under which 40, you must put the 100, and it maketh $\frac{40}{100}$, which abbreuiated, commeth to $\frac{2}{5}$. So the iust price of one pound after 5 score to the hundred, maketh 5 s. 6 $\frac{2}{5}$ d. One example more, and so I will leaue this rule.

If 100 cost $10\frac{3}{4}$ d. what	9874?	
6 d	246 — 17	
4 d	164 — 11 — 4	
$\frac{2}{4}$ d	20 — 11 — 5	
$\frac{1}{4}$ d	10 — 5 — $8\frac{1}{2}$	
Maketh li	4 | 42	5 $5\frac{1}{5}$
	20	
ſ	8 | 45	
	12	
d	5 | $45\frac{1}{5}$ | 91	parts of a peny.
	100 | 100	

 Also

Also the like may bee done of the usuall weights heere in England (which is 112, for every hundreth weight) in case you know the aliquot parts of a hundreth weight, which are these, 56 li. 28 li. 14 li. and 7 li. For 56 li. is the $\frac{1}{2}$ of 112 li. 28 li. is the $\frac{1}{4}$ of 112 li. 14 li. is the $\frac{1}{8}$ and 7 li. is $\frac{1}{16}$ part.

Therefore for 56 li. take the $\frac{1}{2}$ of the summe of money that 112 li. weight is worth.

For 28 li. take the $\frac{1}{4}$ of the summe of money that 112 li. weight is worth.

For 14 li. take the $\frac{1}{8}$ of the summe that 112 li. is worth.

And for 7 li. the $\frac{1}{16}$ of the summe of money that 12 li. is worth.

As for example; At 17 li. 19 s. the hundreth pounds weight, that is to say, the 112 li. what shall 3 quarters and 7 pound cost?

1 C.——17 li.——19 s.				3 q;——7 li.
2 quarternes	8	19	6	
1 quarterne	4	9	9	
7 pounds	1	2	$5\frac{1}{4}$	
Maketh pounds	14	11	$8\frac{1}{4}$	

The

The second Chapter intreateth of the Reduction of diuers measures to others value by Rules of Practise.

Now will I shew a few examples of Practice in reducing of measurs, as Ells, Yards, Braces, Pawnes of Genes, &c. Much more would I haue touched, but that I feare the booke will rise to too great a volume. 14 Rule.

In 864 Ells of Antwerpe, how many yards of London?

864	864
432	216
216	648

maketh 648 yards of London.

Item *in these and such like questions of Flemmish measure, to be brought into yards English, first take the* ½ *of the giuen number, as appeareth in the first example towards your left hand. Then take the* ½ *of that product, or the* ¼ *of the giuen number, and adde those 2 products together, they shall bee yards English; as by the example you may perceiue.*

The

The second example toward your right hand is yet briefer than the first, whose worke is this; Take the $\frac{1}{4}$ of the deliuered number, and that product subtract out of the giuen number, and the rest sheweth your desire. Of these two wayes vse which you thinke best.

The proofe.

In 648 yards of London,
How many Ells of Antwerpe?

	648	
	216	
maketh	864	Ells of Antwerpe.

15 Rule. *Item for the vnderstanding of this worke, first take the $\frac{1}{3}$ part of the yards of* London, *which found, adde that $\frac{1}{3}$ part and the yards together, as appeareth by the practise, and the product sheweth the Ells of* Antwerpe.

Item, in 320 yards of London,
How many Ells of Antwerpe?
maketh $426\frac{2}{3}$ Ells:

	320 yards.			Proofe.
$\frac{1}{3}$	$106\frac{2}{3}$			$426\frac{2}{3}$ Ells.
	$426\frac{2}{3}$ Ells		$\frac{1}{4}$	$106\frac{2}{3}$
				320 yards.

Other Reductions.

16 Rule. *Item, you shall vnderstand, that forasmuch as sixe braces of Millan make fiue Ells of Antwerpe, whereupon according to the* Rules of Practise, *you may reduce the one into the other, by the like reasons aforesaid, in taking the $\frac{1}{6}$ part, and then subtract*

subtract the same, to make Ells of Antwerpe. And againe by the contrary, taking the $\frac{1}{5}$ part with adding the giuen number, to turne the Ells to Braces, as for example.

In 876 Braces, how many Ells of Antwerpe?

876	The contrary.
146	730 Ells Flemmish.
Ells 730 Antwerpe.	146 Braces.
	876 Braces.

Ells 730 Antwerpe.

182$\frac{1}{2}$.

Yards 547$\frac{1}{2}$ English.

Thus appeareth, that 876 Braces by Practice, make 730 Ells Flemmish, which Ells Flemmish reduce into English yards.

So againe vpon the same first question of Braces, I would know how many yards English they make.

After the rate that 100 Braces are worth 62$\frac{1}{2}$ yards.

876 Braces.

438

109$\frac{1}{2}$

I answer, 547$\frac{1}{2}$ yards.

ITem, to the vnderstanding of this worke, and such like, first take the $\frac{1}{2}$ of the giuen Braces, and after take the $\frac{1}{4}$ of that halfe, or the $\frac{1}{8}$ of the giuen number, and adde them together, and the products are also yards English. 17 Rule.

Item,

Item, three Ells of Rochell make 5 Ells at Lisbone. So likewise three Ells at Lyons make 5 Ells at Antwerpe.

To worke these and such like, double the Ells of Lyons, and the Ells of Rockell, and from their products subtract the $\frac{1}{6}$, and the rest shall bee the Ells of Antwerpe, or the Ells of Lisbone.

1723

Example.

In 63 Ells of Lyons how many Ells of Antwerpe?

	63
	63
	126
$\frac{1}{6}$)	21

Ans. 105 Ells Ant.

In 100 Ells of Rochel, how many Ells of Lisbone?

	110
	200
$\frac{1}{6}$	$33\frac{2}{3}$.

Ans. $166\frac{1}{3}$ Ells of Lisb.

18 Rule. Touching the proofe or returne of these and such like questions, for a generall rule, you shall first take the $\frac{1}{5}$ of the giuen number: and adde that $\frac{1}{5}$ and the giuen number together, and the $\frac{1}{2}$ of that product shall be your desire.

Example.

In 105 Ells of Antwerpe, how many Ells of Lyons?

	105
$\frac{1}{5}$)	21
$\frac{1}{2}$)	126

An. 63 Ells of Lyons.

In $166\frac{2}{3}$ Ells of Lisbone, how many Ells of Rochell?

	$166\frac{2}{3}$
$\frac{1}{5}$	$33\frac{1}{3}$
$\frac{1}{2}$)	200

An. 100 of Roch.

The

The third Chapter teacheth of the Order and worke of the Rule of 3 in broken numbers after the Trade of Merchants, digressing something from M. *Records*, which is comprehended in three *Rules*.

NOw, that I haue somewhat intreated of the Rules of Practice, I will giue a few instructions, after my simple order, for the working of the Rule of three in broken numbers, wherein I shall neede to say the lesse, because I hope the studious learner, that hath trauelled any thing in the Grouuds of Arts, *is not vnfurnished of knowledge capable to vnderstand mee.*

But before I deliuer any instructions for broken numbers, I will propone a question which shall bee wrought three sundry wayes, thereby to shew, as it were, three degrees of Comparison: how farre the Rule of three in broken, for more speede of worke, differeth from the whole, which I rather set downe for a view, that the studious heerein may bee more desirous to attaine broken, leauing any more to discourse in dialogue forme, but onely to giue instructions where neede is: and in the rest to put forth the questions with their answers.

My

My first question is this.

The first way.

If one yard cost 6 s. 8 d. what are 789 worth at that rate?

1——6 s.——8 d——789
12 80
80 63120 d

Heere the product of the summe are pence, according to the nature of the middle number.

1 1
1 3 7 0   1
6 3 1 2 0 (5 2 6 0 (263
1 2 2 2 2   2 2 2 0
1 1 1

I answer——263 li.

The second way.

1——6 $\frac{2}{3}$ s——789
20
3 20
15780 s.

Here the product of the summe are s. according to the nature of the middle number.

1
1 5 7 8 0 (5 2 6 0 (263 li.
3 3 3 3   2 2 2 0
li.

The third way.

1——$\frac{1}{3}$——789
3 1 1
789

Here

Here the product is pounds, according to the title of the second number.

~~1~~
~~789~~ (263
~~333~~

I answer. 263 li.

Now that you haue seene the three former vertues of the Rule of three, whose products haue first brought forth d. next s. and lastly li. I will deliuer three notes in order following: and with them a dozen questions that shall shew the worke of the Rule of three in broken numbers or Fractions.

1 The first foure shall be sundry questions of a Fraction comming in the second place. Note these three rules.

2 The second foure shal be of two Fractions comming in the second or third place.

3 The third foure of Fractions in all three places.

Notes vpon the first Rule for a Fraction comming in the second place.

My first question is this.

If one yard cost mee 3 s. 4 d. what are 756 worth at that price? 1 Rule.

In setting downe the question to performe the worke, I turne foure pence into the part of a shilling, which is $\frac{1}{3}$ and then the question standeth thus: The first variety.

$1 \text{———} 3\frac{1}{3} \text{———} 756.$

To

To the ready working of this queſtion, and all ſuch other like: my firſt note is this, which take for a generall rule; that when any one Fraction ſhall come, either in the ſecond or third place, that the Denominator of that Fraction or Fractions, muſt alwayes bee brought vnto the Number, or Numerator of the firſt place; and thereby multiply the one into the other.

A generall rule.

And this benefit is alwayes gotten by the vertue of bringing the Denominator of the ſecond Numbers Fractions vnto the firſt place: For the Fraction in the middle number is now releaſed: and the product that commeth of the multiplication, is of the nature and like denomination of the whole number in the ſecond place which here are ſhillings.

Note this.

Whereupon now to worke the Queſtion, I bring 3, the Denominator of the Fraction in the ſecond place, vnto my firſt Number 1, with a line ſet vnder thus $\frac{1}{\ }$, and the 3 vnder it thus, $\frac{1}{3}$: ſaying once 3, is 3 my Diuiſor: that done, reduce $3\frac{1}{3}$ ſaying, 3 times 3 is 9, and the other 1 ouer 3 make 10: my ſecond number in the rule of three, by which 10 I doe multiply my laſt number 756, as appeareth by the worke thereof, and it yeeldeth 7560 ſhillings my Diuidend.

Then diuiding 7560 by 3 my Diuiſor, it yeeldeth in quotient 2520 pounds, which maketh 126 pounds, as appeareth here moſt plainely, both by the example and the worke.

At

At 3 s. 4 d. the yard, what 756 yards?

$$\begin{array}{ccccc} \frac{1}{3} & \text{———} & 3\frac{1}{3} & \text{———} & 756 \\ & & 10 & & 10 \\ & & & & \overline{7560\text{ s.}} \end{array}$$

```
1        1
7560   2520
3333   2220   (126.
```

I answer 126 li

Yet otherwise vpon the same question, altering the price now into the proportion it beareth to a pound, for the 3 s. 4 d. is $\frac{1}{6}$ part of a pound: which example first standeth thus, as appeareth on the left hand, and afterwards wrought as appeareth on the right hand.

$$1 \text{ — } \frac{1}{6} \text{ — } 756 \qquad \begin{array}{ccccc} 1 & \text{———} & \frac{1}{6} & \text{———} & 756 \\ 6 & & 1 & & 1 \\ & & & & \overline{756\text{ pounds}} \end{array}$$

The second variety.

As soone as I haue carried 6 the denominator of my middle number vnto my first place, as before hath beene taught, I pull downe 1, the numerator of 6, with a line vnder 6, thus, $\frac{6}{1}$, and that one in custome I pull downe in sight; being the figure that I will multiply my third or last number by, according to the tenor of the rule of three. And because one can neither multiply nor yet diuide (though here it is set downe in forme of multiplication, the rather for your understanding (the product of the multiplication according to the decla-

ration of this my first Rule or note, is conuerted into the title of my second number, which here are pounds. Now followeth the diuision performed in my Divisor 6, to make an end of that question.

x x
7 5 6 (1 2 6. *which maketh* 1 2 6 li. *as before.*
6 6 6

And thus much for the variety in working that question,

And now followeth another.

My second Question.

If one yard of Cotton cost $8\frac{1}{4}$ d. what 895?

1 ——— $8\frac{1}{4}$ ——— 859
4 33 33
2577
2577
28347

x
x|3 x x x|1 li. s d
x8x47(7086(590 (29—10—$6\frac{3}{4}$
4444 4 xxxx xx0

This question was also wrought like the first, and bringeth forth 29 li-10 s-$6\frac{3}{4}$ d. *the price of* 859 *yards.*

My

My third Queſtion.

If 7 pounds of any thing coſt 3 li——10 s, what comes 987 pounds to?

li.

7———$3\frac{1}{2}$———987

2———7   7

14   6909

00
~~1~~ ~~4~~ ~~1~~
~~2~~ ~~3~~ ~~4~~ |7
~~6~~ ~~9~~ ~~0~~ ~~9~~ (493 $\frac{7}{14}$ | $\frac{1}{2}$
~~1~~ 444
~~1~~ ~~1~~

I anſwer, 493 li—10 s.

Notes vpon my ſecond Rule for two Fractions comming in the ſecond and third place.

My firſt queſtion is this.

IF one Ell coſt 13 s——4 d. what halfe a quarter or $\frac{1}{8}$ of an Ell?

Anſwer. *Firſt bring* 13 s—4 d. *into the parts of a pound, which is* $\frac{2}{3}$, *and then will the queſtion ſtand thus:*

1——$\frac{2}{3}$ li——$\frac{1}{8}$

ITem, for the performance of this worke, doe as before was taught in the firſt Rule: firſt *bring* 3 *the* denominator *of the ſecond* fracti-

on vnto your *first number* 1. setting a line vnder it thus: 1 Saying once 3 is 3, that done, bring 8 the *denominator* of the *third fraction*, setting it vnder 3, and multiply them together, saying, 3 times 8 maketh 24, which 24 is your *diuisor*. (Now haue you done with the *denominator* 8) Therfore you shall put a line vnder, thus, 3 And the like line also vnder 8, setting or pulling downe vnder them their owne *numerators*, that is, 2 vnder 3, and also 1 vnder 8, as appeareth in the example, which *numerators* for a generall rule euermore to be pulled downe of custome in sight, to multiply the one by the other, according to the tenour of the *Rule of three*. Then I multiply the one by the other, saying, once 2 is two, which signifyeth 2 li. being of the nature and like denomination of the middle number, which 2 li. is to bee reduced into shillings, otherwise it cannot bee diuided by my first number 24.

Then diuiding 40. by 24, the quotient bringeth forth 1 $\frac{2}{3}$. So much is $\frac{1}{8}$ of an Ell worth after that rate. Otherwise although 2 pound could not be diuided by 24, yet it might haue been abbreuiated to $\frac{1}{12}$ of a pound: which is worth 1 s. 8 d, as before.

li.

$1 \text{———} \frac{2}{3} \text{———} \frac{1}{2}\ (\ 1$

3	2	1	$\cancel{24}$ (6
8	20		$\cancel{40}$ ($1\frac{2}{3}$ s.
24	40		$\cancel{2}\cancel{4}$

Second question.

IF *one pound of any weight cost* 13 *shillings* 4 *pence, what are* $\frac{7}{8}$ *of the pound worth after that rate?*

Answer. Reduce the 13 shillings 4 pence into the parts of a pound: which is $\frac{2}{3}$, and then will the question stand thus.

li.

$1 \text{———} \frac{2}{3} \text{———} \frac{7}{8}$

ITem *for the vnderstanding of this, if you marke well the last example, this and the rest lyeth open, and needs small instruction. For as you did last, so againe, bring the* denominator *of the second and third* Fraction, *vnto the first figure* 1, *multiplying the one into the other, which maketh also* 24, *your* Diuisor.

Then making a line vnder 3, *thus,* $\underline{3}$ *and a line vnder* 8, *thus,* $\underline{8}$ *and pulling downe their* Numerators *vnder each figure, that is* 2 *vnder* 3, *and* 7 *vnder* 8, *which as I said before for a generall rule I pull downe of custome in sight, to bee the two numbers, that of duty ought to bee multi-* Note.

 plied

plyed together, which done, I bring 2, being the lesser figure vnder 7, multiplying them together, it maketh 14, which are of the nature of the middle number: that is to wit, pounds, which 14 cannot aptly bee diuided among 24: therefore are reduced into *shillings*, as is plainly to be seene in the example: then 280 *shillings* parted among 24, yeeldeth for his *quotient* 11 s. 8 d. your desire, and the iust price of $\frac{7}{8}$ of an *ell*. Otherwise 14, though it could not bee diuided by 24, might by *mediation* or *diuision* in broken *numbers* haue beene diuided or abbreuiated to $\frac{7}{12}$, which in effect being reduced to his known parts, maketh 11 s. 8 d. as before. But my good will and meaning is to aide young beginners: therefore haue I reduced the 14 *pound* into *shillings*, which is the easier way.

Now followeth the Example.

1 —	2 —	7	1
3	2	8	~~2~~
8	2	7	~~4~~ (6
24		2	~~280~~ (11 $\frac{2}{3}$ s.
		14	~~244~~
		20	~~2~~
		280 s.	I answer, 11 $\frac{2}{3}$ s.

The

The third Example.

If one yard cost mee 2 s.—6 d. what $345\frac{3}{4}$ yards?

Answer. First put 6 d. into the parts of a shilling, and then the question standeth thus;

$$1 —— 2\frac{1}{2} —— 345\frac{3}{4}$$

Item, to the ready understanding of this, and all such like, according as before hath beene declared, bring the *Denominators* of the second and third *Fractions* unto the first place, multiplying them the one into the other, all which make 8 for the common *Divisor*. Then next reduce your second number: saying, two times 2 is 4, and 1 is 5; as was taught in the example aforesaid. Lastly, reduce your third number $345\frac{1}{4}$ all into fourths, and they make 1381, which 1381 is to bee multiplyed by 5, according to the tenor of the *Rule of three*: which done, maketh 6905 s. and diuided by 8, your Diuisor yeeldeth in Quotient $863\frac{1}{8}$ s. which maketh in pounds 43 li. 3 s. $1\frac{1}{2}$: and so much are the $345\frac{1}{4}$ yards worth at that price.

The same question wrought again by 2 shillings 6 pence, is now conuerted into the parts of a pound, and standeth thus;

$$1 —— \frac{1}{8} —— 345\frac{1}{4}$$

Item, After I haue brought here my second and third *Denominator* vnto my first place, and found 32 to bee my *Diuisor*; hauing thus fini-

ſhed my firſt place with all things unto him belonging (which is meant of bringing and multiplying the Denominators of the ſecond and third fractions into him) I then goe in hand to ſee what is to doe in my ſecond place, where preſently of cuſtome I pull downe my Numerator 1 vnder 8, being the figure in ſight that ſhall multiply my third number.

Then laſtly I reduce 345 $\frac{1}{4}$ all into fourths as afore was practiſed, which maketh 1381, the which 1381 I am to multiply by 1 my ſecond number, they are nothing increaſed, but by the *Metamorphoſis* of my worke they are now 1381 pound, being of the nature of the middle number, as I haue often ſhewed you, which diuided by 32 my diuiſor, yeeldeth 43 pound and $\frac{5}{32}$, which $\frac{5}{32}$ of a pound reduced into knowne numbers, make 3 ſhillings 1 $\frac{1}{2}$ d. as before.

Example.

$$\begin{array}{llllll} 1 & \text{———} \frac{1}{8} \text{———} & \underline{345\frac{1}{4}} & \cancel{1} & & \\ & & & \cancel{1}\cancel{0}\cancel{5} & & \\ 32 & 1 & & \cancel{1}\cancel{3}81 & (43 & \frac{5}{32} \\ & & 1381 & \cancel{5}\cancel{2}\cancel{2} & & \\ & & & \cancel{3} & & \end{array}$$

NOW follow foure other queſtions, which are in all three places broken numbers: or whole and broken together.

Item, Firſt for the finding out of your *Diuiſor*,

uisor, you ſhall take this for a moſt certaine, and generall rule: That you muſt multiply the *numerator* of the firſt *number* in the queſtion, by the *Denominator* of the ſecond: And that *Product* againe by the *Denominator* of the third: And the totall thereof ſhall bee your *Diuisor*.

Secondly, for a generall rule to finde out your *Diuidend*, *Multiply* the *Denominator* of the firſt number by the *Numerator* of the ſecond, and the whole thereof by the *Numerator* of the third. And the totall thereof ſhall euermore be your *Diuidend*.

Now for an example, I propone this queſtion, thereby to make my meaning more plaine; and to ſhew you, as I haue done in the reſt, the manner and order of the worke.

If $\frac{2}{3}$ *of any weight or meaſure coſt* $\frac{5}{6}$ *of a pound, or 20s. what are* $\frac{3}{8}$ *of the like weight or meaſure worth after that rate?*

Example.

$\frac{2}{3}$ —— $\frac{5}{6}$ —— $\frac{3}{8}$.

ITem, for the more plainer underſtanding hereof, and all other the like, in broken numbers: Firſt you ſhall pull downe 2, the *Numerator* of the firſt *Number* or Fraction, with a line under, thus, $3\frac{2}{2}$; that done, according as you haue learned before, bring 6, the *Denominator* of the ſecond *Fraction*, and ſet it vnder 2, multiplying the one into the other,

other, which maketh 12. Then laſtly, bring 8, the *Denominator* of the third *Fraction*, and ſet it under 12, multiplying that 12 by 8, which amounteth to 96, or elſe for more briefe, multiply 6 by 8, ſaying, ſix times 8, makes 48, which 48 ſet under 2, and multiply the one into the other, it maketh 96, as before. And this 96 is the firſt *number* in the Rule of three. That ſhall alwayes for a moſt generall rule be your *Diuiſor*.

Secondly, to worke for your *Diuidend*, you ſhall, (as it hath beene ſufficiently declared before) pull downe 5, the *Numerator* of your ſecond *Fraction*, and ſet it vnder 6, with a line vnder, thus, 6.

That done (as you know) you are to pull downe 3, the *Numerator* of the third *Fraction*, and ſet it vnder 8, with a line vnder it, thus, 8 multiplying the one into the other, according to the tenour of the Rule of three; which maketh 15. Then according to my note, forget not to bring the *Denominator* of the firſt *Fraction*, which is 3, vnder 15 and multiply them together, which maketh 45, which 45 is your *Diuidend*, and are of the nature of *Denomination* of the middle *number*, as I haue taught you before: And therefore are 45 li. which aptly cannot bee diuided by 96. Therefore you ſhall reduce the 45 li. into s. as you ſee performed in the Example, which amounteth to 900 s. which diuided by 96 your Diuiſor,

uisor, it yeeldeth 9 s. and $\frac{36}{96}$ of a shilling, which in lesser termes is $\frac{3}{8}$: which $\frac{3}{8}$ in money maketh 4 $\frac{1}{2}$ d: and so much will the aforesaid $\frac{3}{8}$ cost, as by the worke following shall appeare.

The Example.

James Prescott James James

2	5	3	
3	6	8	
2	5	3	
6		5	3
12		15	9 6
8		3	9 0 0 0 (9 $\frac{36}{96}\|\frac{3}{8}$ s.
96		45	96
		20	
		900	

Otherwise though 45 could not bee diuided by 96, yet by Diuision in broken numbers it might haue been abbreuiated to $\frac{15}{32}$ of a li. which reduced into knowne parts, will make 9 s. 4 $\frac{1}{2}$ d. as before.

Now my second example shall be the proofe of this question.

If $\frac{3}{8}$ yards cost $\frac{15}{32}$ of a pound, or 20 shillings, what shall $\frac{2}{3}$ cost?

Answer. Worke as was taught you before, and you shall haue your desire.

Here

DEFECTIVE ORIGINAL

$$\frac{3}{5} \text{———} \frac{15}{32} \text{———} \frac{2}{3}$$

$$\begin{array}{ccc} 3 & 15 & 2 \\ \underline{32} & \underline{2} & \\ 96 & 30 & \\ \underline{3} & \underline{8} & \\ 288 & 244 & \end{array}$$

Here as appeareth by the worke, the multiplication being ended, 240 is to be divided by 288, which to some perchance may seeme hard, yet notwithstanding is the worke good. Therefore abbreviate 240 by 288, as you see here is practised: and the end of your abbreviation shal come to $\frac{5}{6}$ your desire, $\frac{240}{288} | \frac{30}{36} | \frac{5}{6}$.

Otherwise. $\begin{array}{c|c|c|c|c} 240 & 120 & 60 & 30 & 5 \\ 288 & 144 & 72 & 36 & 6 \end{array}$

Otherwise. $\begin{array}{c|c|c} 240 & 40 & 5 \\ 288 & 48 & 6 \end{array}$

The third Question.

If $\frac{3}{4}$ *Ells cost* 13 s—4 d. *what* 156 $\frac{1}{2}$ *Ells.*

Answer. To worke this question the shortest way: reduce 13 s. 4 d. into the parts of a pound, which is $\frac{2}{3}$.

Then as you did afore, after you have set downe the question, the numerator of the first fraction 3 is pulled downe under 4, and *Denominators* of the other, 2 *Fractions* multiplied

plied into him, which maketh 18, your *diuisor*.

Then the *numerators* of the second *fraction* is pulled downe, vnder 3 of custome now in sight, ready to *multiply* my third number by which is performed as soone as the last numbers 156½ is reduced into halfes.

Then lastly, I multiply that product by 4, the *denominator* of the *fraction*: it yeeldeth 1504, which I diuide by 18, and my quotient is 139 li. and ⅑ of a pound remaining, which is worth 2 s——2⅔ d, And so much will 156½ ells cost, as by the worke following doth appeare.

3	2	156½	~~1~~	
4	3	313	~~47~~	
3	2	2	~~176~~|2	li.
6		626	~~2504~~ (139⅑	
18		4	~~1~~888	
		2504	~~11~~	

The fourth question.

If 2½ *Ells cost* 1⅔ pounds, *what commeth* 29¾ *Ells to?*

Item, to the workemanship of this question, first reduce your second *number* in saying, three times 1 is 3, and 2 is 5. Then bring the *multiplication* of the *Denominators* of the second and third *Fractions* which maketh 12: and *multiply* that a by 5 your first *numerator*,

and

and it maketh 60, which is your *diuisor*.

Then the *reduction* of the second number, which is 5, *multiplyed* by 117 the product of the last *numbers* reduction, make 585, which 585 yet resteth to bee *multiplyed* by 2 the *denominator* of the Fraction in the first place, yeeldeth 1170, which diuided by your *diuisor*, 60, yeeldeth 19 pound, 10 s. as appeareth by the work thereof.

Thus hauing now touched the 12 questions whereof I first pretended, which with diligence and oft practice, I trust are sufficient to aid the desirous vnto the working of any broken numbers, I will now intreate of diuers necessary Rules incident vnto trafficke, as heereafter followeth.

The 4 Chapter treateth of Losse, gaine in the Trade of Merchandise.

IF one yard cost 6 s——8 d. and the same is sold againe for 8 s——6 d. the question is, what is gained in 100 pounds laying out on such commodities?

Answer. The Rule of three direct, applied two manner of wayes to doe the same: the one is to say, If $6\frac{2}{3}$ giue $8\frac{1}{2}$, what giueth 100? *multiply* and diuide, and looke what your quotient bringeth forth aboue your laying out, is the neate gaines and solution

to

to your queſtion : If you follow the worke, your ſolution will bring forth 127 li——10 s. which is 27 li. 10 s. more than your principall, and ſo much is gained in the 100 pounds laying out,

Item, to worke it the other way, which I take the neareſt, ſeeke the difference betwixt the juſt price and the other price, which is one ſhilling 10. pence; then ſay by the rule of three If $6\frac{2}{3}$ s. gaine $1\frac{5}{6}$ s. what ſhall 10 pound gaine? *Multiply* and divide, and you ſhall finde 27 li. 10 s. and ſo much is gained in 100 li. laying out.

You may vſe which of theſe two wayes you thinke good.

The Proofe

If a yard of cloth bee deliuered for 8 s. 6 d. whereupon was gained after the rate of 27 li. 10 s. in 100 pounds laying out: The queſtion is, what the yard coſt at the firſt hand?

Anſwer. Put your gaine 27 pounds 10 s. to 100 pounds, all maketh 127 li.—10 s. Then ſay, If 127 li 10 s. giue but a 100 pounds, what giueth $8\frac{1}{2}$ s? Worke, and you ſhall finde 6 s. 8 d. the true ſolution to your queſtion.

Yet another Example or Proofe upon the firſt queſtion.

If one yard coſt 6 s. —8 d. the queſtion is, what

what price the same is to bee sold againe, for to gaine 27 li. 10 s. in 100 pounds laying out?

Answer. *Say by the Rule of three, if* 100 li. *gaine* 127 li. 10 s. *what giveth* 6 $\frac{2}{3}$ s? *Multiply and diuide, and you shall finde* 8 s. 6 d. *your true solution.*

If one Ell cost 7 s. 8 d. and bee sold againe for 8 s. 6 d. *The question is,* What is gained in 20 pounds laying out in such commodities?

Answer. *Seeke the difference betwixt the just price, and the other price which is* 10 *pence, and then apply the* Rule of three, *as before is taught, saying, If* 7 $\frac{2}{3}$ s. *giue* $\frac{5}{6}$ *shillings, what giueth* 20 li? *Multiply and diuide, and you shall finde* 2 li. 3 $\frac{11}{23}$ s. *and so much is gained in* 20 li. *laying out.*

The proofe also by an example of losse.

A *Merchant hath bought Holland cloth at* 8 s. 6 d. *the Ell, which proueth not to his expectation, whereupon hee is content to lose* 2 li. 3 $\frac{11}{23}$ *in* 20 *pounds laying out, The question is, what price ought to bee made of the Cloth, abating this losse?*

Answer. Doe as before in Gains hath beene taught, putting 2 li. 3 $\frac{11}{23}$ s. to your 20 pound, all together, maketh 22 li. - 3 $\frac{11}{23}$. Then say by the Rule of three. If 22 li. - 3 $\frac{11}{23}$ s giue but 20 li. what shall come of 8 $\frac{1}{2}$ s? worke, and you shall finde 7 s. - 8 d. the just price that the Ell ought

to

to bee sold for after the rate of this losse.

Thus it appeareth euidently, as in company the *Rule* is appliable as well to gaine as losse.

If 20 $\frac{1}{4}$ *yards cost* 26 li. $\frac{1}{3}$ 10 s. *how shall I sell the same againe to gaine* $\frac{1}{3}$ *of the principall, or to make of* 3, 4: *which is all one.*

Answer. By the *Rule of three*, if 3 doe giue 4, what will 36 $\frac{1}{2}$ giue? Multiply and diuide, and you shall finde 48 $\frac{2}{3}$ li. Then say againe, if 20 $\frac{1}{4}$ yards doe giue 48 $\frac{2}{3}$ pounds, as well principall as gaine, what will one yard bee worth at that price? Multiply and diuide, and you shall finde 2 li. 8 $\frac{16}{243}$ s.

If one Ell of Cloth cost mee 8 s. 8 d. *and afterwards I sell* 10 $\frac{1}{2}$ *Ells thereof for* 5 li. 13 s. 4 d. *I would know, whether I winne or lose: and how much vpon the* 100 *pounds of money.*

Answer. See first at 8 s. 8 d. the Ell, what 10 $\frac{1}{2}$ Ells comes to, and you shall finde 4 li. 11 s. and I sold the same for 5 li—13 s—4 d. so that I did gaine vpon the 10 $\frac{1}{2}$ Ells 22 shillings 4 pence. Then if you would know how much is gained in 100 pounds, I say by the *Rule of three*, If 4 li—11 s. did gaine 22 s—4 d. what will 100 pounds gaine? Multiply and diuide, and you shall finde 24 li—10 s—10 d $\frac{10}{91}$. and so much is gained in the 100 pound of money.

If 12 $\frac{1}{2}$ *yards cost mee* 11 *pound fiue shil-*

 lings,

lings, and I sell the yard againe for 16 *shillings, the question is, whether I doe win or lose, and how much in or vpon the pound of money?*

Answer. Looke what the 12 $\frac{1}{2}$ yards come to at 16 s the yard, and you shall finde 10 pound. But they cost 11 pound 5 shillings. So there is lost vpon the whole 1 li. 5 s. Then to know how much is lost in the pound, say by the rule of 3, if 11 $\frac{1}{4}$ pound doe lose 1 $\frac{1}{4}$ pound, what will 1 pound lose? Multiply and diuide, and you shall finde 2 s 2 d $\frac{2}{3}$, and so much is lost in the pound of money.

If I sell the 100 *weight of any commodity for* 4 *pound, whereupon I doe lose after* 10 *pound in the* 100 *pound, I demand how much I shall lose or gaine in the* 100 li? *if in case I had sold the same for* 4 *pound* 10 *shillings.*

Answer. Say, if 90 pound yeeld 100, how much will 4 giue? multiply and diuide, and you shall finde 4 $\frac{4}{9}$. Then say againe, if 4 $\frac{4}{9}$ giue mee 4 $\frac{1}{2}$ what will 100 come to? Multiply and diuide, and you shall finde 101 pound $\frac{1}{4}$ which is more then 100 pound by 1 pound 5 shillings: and so much is gained in the 100 pound.

A Merchant hath sold Currans for the summe of 430 *pound, and hee hath gained therein after* 10 *pound in the* 100 *pound. The question is to know how much he gained in all.*

Answer. Say by the rule of three. If 100 pound doe gaine 10 pound, what will 430 pound gaine? Multiply and diuide, and you shall finde 43, and so much hath he gained in all.

If

If one yard be worth 28½ ſ. *for how much ſhall* 10 *yards be ſold to gaine after* 8 li. 6 ſ 8 d. *in the* 100 *pound*

Anſwer. Firſt adde 8 li—6 ſ—8 d. to 100. Then ſay, if 100 li. doe giue 108⅓ for principall and gaine, what will 28½ ſ principall yeeld? Multiply and diuide, and you ſhall find 30⅞ ſ. Then ſay againe by the Rule of three, if 1 yard doe giue 30⅞ ſ (which is as well the principall as the gaine) what ſhall 10 yards giue? Multiply and diuide, and you ſhall finde 15 li 8 ſ 9 d. And for the ſame price ſhall the 10 yards be ſold, for to gaine after the rate of 8 li—6 ſ—8 d. vpon the 100.

A branch or proofe out of this Queſtion.

A Merchant hath ſold clothes for 15 li—8 ſ—9 d. *and he hath gained in the whole, the ſumme of* 1 li.—3 ſ—9 d. *The queſtion is, to know how much he hath gained in the* 100 *pound?*

Anſwer. To know this, firſt rebate the gaines from the price, and there will remaine 14 li. 5 ſ 0 d. Then ſay by the rule of three direct, if 14 li. ¼ giue mee 1 li. 3¾ ſ. what will 100 li. giue? Multiply and diuide, and you ſhall finde 8 li. 6 ſ--8 d. the effect deſired, the proofe is apparant in the queſtion before.

Yet another Branch or Proofe of the first Question.

If 10 *yards bee deliuered for* 15 li. 8 s. 9 d. *whereupon was gained after the rate of* 8 li. 6 s. 8 d *vpon the* 100 *pound, the question is, what the yard did cost at the first hand.*

Answer. First, say by the Rule of three, if 10 with principall and gain yeeld 15 li. 8$\frac{3}{4}$ shillings, what shall 1 yeeld? Multiply and diuide, and you shall finde 30$\frac{7}{8}$ shillings. Then say againe by the Rule of three, if 108 $\frac{1}{3}$ principall and gaine giue but 100, what shall 30$\frac{7}{8}$ s. of principall and gaine yeeld? Worke, and you shall finde 21$\frac{1}{3}$ s. And so much did the yard cost at the first penny.

If one yard cost 36 s. *how much shall* 12 *yards be sold for to gaine after the rate of* 10 li. *in the* 100?

Answer. First, say, If 100 giue 110 pound principall and gaine, what will 36 s. giue? Multiply and diuide, and you shall find 39$\frac{3}{5}$ s. Then say againe by the rule of three, If one yard of principall and gaine yeeld 39 $\frac{3}{5}$ shillings, what shall 12 yards gaine? Multiply and diuide, and you shall finde 23 li.——15$\frac{1}{5}$ s. which $\frac{1}{5}$ s. in knowne number, is 2$\frac{2}{5}$ d. And for the same price shall the 12 yards be sold, to gaine after the rate of 10 in the 100.

The Proofe.

If 12 *yards be sold for* 23 li — 15 2 $\frac{2}{5}$ d. *where-*

upon

upon is gained after 10 li. *in the* 100. *The question is, what the yard cost at the first penny*?

Answer. First say, If 12 giue 23 li. 15$\frac{2}{3}$ s. what one yard? Multiply and diuide, and you shall finde 39$\frac{3}{5}$ s. Then say againe by the Rule of 3, if 110 pounds giue but 100, what shall 39 $\frac{3}{5}$ s. giue? Worke, and you shall finde 36 s: the iust price of the yard at the first hand.

Item, *When one Merchant selleth wares to another, and hee giueth to the buyer* 1 li. 6 s. 8 *d. vpon the score, or* 20 li. *The question is, How much shall the buyer gaine vpon the* 100 *pound after that rate.*

Answer. First adde 1 li—6 s—8 d. vnto 20 li. and they are 21 $\frac{1}{3}$. Then say, if 20 pound giue 21$\frac{1}{3}$, what shall 100 giue? Multiply and diuide, and you shall finde 106$\frac{2}{3}$. So the buyer getteth after the rate of 6$\frac{2}{3}$ li. vpon the 100 pound.

Gentle Reader, other necessary questions appertaining to Losse and Gaine, you shall haue in the eight Chapter of this Treatise.

The fifth Chapter entreateth of Losse and Gaine vpon time, wrought by *the double rule of three, or by the Rule composed:* which is contained in foure speciall selected branches or questions of diuers formes, *each one of them springing from the first* question, and each one of them also, being a proofe to other, &c.

I*F one yard cost mee* 2 s—8 d. *ready money, and after I sell the same againe for* 2 s—10 d. *to be paid for it at the end of three moneths: the question is, what I gaine vpon the* 100 li. in 12 *moneths?*

Answer. First say, if $2\frac{2}{3}$ gaine $\frac{1}{6}$, what shall 100 li gaine? Multiply and diuide, and you shall finde $6\frac{1}{4}$ li. Then say againe, by the Rule of three, if three moneths gaine $6\frac{1}{4}$ pound, what shall 12 moneths gaine? Worke, and you shall finde 25 li. and so much shall I gaine in 12 moneths after that rate.

Item, you may also worke it all at one working by the first part of the rule of 3 composed, saying, if $2\frac{2}{3}$ d in three moneths doe gaine $\frac{1}{6}$ of a shilling (which is 2 d.) what will 100 li. gaine in

in 12 moneths: which for thy further encouragement, the worke of this one example I haue here put downe, to verifie that I affirme in the first part of this *Ground of Arts*, that this Rule, and so all others, more reioyceth in *Broken* then in *Whole*.

ſ	monethſ	ſ	li	mo.
$2\frac{2}{3}$ —	3 —	$\frac{1}{6}$ —	100 —	12
8		1	20	
~~3~~	~~2~~		2000	
24	~~72000~~	~~1~~	3	
6	~~144444~~ (~~500~~ (25		6300	
144	~~144~~ . ~~220~~		12	
	~~1~~		72000	

Where the multiplication and the division being ended, maketh 25. li your desire.

If a yard be delivered for 2 s—10 d, *to bee paid at* 3 *moneths, whereupon was gained after the rate of* 25 li *in the* 100 *for* 12 *moneths, the question is now what the yard cost at the first hand?*

Answer. First say, if 12 moneths gaine 25 li, what shall three moneths gaine? Worke and you shall finde $6\frac{1}{4}$li: Then say againe the second time, if $106\frac{1}{4}$li, give but 100, what shall $2\frac{5}{6}$s give? Worke, and you shall find 2 s 8 d. which is the iust price that the yard cost at the first hand.

If one yard of cloth cost mee 2 s——8 *d ready*

money, for what terme ſhall I ſell the ſame againe for 2 s. 10 d. *ſo that I might gaine after the rate of* 25 *pound vpon the* 100 *pound in* 12 *moneths.*

Anſwer. Firſt ſay, if $2\frac{2}{3}$ gaine $\frac{1}{8}$ what ſhall 100 pound gaine ? Multiply and diuide, and you ſhall finde $6\frac{1}{4}$ pound. Then ſay againe for the ſecond work, it 25 pound be come of 12 moneths, what ſhall come of $6\frac{1}{4}$? Worke, and you ſhall finde three moneths, the iuſt terme of time that the Cloth ought to bee deliuered at 22 s. 10 d. to gaine 25 pound vpon the 100 pound in 12 moneths.

If one yard coſt mee 2 s. 8. d. *ready money, for what price ſhall I ſell the ſame againe to be payd at the end of three moneths, ſo that I may gaine after the rate of* 25 *pound in the* 100 *pound for* 12 *moneths.*

Anſwer. Firſt ſay, if 12 gaine 25 li, what ſhall 3 moneths gaine: Multiply and diuide, and you ſhall finde $6\frac{1}{4}$ li. Then ſay for the ſecond worke, if 100 li. giue $106\frac{1}{4}$, what giueth $3\frac{3}{3}$ s? Worke, and you ſhall finde 2 s—10 d, and for that price muſt the yard be ſold to gaine after 25 pound in the 100 pound for twelue moneths.

Many other of theſe queſtions I might here haue deliuered, but for feare the Booke would riſe to too thicke a volume, and ſo to make the price ſo much the dearer, whereby it might not bee ſo portable to my Countrey-men as I wiſh it. But theſe 4 I have of purpoſe framed in this order, hauing relation one to another,

aſſuring

assuring you that what question soeuer may bee proposed within the compasse of this Rule you shall finde by one of these 4, to make a solution. And moreouer, diuers others are yet to be deliuered, where the Creditor giueth diuers dayes of payment, which can neuer be well wrought, nor yet vnderstood, vnlesse you can first finde by art the iust times that all those payments, how different soeuer they bee, ought to bee payd at once: whereupon first I thinke good heere to giue some instructions into such a rule, for it is the onely aid for the finishing of such questions as hereafter shall follow.

The sixth Chapter intreateth of Rules of payment, which is a right necessary Rule, and one of the chiefest hand-maids *that attendeth vpon buying and selling, &c.*

Example.

A Merchant doth owe a summe of money, whereof, the $\frac{1}{2}$ is to bee payd at 6 moneths, and the $\frac{1}{3}$ at 8 moneths, and the rest at a yeere. If he would pay all at one payment, the question is, what time ought to bee giuen him.

Answer. I haue omitted the quantity of the

the summe, for you shall vnderstand, the Rule is appliable, and yeeldeth a true solution to what summe soeuer shall bee proposed: But now for order sake in teaching, I doe imagine the summe to bee 60 pounds, whereupon the manner of this worke is to multiply the proportionate part of the money by the time, as in company. Then 20 being the first payment, and the $\frac{1}{3}$ of 60, which $\frac{1}{3}$ multiplied in broken numbers by 6, his time of payment maketh $\frac{6}{3}$, which in whole numbers, as appeareth by the example in the operation, maketh two moneth: next 30 which is the $\frac{1}{2}$ multiplied by his terme 8, yeelds 4 moneths, then the rest which is 10 li, must needs bee abreuiated into the proportion it beareth to 60, which is $\frac{1}{6}$ which $\frac{1}{6}$ multiplied by his time 12 moneths, produceth $\frac{12}{6}$ maketh two moneths. All which added together, as appeareth in the operation, maketh eight moneths, which is the iust time that all those payments ought to bee paid at once.

$\frac{1}{3}$ by $\frac{6}{1}$ = $\frac{6}{3}$	2 Mon:
$\frac{1}{2}$ by $\frac{8}{1}$ = $\frac{8}{2}$	4 Mon:
$\frac{1}{6}$ by $\frac{12}{1}$ = $\frac{12}{6}$	2 Mon:
	8 Mon:

A Merchant hath 800 li *to pay, the* $\frac{1}{8}$ *thereof ready money, the* $\frac{1}{4}$ *at two moneths, the* $\frac{1}{2}$ *at foure moneths, and the rest at a yeare. The question is, if he would pay all at one payment, what time ought to be giuen him?*

Answer.

$\frac{1}{4}$ $\frac{2}{1}$	
$\frac{2}{4}$	$0\frac{1}{2}$
$\frac{1}{2}$ $\frac{4}{1}$	
$\frac{4}{2}$	2
$\frac{1}{12}$ $\frac{12}{1}$	
$\frac{12}{12}$	1
	$3\frac{1}{2}$

Answer. The ready money is neuer multiplied. then $\frac{1}{4}$ multiplied by two months as you did before, maketh $\frac{1}{2}$, then $\frac{1}{2}$ by 4 produceth 2 moneths, as appeareth here in the operation: But now for the rest of the money you cannot multiply it vntill you haue sought what proportion it beareth to 800 pounds. Therefore you must subtract the ready money, the $\frac{1}{4}$ and the $\frac{1}{2}$ out of the principall. The rest will will bee $66\frac{2}{3}$ li. which you must looke what part it beareth to the principall, which you shall finde to be $\frac{1}{12}$, the same you must also multiply by his time 12 moneths, and it yeeldeth 1 moneth, so all make $3\frac{1}{2}$ moneths, as appeareth in the operation.

A Merchant is to pay 1200 li. *in three termes, that is to wit,* 400 li. *at two weekes: and* 600 li. *at foure moneths: lastly,* 200 li. *at fiue moneths: The question is, in what time they ought to be payed at once.*

Answer. Proportionate the parts, and you shall find that 400 is $\frac{1}{3}$ part; and for 600 you shall finde $\frac{1}{2}$, and likewise 200 is the $\frac{1}{6}$ part, which multiply by their times as before, and you shall haue $\frac{2}{3}$ weekes, more eight weekes, and lastly $3\frac{1}{3}$ weekes, which together maketh twelue weeks, or three moneths your desire.

A Merchant is to pay 600 *pound in three termes, whereof* 100 *pound is payed present, more* 300 *pound at twenty dayes, and the rest at fiue moneths,*

neths, accounting thirty dayes to a moneth. The question is, what time ought these payments to bee payed at once.

Answer. Worke, and you shall finde two moneths.

The seuenth Chapter intreateth of buying and selling in the Trade of Merchandise, wherein is taken part ready Money, and diuers dayes of payment giuen for the rest, and what is won or lost in the 100 li. *forbearance for 12 moneths more or lesse, according to the quantity of money, or proportion of time, &c.*

A Merchant hath bought Satens which cost eight shillings the yard ready money. And hee selleth the same againe to another man for 10s. the yard, but he giueth two dayes for the payment, that is to say, three moneths for the one halfe, and fiue moneths for the other halfe. The question is to know how much the seller doth gaine vpon the 100 pound in 12 moneths after that rate.

Answer. Seeke first by the Rules of payment, at what time those two payments ought to bee payed at once, and you shall finde foure moneths, at which time the second Merchant ought to haue payd the whole entire payment: And

And therefore ſay by the firſt part of the Rule of three compoſed: If 8 ſhillings in 4 moneths doe gaine 1 s, what wil 100 li. gaine in 12 moneths?

ſ	m	ſ	li	m.
8	4	1	10	12
4			20	
32			2000	
			2	
			4000	
			12	
			48000	

$\cancel{1}$
$\cancel{1}\cancel{6}$ $\cancel{1}$8
$\cancel{4}$8$\cancel{0}\cancel{0}\cancel{0}$ ($\cancel{1}\cancel{5}$00 (75
$\cancel{3}\cancel{2}\cancel{2}$ $\cancel{2}\cancel{2}$0
$\cancel{3}$

Multiply and diuide, and you ſhall finde 75 pounds, as appeareth in the example, and ſo much doth the firſt Merchant gaine vpon 100 pounds in 12 moneths.

A Merchant hath ſold 50 Clothes, at 9 ½ li. the peece, to be payd the one ½ at foure moneths, the ⅓ at fiue moneths, and the ⅙ at ſeuen moneths, and the ſellers minde is to take no more but after eight pounds in the 100 for 12 moneths. The queſtion is now, what the firſt Merchant gaineth in the ſale of theſe Clothes after that rate.

Anſwer. Firſt, looke what the 50 Clothes come to at that price, and you ſhall finde 475 pounds. Then ſecondly, according to your direction in the Rules of payment, ſeeke at what time all the payments are to bee performed at once. And you ſhall finde 4 ⅚ moneths. Then thirdly ſay, by the firſt part of the Rule of three compoſed. If 100 li. in 12 moneths gaine 8 li. what will 475 li. gaine in 4 ⅚ moneths? Worke, and you ſhall finde 15 li. and

and $\frac{11}{38}$ of a pound, which is the neat gaine that the first Merchant hath after the rate aforesaid.

A Merchant hath bought Holland at 7 ſ 3 d the Ell ready money, and hee selleth the same againe for 8 ſ 4 d the Ell to bee payed $\frac{1}{4}$ part in ready money, more $\frac{1}{3}$ part at 2 moneths, and the rest at 4 moneths. The question is now to know how much the first Merchant doth gaine vpon the 100 pounds in twelue moneths after that rate?

Answer. According to the direction deliuered you in the rule of payment, the ready money is not to bee multiplied. Then working for the other two payments to finde out the true proportion at what time they ought to bee paid at once, you ſhall finde for $\frac{1}{3}$ at two moneths, $\frac{2}{3}$ of a moneth. And the rest of the money which is $\frac{5}{12}$ multiplied by his terme 4 moneths, yeeldeth 1 $\frac{2}{3}$ moneths, both which added together make 2 $\frac{1}{3}$ moneths, the iust time that both the payments ought to bee performed at once. And therefore ſay by the first part of the Rule of 3 composed, if 7 $\frac{1}{4}$ in 2 $\frac{1}{3}$ moneths doe gaine $\frac{13}{240}$ of a pound, what ſhall 100 pounds gaine in 12 moneths after that rate? worke and you ſhall finde 78 $\frac{172}{203}$ pounds. And ſo much doth hee gaine vpon 100 pounds in twelue moneths.

A Merchant hath bought 30 clothes at 6 pounds the peece for ready money. Afterward hee selleth tenne of them for 7 pound the peece, for three

three moneths terme. And the other twenty hee selleth for 8 pound the piece for 4 moneths terme. The question is now what hee gaineth vpon 100 pounds in twelue moneths?

Answer. First finde the value of the thirty clothes, which amount to 180 pounds. Secondly, seeke what the tenne pieces come to at 7 pounds, and what the twenty pieces come to at 80 pounds; the one comes to 70, and the other to 160. both which together make 230, which is 50 pounds more then they cost. Thirdly, as I haue taught you in the Rule of payment, proportionate the first and second prices vnto the proportion they beare vnto 230, the product of their two prices, and you shall find $\frac{7}{23}$ for the first, and $\frac{16}{23}$ for the latter. Then fourthly, multiply those parts by their times, and you shall haue $\frac{21}{23}$ & $\frac{64}{23}$ both which together maketh three whole moneths, and $\frac{18}{23}$ of a moneth, which is the iust time that both those payments ought to bee paid at once.

Then say by the first part of the Rule of three composed. If 180 pounds in 3 $\frac{16}{23}$ moneths doe gaine 50 pounds, what shall 100 gaine in twelue moneths? Multiply and diuide, and you shall finde 90 $\frac{10}{51}$ pound, and so much doth he gaine vpon 100 pounds in twelue moneths.

A Merchant hath bought cinamon which cost him 9 shillings the pound ready money. The question is now at what price hee ought to sell thee 100 weight. To wit, 112 pounds to bee paid the $\frac{1}{3}$ at two moneths, and the residue

at

at the end of three moneths, so that he may gaine after the rate of ten pounds vpon 100 *pounds for twelue moneths.*

Answer. Seeke first by the Rules of payment at what terme both the payments ought to bee payd at once, where the $\frac{1}{4}$ multiplyed by his terme two moneths, maketh $\frac{2}{4}$ moneths.

Likewise the next payment, which is $\frac{3}{4}$ multiplyed by his terme three moneths, maketh $2\frac{1}{4}$ moneths, both which added together, maketh $2\frac{3}{4}$ moneths, which is the time, that both the payments ought to bee payd at once. Then say by the Rule of three, if 12 moneths doe giue mee 10 pounds, what will $2\frac{3}{4}$ moneths giue? Multiply and diuide, and you shall finde $2\frac{7}{24}$ pounds. Then say againe by the Rule of three, If one pound cost me 9 s. what will 112 pounds cost? Multiply and diuide, and you shall finde 50 li—8 s. Then say once againe: If 100 pound doe giue $102\frac{7}{24}$, what will $50\frac{2}{5}$ pounds giue? Multiply and diuide, and you shall finde 51 li. 11 s.—$1\frac{1}{5}$ d. and for that price ought I to sell 112 pound of Cinamon to bee payd at the two seuerall payments aforesayd, to gaine thereby after the rate of 10 pounds vpon the 100 pound in twelue moneths.

Briefe Rule.

Briefe

Briefe Rules for our hundreth weight here at London, which is after 112 pound for the 160.

Item, *who that multiplieth the pence that one pound weight is worth by 7, and divideth the product by 15. shall finde how many pounds in money 112 pound weight is worth.*

And contrariwise, he that multiplieth the pounds that 112 pounds weight is worth by 15. and divideth the product by 7. shall finde how many pence in money the one pounds weight is worth.

Example.

At 10 pence the pound weight, what is 112 pounds weight worth?

Answer. Multiply 10 by 7, and thereof commeth 70. the which divided by 15, and you shall finde $4\frac{2}{3}$ pounds. And thus the 112 pounds is worth 4 li. 13 s. 4 d. after the rate of 10 pence the pound aforesaid.

At 6 pounds the 112 pounds weight, what is one pound worth?

Answer. Multiply 6. by 15. and thereof commeth 90. the which divide by 7. and you shall finde $12\frac{6}{7}$ d. So much is one pound worth, when the 112 pounds did cost 6. pounds.

The eight Chapter intreateth of Tares and allowances of Merchandize ſold by weight, and of loſſes and gaines therein, &c.

AT 16. pound the 100. Suttle, what ſhall 895. pound Suttle bee worth, in giving 4. pound weight upon euery 100 for treat?

Anſw. *Adde* 4 *vnto* 100. *and you ſhall haue* 104. *Then ſay by the Rule of three: If* 104 *be worth* 16 *pounds, what are* 895 *pounds worth? Multiply and diuide, and you ſhall finde* 237 li. 13. s. 10. $\frac{2}{13}$ d, *and ſo much ſhall the* 895. *pounds weight be worth.*

Item, at 3. s. 4. d. the pound weight ſhall 754 $\frac{1}{2}$ pound be worth, in giving 4 pounds weight upon euery hundred for treat?

Anſwer. See firſt by the rule of three what the 100. pound is worth, ſaying: If one coſt 3. $\frac{1}{3}$ s. what 100? Multiply and divide. and you ſhall find 16 $\frac{2}{3}$ pounds. Then adde 4 unto one 100 and they are 104. Then ſay againe by the rule of three, if 104 be ſold for 16 $\frac{2}{3}$ pounds, for how much ſhall 754 $\frac{1}{2}$ bee ſold for? Multiply and diuide, and you ſhall finde 120 li. 18 s. 3 $\frac{2}{13}$ d. And for ſo much ſhall the 754. $\frac{1}{2}$ pound bee ſold for at 3 s. 4 d. the pound, in giuing 4 upon the 100.

Other neceſſary briefe rules there are for the finding of treats, or caſting up of cheſts of Sugar, &c.

&c. which for that it is a mysterie, I omit: if any lacke instruction that way, they shall find me ready to pleasure them.

Item, if 100 pounds be worth 36 s. 8.d. what shall 860 pounds be worth in rebating 4 pounds upon every hundred for tare and closse?

Answer. Multiply 860 by 4, and thereof commeth 3440, the which divide by 100, and you shall have 34 $\frac{2}{5}$ pounds, abate 84 $\frac{2}{5}$ from 860 and there will remaine 825 $\frac{3}{5}$ pounds. Then, say by the rule of three. If 100 pound cost 36 $\frac{2}{3}$ s. what will 825 $\frac{3}{5}$ cost after that rate? Multiply & divide, and you shal find 15 li. 2 s. 8 $\frac{16}{25}$ d. And so much shall the 860 cost in rebating foure pounds upon every hundred, for tare and closse.

Item, *whether doth hee lose more that giveth* 4 *pounds upon the* 100, *or he that rebateth* 4 *pounds upon the* 100?

Answer. First note, that hee that giveth 4 pounds on 100, giveth 104 for 100. And hee which rebateth 4 pounds upon the 100, giveth the 100 for 96. Therefore say by the rule of three, if 104 bee delivered for 100, for how much shall the 100 be delivered? Multiply and divide, and you shall finde 96 $\frac{2}{13}$, and hee which rebateth 4 in the 100, maketh but 96 pounds of 100, so that he loseth 4 pounds in the 100, and the other which giveth 4 li. upon the 100 looseth but 3 $\frac{11}{13}$ pounds upon the 100. Thus you may see, that hee which rebateth 4 pounds

in the 100. looseth more by $\frac{11}{13}$ pound in the 100 pounds, then the other which gave 4 pounds upon the 100, for tare and closse.

If 100 pounds of any thing cost mee 23 s. 4 d, the question is, how I shal sel the pound, to gain after the rate of 10 pounds, upon the 100 pound

Answ: Say by the rule of three, if 100 pounds give 110 pounds, what shall $23\frac{1}{3}$ s. give? Multiply and divide, and you shall find $1\frac{17}{60}$ pounds. Then say againe, If 100 pound bee worth $1\frac{17}{60}$ pounds, what is one pound worth? Multiply and divide, and you shall finde 3 d. $\frac{2}{25}$. And so much is the pound worth in gaining 10 pounds upon the 100.

Item, *A Grocer hath bought C. weight of commodity for* 6. li. 10 s. *The question is now, to know how many pounds thereof he shall sell for* 33 s. 4. d. *to gaine* 20 s *in the C. weight.*

Answer. Adde 20. s. unto 6 li. 10 s. and they make 7 li. 10 s. Then say, if $7\frac{1}{2}$ pound yeeld mee 112 pound, what shall $1\frac{2}{3}$ pounds yeeld? Multiply and divide, and you shall finde $24\frac{1}{9}$ pound. And so many pound ought hee to sell to gaine 20 s. in his C. weight.

If one pound weight cost 3 s. 4 d. *and I sell the same againe for* 4 s. *what is gained in a* 100 *pound of money laid out in that commodity?*

Answer. You may say, If $3\frac{1}{3}$ s. give 4

what

what will 100 pound gaine? But then when you have found, you must subtract 100 pounds out of the Product, the rest is your neat gaine, or else to produce the neat gaine in your worke at the first. Subtract the iust price out of the over-price, as I taught before in the first beginning of Losse and Gaine, and your conclusion shall bee all one. Multiply and divide, by which of the two waies you thinke good, and you shall finde that hee gaineth 20 pounds, in the 100 pound.

Item, if the pound weight which cost 4.s. *be sold againe for* 3 s.–4 d. *I demand what is lost in the* 100 *pounds of money.*

Answer. Say, If 4 s lose $\frac{2}{3}$ s. what shall 100 lose? Multiply and divide, and you shall finde 16 li. 13 s. 4 d. and so much is lost upon the 100 of money.

Item, if C. weight of any commodity cost 45 *pounds, & the buyer repenting, would lose* 5 *pounds in the* 100 *of mony, I demand how the pounds may be sold, his losse to be neither more nor lesse than after the rate aforesaid of* 5 *by the* 100?

Answer. By the Rule of three, if 100 lose 5, what shall 45 lose? Worke, and you shall finde $2\frac{1}{4}$ pound, which rebated from the principall 45, resteth 42 li. 15 s. Lastly say, if 112 yeeldeth but 42 li. 15 s. what one pound? Multiply and divide, and you shall finde 7. s. 7 d. $\frac{17}{28}$.

And so much is the pound worth after that losse.

A Grocer hath bought 3 peeces of Raisins, weighing 175 $\frac{1}{2}$ pounds, 182 $\frac{1}{4}$ pounds : 191 pounds : tare for each fraile 2 $\frac{3}{4}$ pounds, at 25 $\frac{1}{2}$ s. the C weight. The question is, what they amount to in money?

I answer 6.li.——3 s.——4 $\frac{23}{28}$ d.

A Grocer hath bought three sackes of Almonds weighing 267 $\frac{1}{2}$ pound, Tare 2 pound. 257 $\frac{1}{2}$ pounds tare 2 $\frac{1}{2}$ pound, 252 pound, tare 3 pound, at 2 s. 10 $\frac{1}{2}$ d. the pound, what amount they to in money?

I answer, 110 li.——12 s.——3 $\frac{3}{4}$ d.

The ninth Chapter intreateth of lengths and breadths of Arras and other clothes, with other questions incident unto length and breadth.

IF *a peece of Arras be 7 Ells and $\frac{3}{4}$ long, and 5 Ells and $\frac{2}{3}$ broad, how many Ells square doth the same peece contain?*

Answ. Multiply the length by the breath, that is to say, 7 $\frac{3}{4}$ by 5 $\frac{2}{3}$. And thereof will come 43 $\frac{11}{12}$ Elles : so many Elles square doth the same peece containe.

Item *more, a peece of Arras doth containe 22 Ells square, and if the same were in length 3 $\frac{1}{3}$ Ells,*

I demand how many Ells in breadth the same peece doth containe.

Answer. Diuide 22 Els by $3\frac{1}{4}$ and thereof commeth $6\frac{10}{13}$. So many Els doth the same containe in breadth.

Item *more, a Merchant hath $3\frac{1}{4}$ Elles of Arras, at $1\frac{2}{3}$ Ells broad, which hee will change with another man for a peece of Arras, that is $\frac{7}{8}$ Els square. The question is, how many Ells of that squarenesse ought the first Merchant to haue?*

Answ. Multiply the first Merchants peece his length by the breadth, and you shall finde, it containeth $5\frac{5}{12}$ Ells, which $5\frac{5}{12}$ Ells you shall diuide by $\frac{7}{8}$ and you shall finde $6\frac{4}{21}$ Ells, and so many Ells of that squarenesse ought the latter Merchant to giue the first,

Item, *a Student hath bought $3\frac{1}{2}$ yards of broad cloath, at 7 quarters broad, to make a gowne, and should line the same throughout with Lambe, at a foot square each skin: the question is now how many skinnes he ought to haue?*

Answ. Seeke first the number of yards square that his cloth containeth, which to doe, multiply $3\frac{1}{2}$ his length, by $1\frac{3}{4}$ his breadth, and you shall finde $6\frac{1}{8}$ yards square: then say by the rule of three, if one yard square giue 9. foot, what shall $6\frac{1}{8}$. Worke, and you shall finde $55\frac{1}{8}$ skins.

Item *more, a Lawyer hath a rich peece of seling come home which is 24. foot and three inches long, and 7 foot and $2\frac{1}{2}$ inches high: the Ioyner is to bee paid by the yard square: the question is, how many yards this peece containeth?*

Answer. Multiply his length by his breadth, that is to wit, $24\frac{1}{4}$ foot by $7\frac{5}{24}$ foot, and you shall finde $174\frac{77}{96}$ foot square, which 174 you shall diuide by 9 (for so many foot make a yard square) and you shall finde 19 yards three foot and $\frac{77}{96}$ of a foot, and so many yards doth this peece hold.

Item, *bought a peece of Holland cloth containing 36 Ells $\frac{1}{3}$ Flemish. The question is how many Ells English it makes?*

Answer, You must note, that fiue Ells Flemmish doth make but three Ells English.

Therefore say by the Rule of three, if fiue Ells Flemmish, make but three Ells English, how many Ells English will $36\frac{1}{3}$ Ells Flemmish make? Multiply and diuide, and you shall finde $21\frac{4}{5}$ and so many English doth $36\frac{1}{3}$ Ells Flemmish containe. The like is to bee done of others.

Item, *more, I haue bought 342 Ells Flemmish, of Arras worke, at two Ells broad Flemmish, and I would line the same with Ell broad Canuas of English measure. The question is, how many Ells English will serue my turne.*

Answer. For as much as three Ells English

liſh are worth fiue Ells Flemiſh, therefore put 3 Els Engliſh into his ſquare, in multiplying three by himſelfe, which maketh 9. Likewiſe multiply the Engliſh Ell, which is fiue quarters, euery way into himſelfe ſquarely, and you ſhall finde 25. Then multiply 342 which is the length of the peece, by 2. which is the breadth, and thereof commeth 684. then ſay by the Rule of three, as before: if 25 Ells ſquare of Flemmiſh meaſure, bee worth 9 Ells ſquare of Engliſh meaſure, what are 684 of Flemmiſh meaſure? Multiply and diuide, and you ſhall finde $246\frac{6}{25}$ Ells Engliſh.

The ſame is alſo wrought by the Backer Rule of three, in ſeeking the ſquares contained in the Flemmiſh Ell of two Ells broad (which are 18) and alſo in ſeeking the ſquares contained in the Engliſh Ell (which are 25) then ſay by the Rule of three backward, If 18 quarters require 342 Ells, what ſhall 25 quarters giue? Multiply and diuide by the Rule of three Reuerſe, and you ſhall finde as before $246\frac{6}{25}$ Ells Engliſh?

Item *more, at three ſhillings foure pence the Flemmiſh Ell, what is the Engliſh Ell worth after the rate?*

Anſwer. Say if three quarters giue $3\frac{1}{3}$ s, what giueth fiue quarters? Multiply and diuide, and you ſhall finde 5 s. $6\frac{2}{3}$ d.

Item *more, at* 8 s. 4 d, *the Flemmiſh Ell ſquare,*

square, what is the English Ell worth after that rate?

Answer. According to the reason of the last Question, consider that a Flemmish Ell square is equall to nine quarters of a yard English, and an English Ell square is equall to 25: quarters of a yard. Therefore say by the rule of three, if 9 quarters give $8\frac{1}{3}$ s. what 25. quarters? Work and finde 23 s. $1\frac{7}{9}$ pence. And so much is the English Ell worth.

Item, *more, at* 6 s. 8 d. *the Ell square: what shall a peece of cloth cost that is* $7\frac{1}{2}$ *Ells long, and* $3\frac{1}{4}$ *Ells broad?*

Answer. Multiply the breadth by the length, and you shall finde $24\frac{3}{8}$ Ells square cost $6\frac{2}{3}$ s. what $24\frac{3}{8}$ Ells? Multiply and diuide, and you shall finde 8 pounds, 2 shillings 6 pence, and so much the same peece of cloth cost.

Item *more, a Mercer sold three peeces of Silke. To wit* $24\frac{1}{4}$ $13\frac{2}{3}$ *and* 25. *yards, at* $9\frac{3}{4}$ s. *the yard, and was glad to receiue in part of payment againe, a cloth containing* $34\frac{1}{2}$ *yards at* $7\frac{2}{3}$ *shillings the yard. The question is now, what the Debtor is in the Creditors Debt? Worke, and you shall finde hee oweth the Mercer* 22 *pounds,* 3 *shillings,* $2\frac{1}{4}$ *pence.*

The

The tenth Chapter entreateth of reducing of pawnes of Geanes into English yards.

Note, that 100 Pawnes doe make 26. yards, wherupon three Pawnes $\frac{11}{13}$, *do make one yard and one Pawne after that rate and proportion is* [illegible] *of a yard.*

In 4563 pawnes of Geanes, how many yards English?

Answer. Say by the rule of three, if 100 Pawnes doe make 26 yards, what will 4563 Pawns make? Multiply and divide, and you shall finde 1186 yards $\frac{19}{50}$. So many yards doe 4563 Pawnes make.

Otherwise, take some other number at your pleasure, as 10 pawnes, which is the $\frac{1}{10}$ part of 100, then to finde his proportion, take the $\frac{1}{10}$ part of 26, which is 2 $\frac{3}{5}$, and then say also by the Rule of three, if 10 Pawnes give 2 $\frac{3}{5}$ yards, what will 4563 Pawnes give? Worke, and you shall finde 1186 $\frac{19}{50}$ yards, as before.

More, at 2 s.–6 d. the Pawnes of Geanes, what will the English yard bee worth after the rate?

Answer. Say by the rule of three, if $\frac{13}{50}$ of a yard cost 2 $\frac{1}{2}$ s. what one yard? Multiply and divide, and you shall finde 9 s. 7. $\frac{5}{13}$ d.

More,

More if 346 $\frac{1}{2}$ *pawnes cost* 30 li. 13. s. 4 d. *sterling, what is that the English yard after the rate?*

Answer. Say by the Rule of three, if 346 $\frac{1}{2}$ Pawnes cost 30 $\frac{2}{3}$ pounds, what are 3 $\frac{11}{12}$ pawnes worth (for so many Pawnes make a yard) Multiply and divide, and you shall finde $\frac{9200}{27837}$ parts of a pound, which in knowne numbers is worth 6 s. 9 d $\frac{6271}{90000}$.

The eleventh Chapter entreateth of Rules of Loane and Interest, with certaine necessary questions and proofs incident thereunto, &c.

I*Tem, lent my friend 226 pounds for 5* $\frac{1}{2}$ *moneths simply without any Interest, upon conditiō, to have the like curtesie againe when I need. But when I came to borrow, he could spare me but* 149 l. 8 s. 4 d *The question is now how long time I ought to have the use thereof, to countervaile my friendship before time shewed him?*

Answer. Say by the backer Rule of three, If 226 pounds give 5 $\frac{1}{2}$ moneths, what time will 149 $\frac{5}{12}$ pounds give? Multiply and divide and you shall finde twelve moneths, & so long time ought I to use his money.

The proofe.

Item, *lent my friend* 149 li. 8 s. 4 d. for twelve

moneths

moneths. The queſtion is now how much money he ought to lend me again for $5\frac{1}{2}$ *months to recompence my friendſhip ſhewed him?*

Anſwer. Say by the backer or reuerſe Rule of three, If 12 moneths giue $149\frac{5}{12}$, what ſhall $5\frac{1}{2}$ moneths giue? Worke, and you ſhall finde 326 pounds, and ſo much ought hee to lend mee to requite my gentleneſſe or good turne.

Two other Branches, yet more, for proofe out of the ſame queſtion.

Item, lent my friend 149 li. 8 ſ. 4. d. *for* 12 *moneths, to haue the like friendſhip againe when I need. And comming to borrow of him, hee very curteouſly tooke mee* 326 *pounds (for that hee could well then ſpare the ſame.) The queſtion is now, how long I ought to occupie it, not vſurping friendſhip, but in his due time to reſtore it againe.*

Anſwer. Say by the Rule of three reuerſe, if $149\frac{5}{12}$ pounds giue 12 moneths, what ſhall 326 pounds giue? Multiply and diuide, and you ſhall finde, that at $5\frac{1}{2}$ moneths terme, I ought to reſtore it againe.

Proofe.

Item, *Lent my friend* 326 *pounds for* $5\frac{1}{2}$ *moneths. The queſtion is now, how many pounds hee ought to lend mee for* 12 *moneths to recompence*

pence this pleasure againe?

Scholer. Worke by the rule of three, reverse as you have done before, and you shall finde 149 li——8 s——4 d.

Againe, foure other selected questions, of Loane and Interest, all out of one branch, and each one also a necessary question, and a particular proofe to other.

ITem, *Lent my friend* 430 *pounds at Interest for three months, to receive after the rate of* 8 *pounds in the* 100 *pounds for* 12 *moneths. The question is, what the Interest commeth to?*

You may, if you please, worke it at two workings by the rule of three direct, in saying, if 12 moneths give 8 pounds, what giveth three Moneths? Multiply and divide; and it giveth 2 pound.

Then for the second work say: If 100 pound yeeld 2 pounds, what yeeldeth 430 pound? Multiply and divide: and you shall finde 8 li. 12. s. and so much comes the loane of 430 pounds to for 3 moneths after the rate of 8 pounds in the 100 pounds of 12 moneths.

Otherwise wrought thus by the Rule of three at twice also.

If 100 pound give 8 pounds, what giveth 430 pounds? Multiply and divide, and you shall finde 34 pounds $\frac{2}{5}$. Then againe for the second worke say: If 12 moneths give 34 pounds

pounds $\frac{2}{5}$, what giveth three moneths? Worke and finde 8. li. 12. s. as before.

Otherwiſe yet at one working: By the firſt part of the rule of fiue numbers forward, in ſaying, if 100 pounds in 12. moneths, gaine 8. pounds, what ſhall 430. pounds gaine in three moneths? Multiply the firſt by the ſecond for your Diuiſor, and the other three, the one into the other for the Diuidend, and you ſhall finde eight pounds 12. ſhillings, as aforeſaid.

Proofe.

Item, *A friend of mine receiued of me* 8.*pounds* 12. *ſhillings for the Intereſt and Vſe of* 430 *pounds for three moneths terme: The queſtion is now, what hee tooke in the* 100. *pound for* 12. *moneths after that rate*?

Anſwer. For moſt briefe, ſay by the firſt part or rule of fiue numbers forward: If 430. pounds in three moneths did pay 8. li. 12. s. what doth 100. pounds in 12. moneths take after the rate? Worke, and you ſhall finde 8. pounds, and ſo much hee tooke upon the 100. pounds for twelue moneths.

A third Queſtion and proofe alſo by the Backer Rule of 5. Numbers.

ITem, *lent my friend* 430. *pounds to receiue for the intereſt thereof, after the rate of* 8. *pounds in*

in the 100 *for* 12 *monethes. The question is now how long time my friend ought to giue the vse thereof, that it may bee returned with* 8 li. 12 s. *gaines.*

You may worke it, if you please, by the Rule of three direct at twice, in saying: If 100 pound yeeld 8 pounds, what yeeldeth 430 pound? Multiply and diuide, and find 34 pound and $\frac{2}{5}$.

Then againe for the second worke say, if 34 $\frac{2}{5}$ pounds, giue 12 moneths, what giueth 8 $\frac{3}{5}$ pounds, Multiply and diuide, and you shall find three moneths, and so long time ought my friend to vse it to returne with 8 li-- 12 s. gaine.

Otherwise at one working by the Backer Rule of 5 numbers, in saying: If 100 pounds in 12 moneths doe gaine 8 pounds, how much time shall 430 pounds be a gaining of 8 pounds 12 s? Multiply the first and the second into the last for your dividend, and the third and fourth multiply together for your divisor, and then divide, and you shall finde 3 moneths, the iust time that my friend ought to use it to returne it with 8 li. 12 s gaine.

A fourth derived question out of this Branch, which is a proofe of this last, and also of the other two going before.

I*Tem, how much money ought a Merchant to deliuer after* 8 *pounds in the* 100 *for* 12 *monethes, that in three monethes hee may gaine* 8 li. 12 s?

Answer.

Anſwer. You may alſo, if you pleaſe, worke it by the Golden rule of Three at twice: firſt ſaying, If three moneths giue $8\frac{3}{5}$ pound, what 12 moneths gaine? You ſhall finde $34\frac{2}{5}$. Then ſay againe, If 8 pounds be come of 100 pounds, what ſhall come of 34 li. 8 ? Worke, and you ſhall finde the anſwer to the queſtion, which is 430 pounds, and ſo much ought the Merchant to deliuer.

But moſt briefly it is anſwered by the Backer Rule of fiue numbers, where I argue thus, ſaying: If 100 li. be twelue moneths a gaining of 8 li. then but for three moneths terme onely to take 8 li. 12 s. muſt needs be a good round ſumme to worke it, ſet your numbers thus: 100---12 8---3---$8\frac{3}{5}$: multiplying the firſt into the ſecond, and alſo by 43 the product of the fifth, for your diuidend, and the third and fourth together with 5 the Denominator of your fraction for your Diuiſor: then diuide, and you ſhall finde as before 430 pounds: the true ſolution to your queſtion.

The twelfth Chapter intreateth of the making of Factors, which is taken in two ſorts.

THe firſt is, when the eſtimation of the Factor is taken vpon the ſending of the Merchant, as if the eſtimation of his perſon be $\frac{1}{4}$ it is vnderſtood that he ſhall haue $\frac{1}{4}$ of the gaine, the Merchant the other $\frac{3}{4}$.

The other ſort is, when the eſtimation of

his making is out of the sending of the Merchant, as if the order and agreement betweene them were such, that the Merchant shall put in 800 li, and the Factor for his making shall haue $\frac{1}{4}$: neuerthelesse he shall haue but $\frac{1}{5}$ of the gaine or profit, for the $\frac{1}{4}$ of 800 is 200 (for the estimation of his making) which with the 800 pounds in all make 1000 pounds, whereof the 200 pounds, is $\frac{1}{5}$.

A Merchant doth put in 800 pound into the hands of his Factour, vnder such condition that the said Factour shall haue $\frac{1}{4}$. And after certaine time they finde in profit 124 li. 6 s. 8 d. I demand how much the Merchant shall haue heereof, and how much ought the Factor to haue?

Answer. *When the estimation of the Factour is out of the sending of the Merchant, it maketh,*

li	s	d	
99	9	4	*for the Merchant.*
24	17	4	*for the Factor.*

But if that his estimation be at the sending of the Merchant, then it maketh but,

li	s	d	
93	5	0	*for the Merchant.*
31	1	8	*for the Factor.*

For the Merchant is then to haue $\frac{3}{4}$, *and the Factor* $\frac{1}{4}$.

A

A Merchant doth put into the hands of his Factor 800 pounds, and the Factor 400 pounds to haue the $\frac{1}{5}$ part of the profit: I demand now for how much his person is esteemed, when the same is counted vpon the sending of the Merchant?

Answer. *According to the tenour and order before prescribed in the first Rule, that is, if his estimate bee $\frac{1}{4}$ hee shall haue the $\frac{1}{4}$ of the gaine. Therefore say by the Rule of three direct. If $\frac{1}{4}$ taken put in 400 pound, what is the estimate, or putting in of $\frac{1}{5}$ taking? Multiply and diuide, and you shall finde 320 pounds, and so much is the person of the Factor estimated.*

Otherwise.

To finde the estimation of the person of the Factour, you shall consider, that seeing it was agreed betweene them, that the Factour should take the $\frac{1}{5}$, then the Merchant shall haue the residue, which are $\frac{4}{5}$: wherefore the gaine of the Merchant vnto that of the Factour is in such proportion as 5 vnto 4. Then if you will know the estimation of the person of the Factor: say if 5 give 4. what will 400 give? Multiply and divide, and you shall finde 320 pound. And so much is the person of the Factor esteemed to be worth.

Other conditions then these aforesaid, may also bee betweene Merchants and Factors, without respect either of sending,

or not sending of the Merchant, where most commonly the estimation of the body of the Factour is in such proportion of the stocke which the Merchant layeth in, as the gaine of the said Factour is vnto the gaine of the Merchant. As thus, if a Merchant doe deliuer into the hands of his Factour 400 pound and hee to haue halfe the profit: the person of the said Factour shall be esteemed to be worth 400 pound: and if the Factor doe take but $\frac{1}{3}$ of the gaine, hee should haue but $\frac{1}{2}$ so much of the gaine as the Merchant taketh, which must haue $\frac{2}{3}$, wherefore the person of the Factor is esteemed but the $\frac{1}{2}$ of that which the Merchant layeth in, that is to say, 200 pound.

And if the Factour did take the $\frac{2}{5}$ of the gaine, then the Merchant shall take the residue which are $\frac{3}{5}$, wherefore the gaine of the Merchants vnto the Factour is then in such proportion as 3 vnto 2: whereupon if you will then know the estimation of the person of the Factor, say, If three giue 2, what shall 400 giue? Worke, and you shall finde 266$\frac{2}{3}$ pounds. And so much is the person of the Factor esteemed to be worth.

And if the Merchant should deliuer vnto his Factor 400 pound, and the Factour would lay in 80, and his person, to the end hee might haue the $\frac{1}{2}$ of the gaine, I demand how much shall his person be esteemed?

Answer. *Abate* 80 *from* 400; *and there will remaine*

remaine 320. *And at so much shall his person bee esteemed.*

A Merchant hath deliuered vnto his Factor 900 pounds to gouerne in the trade of Merchandise, vpon condition that hee shall haue the $\frac{1}{3}$ of the gaine, if any thing bee gained, and also to beare the $\frac{1}{3}$ of the losse, if any thing bee lost. Now I demand how much his person was esteemed at?

Answer. *Seeing that the Factour taketh the $\frac{1}{3}$ of the gaine, his person ought to be esteemed as much as $\frac{1}{2}$ of the stocke, which the Merchant layeth in: that is to say, the $\frac{1}{2}$ of 900 pound, which is 450. The reason is, because $\frac{1}{3}$ of the gaine that the Factour taketh, is the $\frac{1}{2}$ of the $\frac{2}{3}$ of the gaine that the Merchant taketh, and so the Factour his person is esteemed to bee worth 450 pounds.*

A Merchant hath deliuered vnto his Factour 600 pound, and the Factor layeth in 250 pound and his person. Now because he layeth in 250 pounds and his person, it is agreed betweene them, that hee shall take the $\frac{2}{5}$ of the gaine. I demand for how much his person was esteemed?

Answer. *For as much as the Factor taketh $\frac{2}{5}$ of the gaine, hee taketh $\frac{2}{3}$ of that which the Merchant taketh, for $\frac{2}{5}$ are the $\frac{2}{3}$ of $\frac{3}{5}$. And therefore the Factors laying in ought to be 400 pound, which is $\frac{2}{3}$ of 600 pound that the Merchant layed in. Then subtract 250, which the Factor did lay in from 400 pound, which should haue beene his*

whole stocke, and there remaineth 150. *pound for the estimation of his person.*

More, a Merchant hath delivered vnto his Factor 800 pound, upon condition that the Factor shall haue the gaine of 160 pound, as though he laid in so much ready money: I demand what portion of the gaine the said Factor shall take?

Answer. *See what part the* 160. *(which the Factor layed in) is of* 960. *which is the whole stocke of their company, and you shall finde* $\frac{1}{6}$. *And such part of the gaine shall the Factor take.*

But in case, that in making their covenants, it were so agreed betweene them, that the Factor should haue the gaine of 160 *pound of the whole stocke which the Merchant layeth in, that is to say, of the* 800 *pound: then should the Factor take* $\frac{1}{5}$ *of the gaine: for* 160 *is* $\frac{1}{5}$ *of* 800 *pound.*

The thirteenth Chapter intreateth of Rules of Barter, and exchanging Merchandize, which is distinct into seuen Rules, with diuers other necessary questions incident thereunto.

The first Rule.

TWo Merchants willing to change their Merchandize the one with the other: The one

one hath 24 broad cloathes at 10 li--10 s. the peece: The other hath Mace at 12 ſ, the pound. The queſtion is, how many pounds of Mace hee ought to giue for his Cloath, to ſaue himſelfe harmeleſſe, and be no loſer?

Anſwer. *Seeke firſt by the Rule of three, what the 24 Clothes coſt at 10 pound 10 ſhillings the peece, and you ſhall finde 252 pound. Then to finde the quantity of Mace, ſay againe by the Rule of three, If 12 ſhillings buy one pound, what ſhall 252 pound buy me? Worke, and you ſhall find 420 pound of Mace: and ſo many pound ought he to giue for his Clothes.*

The Proofe.

Two barter. The one hath 420 pounds of Mace at 12 ſ. the pound, to barter or change broad Cloathes at 10 pounds 10 ſhillings the peece. The queſtion is, how many broadcloths he ought to giue for all his Mace?

Anſwer. *Firſt ſay, if one coſt* 12 ſ. *what* 420? *you ſhall finde* 5040 ſ. *Then ſay againe, If* $10\frac{1}{2}$ *pounds giue* 1 *cloth, what ſhall* 5040 *ſhillings giue? Worke and you ſhall finde* 24 *clothes, your deſire.*

The ſecond Rule.

Two change Merchandiſe for Merchandiſe: The one hath Pepper at 2 ſhillings 4 pence the pound, to ſell for ready money.

But in barter hee will haue lesse then three shillings the pound. And the other hath Holland at fiue shillings 6 pence the Ell ready money. The question is now at what price hee ought to deliuer the Ell in the barter to saue himselfe harmelesse.

Answer. *Say by the Rule of Three direct 2 $\frac{1}{3}$ ready money giue 3 s, in barter, what sh. 5 $\frac{1}{2}$ giue in barter? You shall finde 7 $\frac{1}{14}$ s, and at that price ought the second Merchant to sell his Holland in barter.*

The Proofe.

Two barter. The one hath Holland at 5 s. 6 pence the Ell to sell for ready money. And in barter hee will haue 7 $\frac{1}{14}$ s. The other hath Pepper at 2 s——4 d, the pound, to sell for ready money. The question is now, how he ought to sell in barter?

Answer. *Say by the Rule of Three direct, if 5 $\frac{1}{2}$ ready money giue 7 $\frac{1}{14}$ s, in barter, what ought 2 $\frac{1}{3}$ s. to take in barter: Multiply and diuide, and you shall finde 4 s. your desire.*

The third Rule.

Two barter: The one hath cloth of Arras at 30 s the Ell ready mony, but in barter he will haue 35 $\frac{1}{2}$ s. And the other hath White-wines which hee deliuered in barter for 16 pounds the Tunne. The question is now, what his wines

wines cost the Tunne in ready money.

Answer. *Say by the Rule of three direct, if* $35\frac{1}{2}$ *s, in barter, giue but* 30 s. *ready money, what did* 16 *pound in barter cost? Worke, and you shall finde* 13 li--10 s $\frac{30}{71}$. *And so much cost his Wines for a Tunne ready money.*

The Proofe.

Two barter merchandise for merchandise: The one hath white wines at 13 li--10: $\frac{30}{71}$ s. the Tunne to sell for ready money. But in barter hee deliuered it for 16 pounds: The other, to make his match good and saue himselfe harmelesse, deliuereth Arras at $35\frac{1}{2}$ s. the Ell. The question is now, what an Ell of his Arras cost in ready money?

Answer. *Say by the Rule of Three direct; If* 16 *pounds in barter giue but* 13 li-- 10 $\frac{30}{71}$ s. *in ready money, what shall* $35\frac{1}{2}$ s. *yeeld in barter? Worke, and you shall finde* 30 s. *your desire.*

The fourth Rule.

Two barter. The one hath Kerseys at 14 *pounds the peece ready money. But in barter hee will haue* 18 *pounds. And yet hee will haue the* $\frac{1}{2}$ *part of his ouerprice in ready money. And the other hath Ginger at eight groats the pound, to sell for ready money. The question is, how hee ought to deliuer the Ginger by the pound in barter to saue himself harmlesse, and make the barter equall.*

Answer.

Anſwer. Item, for the working of this queſtion, and ſuch other the like, you muſt vnderſtand, if the party ouer-ſelling his wares, require to haue alſo ſome portion in ready money, as $\frac{1}{2}\frac{2}{3}\frac{3}{4}$, &c. Then ſhall you firſt rebate the ſame demanded part, whatſoeuer it bee, from the ouerprice, and alſo from the iuſt price. And thoſe two numbers that ſhall remaine after the ſubtraction is made, ſhall bee the two firſt numbers in the Rule of three. And the iuſt price of the ſame Merchandize ſhall bee the third number, which by the operation of the Rule of three direct, ſhall yeeld you a true ſolution, how, and at what price you ſhall ouerſell that your merchandize, to ſaue your ſelfe harmleſſe, and make the barter equall.

Example.

Take the $\frac{1}{3}$ (of eighteene) which is the ouerprice of his Cloath, which $\frac{1}{3}$ of eighteene is ſixe, which you muſt ſubtract from 18. there reſt 12. And alſo abate it from 14, which is the iuſt price of the Cloth and there remaineth 8 which 8 and 12 are the two firſt numbers in the Rule of Three. Then take eight groates or $2\frac{2}{3}$ ſhillings for the third number. Then ſay by the Rule of three direct. If 8 pounds giue 12 pounds, what

14	18
6	6
8	12

what shall 2 $\frac{2}{3}$ s. give? Multiply and divide, and you shall finde 4 s. And for so much shall the second Merchant sell his Ginger, or his commodity in barter, to ballance the same equall.

The proofe.

Two barter, the one hath five Kerseys at 14 pounds the peece ready money. But in barter hee will have 18 pounds. And yet hee will have the $\frac{1}{3}$ part of his overprice in ready money: And the other hath Ginger, which he having cunning enough to make the barter equall, delivered in barter for 4. s. the pound. The question is now, what his Ginger cost him ready money?

Answer. After you have made the subtraction, abating 6 the $\frac{1}{3}$ part of 18, both from 18 and 14 (as before was taught you:) then wil there remaine eight and 12 for your two first numbers in the Rule of three. Then say, If twelve give eight, what shall come of 4 the over-price of the pound of Ginger, multiply and divide, and you shall finde 2 shillings 8 pence your desire.

Two Merchants barter merchandise for Merchandise. The one hath Devonshire *whites at* 7 li. 13 s. 4. d. *the piece ready money: but in barter hee doth them away for* 8. li - 3 - s - 4 - d. *and yet hee will have the $\frac{1}{3}$ part of his price in ready money. And the other hath Cottens at 3 pounds the peece ready money. The question is now, at what price he ought*

ought to ſell or exchange his Cottens in barter to ſaue himſelfe harmeleſſe, and make the barter equall?

7———13———4　　　8———3———4
2———14———$5\frac{1}{4}$　　　2———14———$4\frac{5}{8}$
———————————　　　———————————
4———18———$10\frac{2}{3}$　　　5———8———$10\frac{2}{3}$

Anſwer. Firſt ſeeke the $\frac{1}{3}$ part of 8 l———3 ſ. 4 d. which is, 2 li———14———$5\frac{1}{3}$ d. which rebated from 8 l———3 ſ———4 d. there reſteth as appeareth by the example abouesaid, 5 l--8 ſ——$10\frac{2}{3}$ d. which is $\frac{2}{3}$ parts of 8———3———4 d alſo rebated from 7———13———4 d. there reſteth, 4 l———18 ſ———$10\frac{2}{3}$ d. the two firſt numbers in the Rule of three, and the 3 pounds, which is the neat price of the peece of Cotten, is the third number; Then ſay by the Rule of three direct, as was taught before. If 4 li--18 ſ--$10\frac{2}{3}$ d. giue 5 li--8 ſ--$10\frac{2}{3}$ d. what ſhall 3 pounds giue? multiply and diuide, and you ſhall finde 3 li. 6 ſ--$0\frac{72}{80}$ pence, the iuſt price that hee ought to deliuer his Cottens in barter.

The fifth Rule.

Two Merchants will change merchandiſe for merchandiſe. The one hath Kerſeys at 43 ſ. *the peece to ſell them for ready money. And in barter, hee will ſell them for* 56 ſ. 8 d. *and hee will gaine after* 10 *pounds vpon the* 100 *pounds. And yet hee*

will

will haue the ½ of his ouerprice in ready money. The other hath flaxe at 3 d. the pound ready money. The question is now, how hee shall sell the pound of his flaxe in barter?

Answer. See first at 10 pounds vpon the 100 pounds what the 56⅔ s. commeth to, in saying (by the Rule of three direct) if 100 pounds give 110 pounds, what 56⅔ s? Multiply and diuide, and you shall finde 3 li--2 s--4 d. of which the ½ that he demandeth in ready money, is 1 li. ——— 11 s——— 2 d. the same 31 s——— ——— 2 d. abated from 40 s. and also from 56 s——— 8 d. there will remaine 8 shillings 10 d. and 25 s——— 6 d. for the two first numbers in the Rule of three, and 3 pence the price of the pound of flax for the third number. Then multiply and diuide, and you shall finde $8\frac{35}{53}$ d. And for so much shall hee sell the pound of flaxe in barter.

The sixth Rule.

Two are willing to exchange merchandise: the one hath Norwich Grograns at 25 s. the peece ready money: and in barter hee will haue 30 s. and hee will haue the ⅓ part of his ouerprice in ready money. The other hath Norwich Stockings at 40 s. the dozen to sell for ready money. But in as much as the first Merchants Grogranes are no better, hee would deliuer them so to ballance the barter, that hee may gaine after 10 pounds in the 100 pounds. The question is now, how he shall sell

sell his Hose the dozen in barter according to his request.

Answer. Say, if 100 giue 100; what shall 40 s. giue, which is the iust price of the dozen of stockings? Multiply and diuide, and you shall finde 44 s. Then take the $\frac{1}{4}$ of 30 s. which is 7 s. 6 d. and subtract it from 25 s. and also from 30 s. and there will remaine 17 s——6 d. and 22 s——6 d. for the two first numbers in the Rule of three, and 44 shillings, which is the iust price (with his gaine in the dozen of stockings) for the third number. Then multiply and diuide, and you shall finde 56 s. $6\frac{6}{7}$ d. and for so much hee is to sell his dosen of stockings in barter.

The seuenth Rule.

Two Merchants will change their merchandise one with the other: The one hath 720 Ells of Cambricke at 5 s. the Ell to sell for ready money, but in barter hee requireth 6 s, 8 d. And yet notwithstanding hee loseth by it after 10 pounds vpon the 100 pounds, whereupon hee requireth one halfe of his ouer-price in ready money: and the other Merchant hauing skill enough to make the barter equall, deliuered English Saffrons at 30 s. the pound. The question is now, what his Saffrons cost the pound in ready money?

Answer. You must first seeke what is lost vpon

vpon the 100 pound, which to doe, you may ſay, (if you pleaſe) if 100 pound loſe 10, what ſhall $6\frac{2}{3}$ loſe? Worke, and you ſhall finde $\frac{2}{3}$ ſ. (or 8 d.) which muſt be rebated from 6 ſ--8 d. ſo reſteth 6 s. ſtill. Or you may ſay, if 100 pound giue mee but 90 pounds, what ſhall 6 s. 8 d. giue? Worke this way either, and you ſhall finde alſo as before directly in your quotient 6 s. your deſire. Then are you next to caſt vp what the 720 Ells of Cambricke commeth to at 6 s. 8 d. the Ell, and you ſhall finde 240 pounds: the $\frac{1}{2}$ whereof the Cambricke-Merchant will haue in ready money, (which is 120 pounds): Nextly you muſt caſt what the Cambricke commeth to after his loſſe in the 100 pound, which, as you found, is but 6 s. an Ell, and you ſhall finde 216 pounds: Now muſt you ſubtract his ready money (which is 120 pounds in all,) out of 240 pound, and alſo out of 216 pound, and there will remaine 120 pounds, and 96 pounds for your firſt two numbers in the Rule of three, and 30 ſhillings is the ouerprice of your Saffron for the third number: Then multiply and diuide, and you ſhall find 24 ſhillings. And ſo much did his Saffrons coſt in ready money.

Two Merchants barter. The one hath 50 *cloths to put away for ready money at* 11 *pounds the cloth, and in barter putteth them away for* 12 *pounds, taking Holland Cloth at* 20 *d. the Flemmiſh Elle, which was worth no more but* 18 d. *The queſtion is now, what Holland payeth for the Cloth, and*

and what hee winneth or loseth by the bargaine?

Answer. 50 Clothes at 11 pounds the cloth commeth to 550 pounds, and put away at 12 pounds the peece, maketh 600 pound. Then to finde what Holland payeth for the Cloth, say by the rule of Three direct, If 20 *d.* buy one Ell, what 600 pounds? Worke and you shall find 7200 Ells. Now to finde the estate of his gaine or losse, you must seeke what his 7200 Elles commeth to at 18d. the Elle: Worke by the Rule of Proportion direct, and you shall finde 540 pounds, which is not so much as his cloths were worth in ready money by 10 pounds: and so much lost the first Merchant by his Exchange.

A Venetian hath in London 100 *peeces of silke, to put away for ready money at* 3 *pounds the peece. But in barter hee deliuered them for* 4 *pounds the peece, taking Wools of a Felmonger at* 7 li. -- 10 s. *the C. weight, which was worth no more but* 6 *pounds the C. ready money. The question is now, what Wools payeth for the silkes, and which of them winneth or loseth by the barter?*

Answer, 100 peeces of silke at 3 pound, is in all 300 pounds, and 4 pounds, is 400 pounds. Then to finde what Wools payeth for the silke, say by the Rule of Three direct: If $7\frac{1}{2}$ buy me 100 weight, what 400 pound? Worke, and finde

finde 53 $\frac{1}{3}$C. weight of wooll. Now to finde the estate of their gaine, and losse, cast vp his wooll at 6 li. the C. (for so much they were worth ready money) and you shall finde 320 pound, which is 20 pound more then the silkes were to bee sold for ready money, whereupon the Venetian gained 20 pounds by the Barter.

A Merchant hath 53 $\frac{1}{3}$ weight of wooll at 6 pounds the C, to sell for ready money, but in barter hee will haue 7 pounds 10 s.: and another doth barter with him for Silkes, which are worth 3 pounds a peece ready money. The question is now, how hee ought to deliuer his Silkes the peece in barter, and how many payeth for the woolls.

Answer. Say by the Rule of proportion, (or by the Rule of three direct) if 6 pounds for C, weight ready money yeeld mee 7 li. 10 s. what will 3 pound yeeld, which is the iust price of a peece of Silke in barter, to make the Trucke equall? Worke, and finde 3 li. 15 s. the price of a peece of Silke in barter: Then say, If 3 li. 15 s. require one peece of Silke, how many peeces of Silkes are bought with 400 pound, which is the value of the 53 $\frac{1}{3}$C. weight of wooll at 7 li. 10 s? Worke by the Rule of three direct, and you shall finde 160 peeces of silke and $\frac{2}{3}$ of a peece, and so many of Silke pay for the wooll, and neither party hath aduantage of other.

Two men will change merchandise the one with the other. The one of them hath Beere at 6. s. 8.d.

the barrell to sell for ready money, but in barter hee will sell the barrell for 8 s. *and yet hee will gaine moreouer after* 10 *pound, vpon the* 100 *pounds. And the other hath white Spanish wooll at* 20 s. *the Roue, to sell for ready money. The question is now, how hee shall deliuer the Roue of Wooll in barter to saue himselfe harmelesse.*

Answer. Say, it $6\frac{2}{3}$ s. which is the iust price of the barrell of beere, bee sold in barter for 8 shillings: for how much shall 20 shillings (which is the iust price of the Roue of Wooll) bee sold in barter? Worke by the rule of three direct, and you shall finde 24 s. Then for because the first Merchant will gaine after 10 pounds vpon the 100 pounds, hee maketh his 100 pounds, 110 pounds. And therefore say by the Rule of three, If the second Merchant of 110 pounds doe make but 100 pounds, how much shall hee make of 24 s? Multiply and diuide, and you shall finde 21 s. 9 d $\frac{9}{11}$ of a penny. And for so much shall hee sell the Roue of wooll to bee deliuered in barter, to the end the first Merchant may giue 10 in the 100.

Two Merchants will change their Commodities the one with the other. The one of them hath white Paper at 4 s. *the reame, to sell for ready money. And in barter he will doe it away for* 5 s. *and yet he will gaine moreouer after the rate of* 10 *pounds vpon the* 100 *pounds. And the other hath Mace at* 14 s. 6 d. *the pound-weight to sell in barter. Now I demand what the pound did cost in ready money.*

Ans.

Answ. Say, if 5 s. (which is the over price of the Paper in barter) be come of 4 s. the just price, of how much shall come $14\frac{1}{2}$ shillings, which is the surprise of the pound of Mace in barter? Multiply and divide, and you shall finde $11\frac{3}{5}$ s. Then for because the first Merchant of Paper will gaine after 10 upon the 100, Say if 100 doe give 110, what shall $11\frac{3}{5}$ shillings give? Worke, and you shall finde 12 s---$9\frac{11}{25}$ d. and so much did the pound of Mace cost in ready money.

The fourteenth Chapter entreateth of exchanging of money from one place to the other.

EXchange is no other thing, then to take or receive money in one City, to render or pay the value thereof in another City, or else to give money in one place, and receive the value thereof in another, at terme of certaine daies, moneths, or faires, according to the diversity of the place.

But this practice chiefly consisteth in the knowledge of the money or Coines in divers places, of which for thy benefit (after a few examples given to the introduction of this worke) I will set downe certaine notes of the diversity of the common and usuall coines in most places in Christendome for traffique.

And first I will begin at *Antwerpe*, where they use to make their accounts by *Deniers de*

 grosse,

grosse, that is to say, pence Flemmish, whereof 12 doe make 1 s. Flemmish, and 20 s. doe make one pound *de grosse*.

Item, a Merchant delivered at Antwerpe 400 *pounds Flemmish to receive in* London 20 s. *sterling, for every* 23 s.——4 d. *Flemmish: The question is now, how much sterling money is to be received at* London *for the* 4000 *pounds of Flemmish?*

Answer. Say by the rule of three, If 23⅓ Flemmish give 20 s. sterling, what 400 pounds Flemmish? Worke, and you shall finde 342 li——17 s——1⅗ pence, and so much sterling shall I receive in *London* for the said 400 pounds Flemmish.

Otherwise also wrought by Rules of practice in taking the ⅐ of the Flemmish money delivered, and abating the same from the principall, the rest is English money, as before.

400 li	0 s	0 d.
57	2	10$\frac{2}{7}$
342	17	1$\frac{5}{7}$ sterling

A Merchant at London *delivered* 200 li. *sterling for* Antwerpe, *at* 23 s. —— 5 d. *Flemmish the pounds sterling: the question is, how much he must receive at* Antwerpe.

Answere. Say by the Rule of three, if 1 pound sterling give 25 s. 5 d. Flemmish, what

200

200 li. sterling? Worke, and thou shalt finde 234 li.--3 s--4 d. So many pounds Flemmish shall he receive at *Antwerpe* for the said 200 pounds sterling.

Otherwise by practise.

1——23--5--200			
3 s-4 d.	33	6	8
1 d.		16	8
maketh sterling	234 l.	3 s.	4 d.

In London 20 *pound sterling is delivered by Exchange, for* Antwerpe, *at* 23 s. 9. d. *Flemmish the pound sterling: the question is, at what rate the Flemmish money ought to be returned to gaine* 4 *pounds upon the* 100 *pound sterling at* London.

Answer. First say by the Rule of three direct: If 1 pound sterling give $23\frac{3}{4}$ Flemmish, what 200 pounds sterling? Multiply and divide, and you shall finde 237 pounds 10 shillings. The which to returne to gaine 8 pounds sterling in *London*, say by the backer Rule. If 200 pounds sterling require the exchange 23 s. 9 d. Flemmish, what the exchange to make 208 li. sterling? Worke by the Rule, and finde 22 s.-10 $\frac{1}{13}$ d. Flemmish, the effect in the question required.

If I take up money at Antwerpe *after* 19 s. 4 d. *Flemmish, to pay for the same at* London 20. *shillings,*

shillings sterling, and when the day of payment is come, I am forced to returne the same money againe in London, *to pay my Bill of Exchange: so that for* 20 *shillings which I take up here at* London, *I must pay* 19 s. 6 d. *at* Antwerpe, *I demand whether I doe win or loose, and how much in or upon the* 100 *pounds of money?*

Answer. Say by the Rule of three: If $19\frac{1}{2}$ give $19\frac{1}{3}$, what will 100 pounds give? Multiply and divide, and you shall finde *99* li. $2\frac{106}{117}$ s. which being abated from 10 pounds, there will remaine 17 shillings $\frac{11}{117}$, and so much I doe lose upon the 100 pounds of money.

If I take up at London 20 *shillings sterling, to pay at* Antwerpe 22 s. 4 d. *and when the day of payment is come, my Factor is constrained to take up money againe at* Antwerpe, *wherewith to pay the aforesaid summe, and there he doth receive* 23 s. 4 d. *Flemmish, for the which I must pay* 20 s. *at* London: *The question is now, whether I doe winne or lose, and how much upon the* 100 li. *of money after that rate.*

Answer. Say by the Rule of Proportion, If $22\frac{1}{3}$ s. give $23\frac{1}{3}$ s. what will 100 pounds give? Multiply and divide, and you shall finde 104 pounds *9* shillings $\frac{37}{87}$, from the which abate 100 pounds, and there will remaine 4 pounds 9 shillings $\frac{37}{47}$, and so much is there gained upon

on the 100 pounds of money.

In Antwerpe *is deliuered* 200 *pounds Flemmish by exchange for* London *at* 20 *shillings sterling for euery* 23 *shillings* 4 d. *Flemmish. The question is, at what rate the same is to bee returned to gaine* 10 *pounds vpon the* 100 *pounds Flemmish in* Antwerpe.

Answer. First say by the Rule of three, if 23 $\frac{1}{3}$ Flemmish giue 20 s, what shall 200 pounds gaine? Worke, and you shall finde 171 pounds 8 s. 6 $\frac{6}{7}$ d. Then say againe by the Rule of three direct, if 171 li--8 s-- 6 $\frac{6}{7}$ s. sterling giue mee 210 pounds Flemmish, what shall 20 s, sterling giue? Worke, and you shall finde 24 s. 6 d. Flemmish. And at the same rate ought the same to bee returned at *Antwerpe* to gaine 10 pounds vpon the 100 Flemmish.

A Merchant of Antwerp *deliuereth* 234 *pound* 3 s. 4. d. *Flemmish, to receiue at* London 200 *pounds sterling: The question is now, how the exchange goeth after this rate?*

Answer. Say by the Rule of three direct, if 200 giue 20, what 234 $\frac{1}{6}$? Multiply and diuide, and you shall finde 23 s--5 d. And for so much goeth the exchange.

Item, *the exchange from* London *into* France, *is not like as it is in* Flanders, *but is deliuered by the French Crowne, which is worth* 50 *soulx Turnois the peece.*

Whereupon also you must note, that in

France they make their accounts by Frankes Soulx, and Deniers Turnois, whereof 12 Deniers make one Soulx Turnois, and 20 Soulx maketh 1 pound Turnois, which they call a Liure or Franke. But the Merchants, to make their accounts, doe vse French Crownes, which is currant among them for 51 Soulx Turnois. But by exchange it is otherwise, for it is deliuered but for 50 Soulx Turnois the Crowne, or as the taker vp of the money can agree with the deliuerer. And note that this △ Character representeth the Crowne by exchange, and is euer 50 Soulx Turnois or French money.

A Merchant deliuereth at London *240 pounds sterling, after 5 shillings 6 pence the crowne, to receiue at* Paris *50 Soulx Turnois for euery Crowne. I demand how much Turnois or French money payeth the bills for the said 240 pounds sterling.*

Answer. Say by the Rule of three, If $5\frac{1}{2}$s. sterling giue me 50s: Turnois, what shall 240 pounds sterling giue? Reduce the pounds into shillings, then multiply and diuide: and you shall finde 2181 Liures, 16 Soulx, 4 Deniers, and $\frac{4}{11}$ Turnois; and so much payeth the Bills at *Paris* for the 240 punds sterling.

A Merchant deliuereth at Roan *or elsewhere in* France 1430 *pounds or frankes, the which francke*

francke or pound is 20 *soulx, or a pound Turnois, to receiue in* London 6 s. 4 d. *sterling for euery* △ *of* 50 *soulx Turnois. The question is, how much sterling money I ought to receiue at* London *for my* 1430 *pound Turnois.*

Answer. Say, if 2 $\frac{1}{2}$ pounds giue mee 6 $\frac{1}{3}$ s what will 1430 giue mee? Worke, and you shall finde 3622 $\frac{2}{3}$ shillings sterling, which maketh 181 pounds 2 s. 8 d. and so much money is to bee receiued at *London*, for the said 1430 Liure Turnois, after 6. s. 4 d. for euery △ of 50 soulx.

In London *is deliuered* 200 *pound sterling by exchange for* Paris, *at* 5 s. 9 d. *the* △ *of* 50 *soulx Turnois. The question is, at what price the said* △ *is to bee returned to gaine* 6 *pounds vpon the* 100 *pound sterling at* London.

Answer. First say (by the Rule of three direct) if 5 $\frac{3}{4}$ s. sterling giue 50 soulx Turnois; what shall 200 pound sterling giue? Worke, and you shall finde 1739 franckes or liures, 2 $\frac{14}{23}$, soulx. Then the which to returne and gaine 6 pounds vpon the 100 pounds in *London*, say by the Rule of three direct, if 1739 francks 2 $\frac{14}{23}$ soulx yeeld 212 pounds, what the △ of 50 soulx? worke and finde 6 s. 1 $\frac{7}{30}$ d. the effect required in the question.

A Merchant deliuered in London 160 *pounds sterling, to receiue in* Biskay *for euery* 5 s. 6 d. *one Ducket of* 374 *Maruides. The question is, how many Maruides ought I to receiue at* Biskay?

Answer.

Anſwer. Say, if 5 $\frac{2}{3}$ s. ſterling giue 374 Maruides: what ſhall 160 pounds ſterling giue? Multiply and diuide, and you ſhall find 217600 Maruides, and ſo many I ought to receiue at *Biskay* for my 160 pounds ſterling.

A Merchant deliuered in Baion 4000 *Maruides, to receiue in* London 5 s. 8 d. *ſterling for euery Ducket of* 374 *Maruides. The queſtion is now, how much ſterling money payeth the bills of Exchange for the ſaid* 4000 *Maruides?*

Anſwer. Say, if 374 Maruides make one Ducket, what 4000 Maruides? Multiply and diuide, and finde 106 Duckets $\frac{178}{187}$.

Then ſay againe, if 1 Ducket giue 5 $\frac{2}{3}$ s. what giueth 106 $\frac{178}{177}$ Duckets? Worke, and finde 30 li--6 s. $\frac{34}{561}$.

¶ Otherwiſe it is wrought more briefe at one working, as in the laſt queſtion before, in conſidering that 5 s---8 d. containeth one Ducket, or 374 Maruides. Therfore ſay by the Rule of three, if 374 Maruides giue 5 $\frac{2}{3}$ s, what 40000 Marnides? Worke, and you ſhall alſo finde in your quotient 30 li--6 $\frac{34}{561}$ s. And ſo many pounds ſterling is to bee receiued for the 40000 Marueids.

In London 200 *pounds deliuered by Exchange for* Vigo, 374. *Marueids the Ducket of* 5 s. 10 d. *ſterling,*

ſterling, maketh 256457 $\frac{1}{7}$ *Marveides : the which to returne and gaine* 10 li. *upon the* 100 *pounds in* London, *ſay by the Rule of three direct, if* 220 li. *require* 256457 $\frac{1}{7}$ *Marveides, what* 5 s. 10 d? *Worke, and finde* 340 *Marveides, the price of every Ducket in returne, which is the effect in the queſtion required.*

These may ſeeme ſufficient for inſtructions.

N*Otwithſtanding for thy further aid and benefit, hereafter follow ſix ſpeciall and moſt briefe Rules of practiſe, for Engliſh, French, and Flemmiſh money.*

1	teacheth	How to turn Flem. to Eng. ſterling.
2		How to turn Eng ſterling to Flem.
3		How to turn Flemmiſh to French.
4		How to turn French into Flemmiſh
5		How to turn ſterling into French.
6		How to turn French into ſterling.

The

The fifteenth Chapter intreateth of the said sixe Rules of breuity, and of valuation of English, Flemmish, and French money, and how each of them may easily be brought to others value.

Rule 1. *How briefely to reduce pounds, shillings, and pence Flemmish into pounds, shillings, and pence English sterling.*

IT is to bee noted, that 7 pounds Flemmish maketh but 6 pounds sterling: 7 s, Flemmish maketh 6 s. sterling, and 7 d, Flemmish 6 d. sterling: so that 7 yeeldeth but 6. Wherein is euident that then is lost $\frac{1}{7}$, (if it may bee so called) when it is reduced into English money: wherefore to know how much 233 l.---13 s--4 d. Flemmish maketh English, you must subtract from it $\frac{1}{7}$, beginning with the pounds, &c. and that which resteth after this subtraction, is the summe required: so that 233 li---13 s 4 d. Flemmish, maketh 200 li----5 s------8$\frac{4}{7}$ d. sterling.

Example:

	li	s	d
	233	13	4
$\frac{1}{7}$	33	7	7$\frac{3}{7}$
	200	5	8$\frac{4}{7}$ *ster.*

Another example.

	li	s	d
	311	0	0
$\frac{1}{7}$	44	8	6$\frac{6}{7}$
	266	11	5$\frac{1}{7}$

To

To reduce pounds, shillings, and pence sterling, into pounds, shillings, and pence Flemmish. Rule 2.

Rule 2

Note that a pound sterling maketh 1 li-3 s. 4d. Flemmish: that is, $1\frac{1}{6}$ ll. 1 s. sterling maketh $1\frac{1}{6}$ s. Flemmish, and 1 d. sterling maketh $1\frac{1}{6}$ d. Flemmish. So that there is gained (if it may be so called) $\frac{1}{6}$ of the summe being thus reduced to Flemmish, for of $\frac{6}{6}$ is made $\frac{7}{6}$ which is one whole and $\frac{1}{6}$. Then to know how much 237 li--7 s-6 d. sterling maketh Flemmish, subtract from your sterling, the $\frac{1}{6}$ of the whole summe, and adde it to the same summe, and it maketh 276 li——18 s——4 d. which is the summe required.

Another Example

Example.

	li.	s	d	
	237	7	6	*ster.*
$\frac{1}{6}$	39	11	3	
	276	18	9	*Flem.*

Another Example.

	li	s	d
	337		
$\frac{1}{6}$	56	3	4
	393	3	4

Example

Example

To reduce pounds, shillings, and pence Flemmish, into pounds, shillings, and pence French. Rule 3.

Ye shall note, that the equality of Flemmish and French money is this, that is to say, the pound Flemmish, maketh 7 pound $\frac{1}{2}$ French, or Turnois. 1 s. Flemmish maketh $7\frac{1}{2}$ s. French, and a groat Flemmish, maketh $7\frac{1}{2}$ d French.

Wherefore to know how much 143 li--4 s. 9 d.

9 d. Flemmish maketh French, ye must multiply the whole number twice by 6, beginning at pence, and so forward, and the product of your second multiplication divide by 5, so the worke is finished. Or multiply the said summe by 7, and take out of it $\frac{1}{5}$, adding it to the product of your multiplication by 7, and that is your number required. So that as well by the one as by the other, 143 li —— 4 s —— 9 d, Flemmish, maketh 1031 li —— 6 s —— $2\frac{2}{3}$ d, French or Turnois.

Example.

li	s	d	
143	4	9	*Flem.*
		6	
859	8	6	
		6	
5156	11	0	*Fren.*
$\frac{1}{5}$ 1031	6	$2\frac{2}{5}$	*Fren.*

The same otherwise.

li	s	d
143	4	9
		7
1002	13	3
$\frac{1}{5}$ 28	12	$11\frac{2}{5}$
1031	6	$2\frac{2}{5}$

Another Example.

143 l. *Flem.*
6
858
6
$\frac{1}{5}$ 5148 *Fren.*
1029 li $\frac{3}{5}$ or 12 s. *Fren.*

Or thus:

143
7
1001
$\frac{1}{5}$ 28 —— 12
1029 li - 12

To

To reduce pounds, ſhillings, and pence French, into pounds, ſhillings, and pence Flemmiſh. Rule 4.

Multiply 233 li——8 ſ——4 d French by 5, and divide the Product twice by 6, that is, the ſaid number by 6, and the product or quotient againe by 6, and the quotient of this ſecond Diviſion is the thing required. So that 233 li--8 s--4 d French, maketh 32 li--8 s--$4\frac{1}{3}$ d Flemmiſh.

Example.				Another.			
	li	s	d	li	s	d	
	233	8	4 *Fren.*	753			*Fren.*
			5	5			
	1167	1	8	3765			
$\frac{1}{6}$	164	10	$3\frac{2}{6}$	$\frac{1}{6}$ 627	10		
$\frac{1}{6}$	32	8	$4\frac{5}{18}$ *Flem.*	$\frac{1}{6}$ 104	11	8	*Flem.*

To reduce pounds, ſhillings, and pence ſterling, into pounds, ſhillings, and pence, French, or Turnois. Rule 5.

The pound ſterling maketh 8 l· 8 s. French, that is to ſay, $8\frac{2}{5}$ pounds: the ſhillings, maketh $8\frac{2}{5}$ ſhillings, and the penny $8\frac{2}{5}$ d French. Wherefore to know what 231 l——1 s——4 d. ſterling maketh French, yee muſt multiply your whole ſumme by 42, that is, by 7, and the product of it by 6, and diuide this ſecond product by

by 5, and that is the summe required.

Otherwise, multiply the summe sterling by 8, and adde twice to the Product ⅛, and it shall produce the summe required. So that both waies 231 li——13 s.——4 d. sterling, maketh 1946 li French, as hereunder followeth.

Example. The same otherwise.

		li	s	d		li	s	d
Sterl.		231	13	4	ster.	231	13	4
				6				8
		1390	0	0		1853	6	8
				7		46	6	8
						46	6	8
	⅕	9730	0	0				
						1946	0	0
French.	⅕	1946	0	0 Fren.				

Another example The same.

Sterl.		753		Ster		753
		6				8
		4518				6024
		7			⅛	150——12
		31626			⅛	150——12
French.	⅕	6325——4 Fren.				6325——4

To reduce pounds, shillings, and pence, French, into pounds, shillings, and pence, sterling.

To

To know how much 1256 li. 12 s. 6 d. *French* maketh in sterling mony: multiply the sum by 5, and divide the product by 7 and 6 at twice, and the last quotient shall be the thing required, that is to say, 1256 li. 12 s. 6 d. maketh 149 pounds, 11 s. 11 $\frac{4}{7}$ d. sterling. *Rule 6.*

Example.	Another Example.
li s d *French.*	li s d
1256--.--12--6	2531 0 0 *Fren.*
5	5
6283---2---6	12655
$\frac{1}{6}$ 1047---3--9	$\frac{1}{6}$ 2109--3.-4-
$\frac{1}{7}$ 149-11-11 $\frac{4}{7}$ *Ster.*	$\frac{1}{7}$ 301--6--2 $\frac{6}{7}$ *Ster.*

Note, that when any money is given by exchange at *London* for *Roan* at 71 $\frac{1}{2}$ d. or rather 71 $\frac{1}{7}$ for the Crowne of 50 s. French, there is neither gaine nor losse: for it is one mony for another, accounting 8 li. 8 s. French, for one pound sterling. So the giver loseth the time of paiment, which is abont 15 daies, and hee that taketh it, hath the gaine of the same.

They of *Roan* that put forth or take money by exchange for *London*, ought to have like consideration.

Item, when any man giveth at *London* 64 pence $\frac{1}{3}$, or rather 64 $\frac{2}{7}$ d. to have at one of the Fayres of *Lyons* a Crowne *de Marc*, hee

N n that

that ſo giveth the money, loſeth the time, and he that taketh it, gaineth the ſame: for 62. pence $\frac{6}{7}$ is equall in value to 45 ſ. French. Hee that putteth or taketh money at *Lyons* for *London*, ought to conſider the ſame.

Item, when any deliver in *Antwerpe* 75 pence, to receive at *Lyons* a Crowne *de Marc*, hee that putteth it forth, looſeth the time, and hee that taketh it gaineth the ſame. For 75 groats Flemmiſh, is equall in value to 45 s. French.

Thus for this time I make an end of the practiſe of Exchange, and the inſtructions thereunto belonging, and according to my promiſe: yet further to gratifie ſuch as are deſirous to know the common Coynes uſed for traficke among Merchants in theſe Cities following, a briefe declaration of their Moneyes, and the reckonings, and account of them.

The

The sixteenth Chapter containeth a declaration of the valuation and diversitie of Coynes of most places of Christendome for trafficke; And the manner of exchange in those places from one Citie or Towne to another: which knowne, is right necessary for Merchants, by meanes whereof they do finde the gaine or losse upon the exchange.

ITem, forasmuch as the greatest diversitie of money of Exchange is at *Lyons*; therefore I will beginne duely of the money of that place.

At *Lyons* they use Franckes, Soulx, and deniers Tournois. A Francke maketh 20 Soulx, and one Soulx 12 Deniers: but the Merchants to keepe their bookes of Accounts, doe use French Crownes of the Marke at 45 Soulx the peece, and doe divide it into 20 Soulx. 1 Soulx is 12 Deniers.

Item, a Marke of Gold maketh 65 △ of the Marke, which serveth for exchange, and divide it into 8 ounces. The ounce into 24 pence or deniers, the denier into 24 graines, and so the summe or whole by imagination or gesse. △ This marke standeth for a Crowne.

Also at *Lyons* there are foure Faires in a yere, at the which they doe commonly exchange, which are from three moneths to three months.

At *Geanes* they use the Soulx : one Ducket maketh 3 pound.

At *Naples* they use Duckets, Taries, and Graines, the Ducket maketh five Taries, and one Tarie twenty Graines: but they take 6 Duckets (which maketh thirtie Taries) for the ounce.

A Ducket maketh ten Carlins, and a Carlin ten Graines, so that 2 Carlins make a Tarie, and 100 Graines make a Ducket.

At *Rome* they use the Duckets of the Chamber: one Ducket is worth 12 Guilis, and one Guilis 10 Soulx.

At *Venice* they use Duckets currant at 124 soulx a piece, or 24 Deniers, and one Denier maketh 32 Picolis.

At *Palerme* and *Messine* they write, after ounce, tarie, and graines, and one ounce is worth, 6 Duckets of 30 taries, and 1 tarie is 20 graines, and 1 graine 6 Picolis, 1 Ducket is also worth 24 Carlius.

At *Millan* they use li. s. d. of Ducket Imperials, and △ of exchange is worth 4 li.

At *Lucques*, *Florence*, and *Ancone*, they use the △ of Gold: in Gold the French Crowne is worth 7 li. but at *Boloigne* 3 li. 10 s.

At *Barcelone* they use the Soulx: the ducket of exchange is worth 22 Soulx.

At *Valence* and *Saragosse* they use the Liver, Soulx, and Denier: the French Crowne of exchange is worth 20 Soulx, and one Soulx is 12 Deniers.

At

At the Faires of *Castil* they use the Marveides, the Ducket is worth 375 Marveides.

At *Lisbone* they use the Raies, one Ducket of exchange is worth 400 Rayes.

At *Noremburge*, *Franckeford*, and *August* in *Germany*, they use the *Krentzars*, whereof 60 make a Floren.

At *Antwerpe* they use li.--s. and d. de Gros, and they exchange into the denier de Gros, to wit, our English peny.

At *London* they use the li. the s. and d. sterling, and they exchange in pence sterling.

The exchange of Lyons *at sundry places.*

Item, at L*yons* there is exchange in three sorts, at the Cities and Townes following.

First, they deliver at L*yons* one Mark to have or receive at *Naples* almost 41½ Duckets, at *Venice* 70 Duckets currant: at *Rome* 63 Duckets of the Chamber: at L*ucques* and *Florence* 65 △ of gold, at *Millan* 82 △.

And contrariwise, at the said Cities aforesaid, they doe give so much of money, to have a Marke at L*yons*.

Secondly, they give at L*isbone* one △ of Marke of 45 soulx Turnois a piece, to have at *Geanes* almost 68 soulx, at *Palerme* and *Messine* almost 24 Carlins, at *Barselone* 22 soulx, at *Valence* or *Saragosse* 20 soulx, at the Faire at *Castill* 350 Marueides, at L*isbone* 360 *Rayes*, in *Antwerpe* 57 Deniers *de Gros*, and at L*ondon* 70 d. sterling.

And contrariwise, they give in the said Cities almost as much of their money to have a French Crowne of the Marke at *Lyons.*

Thirdly, they do give at *Lyons* a △ of the Sun, to have almost 93 *Krentzars* at *Franckeford, Augsburge, Noremberge,* or other Cities in *Almaine.*

Also at *Lyons* onely they do pay they change the $\frac{2}{3}$ in Gold, and $\frac{1}{3}$ in Money, or else all in Money, in giving 1 $\frac{1}{2}$ for the hundreth.

Changes at Naples and other Townes.

Item, at *Naples* they give or deliver almost 112 Duckets, to receive at *Rome* 100 Duckets of the Chamber at the old value.

Through *Lucques* and *Florence* they deliver 100 Duckets Carlins, to receive there almost 86 △ of Gold.

Through *Palerm* and *Messine*, one Ducket of 5 Tary, to receive there almost 164 graines.

Through *Millan*, one ducket, to receive there almost 90 Soulx.

Through *Geans* one Ducket to receive there almost 65 soulx. The whole summe to be paied within ten daies after the sight of the Bill of Exchange.

Also at *Naples* they deliver one Ducket to receive in *Antwerpe* almost 67 d. or Deniers de *Gros*, within two moneths. At *London* almost 60 d. sterling in three moneths. At *Barselone* almost 120 soulx within two moneths.

At

At *Valence* almoſt 18. Soulx within two Moneths. At *Lisbone* 333. Rayes within three moneths: and at the Faire at *Caſtill*, almoſt 340. Marucides at the ſame Faire.

Change of Venice to other places.

At *Venice* they deliver 100 Duckets currant to receiue in *Almaine* almoſt 140. *Florens* at 60. *Krentzers* the peece.

At *Lucques* and *Florence* almoſt 108. △ of Gold in 10. dayes.

Likewiſe at *Venice* they deliver a Ducket currant to receiue at *Palerme* and *Meſſine* almoſt 21. Carlins, at *Millan* almoſt 93. Soulx; at *Geanes* almoſt 62. Soulx, the whole at ten dayes end.

Of the Pair of Pari.

As touching the Exchange, it is neceſſary to underſtand or know the *Pair*, which the Italians call *Pari*, which is no other thing then to make the money of the change of one City or Towne, to or with the money of another, by meanes whereof they doe finde the gaines or loſſe vpon the Exchange.

Example.

Item, having receiued letters of credit of one of *Antwerpe*, that the △ of the Sunne is there worth 7 ſoulx: The queſtion is, what the ſame is worth at *London*, when the *Pai* of Exchange goeth for 23. ſhillings?

Answer. *Say, if* 23 *give but* 20, *what giveth* 7? *Work, and finde* 8 1. 1, $\frac{1}{23}$ d. *and so much is the* △ *of the Sunne worth at* London.

The seventeenth Chapter containeth *also a Declaration of the diversitie of the weights and measures of most places of Christendome for traffique. At the end of which discourse are two Tables, the one for weight, and the other for measure, proportionate and reduced to an equality of our English measure and weight, by the aid whereof, the ingenious may easily by the Rule of three, convert the one into the other at pleasure, &c.*

AT *London*, and so all *England* thorow, are used two kindes of weights and measures, as the *Troy* weight, and the *Haberdepoise.* From the *Troy* weight is derived the proportion and quantity of all kinde of drie and liquid measures, as pecks, bushels, quarters, &c. wherewith is bought and sold all kinde of graine and other commodities met by the Bush. And in liquid, ale, beere, wine, oyle, butter, honey, &c. upon these grounds and statutes is bread made, and sold by the *Troy* weight: and so is gold, silver, pearle, precious stones, and Iewels. The least quantity of this *Troy* weight is a graine: 24 of these graines make a penyweight, twenty penny weights an ounce, aud

12. ounces a pound, 2. pounds or pints of this weight maketh a quart. And so ascending into bigger quantities, is produced the measures whereby are sold our other naturall sustenance : *viz.* Ale or Beere, with all other necessary commodities, as Butter, Honey, Herrings, Eeles, Sope, &c. All which last before rehearsed, though their measures (wherein they are contained) bee framed and derived from the *Troy weight*, yet are they in traffique with divers commodities, as Lead, Tinne, Flax, Wax, with all other commodities both of this Realme, and of other forraigne Countreyes whatsoever, bought and sold by the *Haberdepoise weight* after sixteene ounces to the pound, and 112. pound to the C. weight. And to every C. is allowed but 12. pound weight at the common beame. From hence is also derived the weight of *Suffolke* Cheese, which containeth 32. cloues, 8. pound to a cloue, and weigheth in all 256. pounds. And also the Barrell of *Suffolke* Butter is, or should bee of like weight with the weight of Cheese, *viz.* 256. pounds: More 14. of these pounds make a Stone, and 26. Stone, containeth a Sacke of English wooll, Forraigne wools, to wit, French, Spanish, and Estrich, is also sold by the pound or C. weight, but most commonly by the Roue, 25. pound to a Roue : other commodities of Tale, are bought and sold by the C. fiue score to the C. Except headed ware, to wit,

See further of these Weights and Measures in Reduction, beginning pag. 133.

wit, Cattell, Nayles, and Fish, which are sold after sixscore to the C. There are also two other sorts of measures, to wit, the Ell and the Yard. By the Ell is usually met, Linnencloth, as Canuas, &c. And by the yard, Silkes, woollen clothes, &c.

Antwerpe.

At *Antwerpe* are also 2. sorts of weights, their gold and siluer weight, and their common weight. Gold and Silver is weighed by the *Marke*, the *Marke* is 8. ounces, the ounce 20. Esterlings, and the Esterling 32. as our graines. The Goldsmiths diuide that into smaller, but not the Merchants: The proofe of gold is made by Karects, whereof 24 maketh a marke of fine gold, the Karect is 24. graines: the proofe of the money is made by *Deniers*: 12. Deneirs is 1. f. fine, that is a *Marke* of fine silver: the Denier also is diuided into 24. grains, and the graine into 4. quarters.

Item, 100. *Marks in* Antwerpe, Troy *weight, maketh at* Lyons 103. *Marks*, 2 ½ *ounces, and* 20. *graines*, 23. *p. At* Noremberg 103 *Marks* 2 ½ *ounces*, 2 *Quints*, 3 *Deniers*; *at* Frankford, 105 *Marks*; *at* Augsburg 104 *Marks*, 3. *ounces*, 1 *quint*. *At* Venice 103 *Marks*, 1 *ounce*, 7 *deniers*, 18 *grains*. *At* London 66 *pounds*.

The Marke of gold or siluer at Antwerpe, Troy *weight which is* 8 *ounces, maketh* 7 ½ *ounces common weight, with which all other merchan-*

dise

dise is weighed. So that the Troy *weight is greater than the common weight by* 6 $\frac{1}{4}$ *in the* C. *By this weight of* Troy, *they also weigh* Muske, Amber, Pearle, &c.

All silkes are bought at Antwerpe, *by the* Burges Ell, *which is greater than the common measure, by which they retaile by two in the hundred. Their common Ell is* $\frac{3}{4}$ *of our yard, and* $\frac{3}{5}$ *of our Ell.*

Lyons.

At *Lyons* is used 3 sorts of weight, whereof the first is the common Towne weight, with which they weigh all kinde of Spicery, and divers other merchandise. The second is called *Geneva* weight, which is 8 in the C greater than the common weight, with which they weigh Silkes, &c. The third is French weight, called commonly the *Marke weight*, and 100 pounds thereof maketh 106 $\frac{1}{4}$ li. *Geneva*, and 114 $\frac{3}{4}$ of their common weight: with which French weight, is weighed all things that payed custome or toll.

At *Lyons* is also used two sorts of Ells or *Aulnes*. The one wherewith they measure grosse cloathes, as canuas. and such like. The other is called the French Ell or *Aulne*, with which they measure all other kinde of merchandise, whereof seven common towne Ells maketh 11 ordinary French Ells.

Roan.

At *Roan* 6 $\frac{1}{2}$ Muides of salt, beeing the measure

ſure of the place, make a C. at *Armuiden* in *Zeland*, and the C. of *Bronage* meaſure of *Armuiden*, maketh at *Roan* 11 Muides, 30 Muides make a laſt of corne, and 16 a laſt of Oates, 100 pound weight there, maketh at *London* 114 $\frac{1}{2}$, and 190 $\frac{1}{4}$ at *Antwerpe*. And 200 ells make at *London* 115 $\frac{1}{4}$.

Noremburge.

A 100 pound weight at *Noremburge* maketh at *London* 111 $\frac{3}{4}$, at *Antwerpe* 107 $\frac{1}{2}$, & 100 ells at *Noremburge* make at *London* 75 $\frac{2}{5}$ at *Antwerpe* 95 $\frac{3}{5}$, &c.

Lisbone.

The C weight at *Lisbone* maketh 4 Roues, every Roue 32 pounds, ſo that their C. weight is 128 pounds, and their pound containeth 14 ounces, and 100 pounds of their weight maketh at *London* 113 $\frac{1}{8}$.

Their Silke, cloth of gold, and woollen is meaſured with a meaſure which they call a cubite, containing about $\frac{3}{4}$ of a Varre of *Caſtile*. Howbeit, their common meaſure is called a Varre, which maketh five *Palmes*, and contayneth 1 $\frac{1}{4}$ of a Varre of *Caſtile*, our Ell of *London* is equall with the Varre of *Lisbone*.

All kinde of Merchandiſe brought from *Flanders*, *Roan*, or *Brittaine*, payeth at *Lisbone*, as a dutie or cuſtome to the King, 20 in the C. which they call the tenth in Merchandiſe and

and the other tenth in money.

Note alſo, that all kind of Merchandiſe comming to *Liſbone* by Land, paieth leſſe in cuſtom, then that that commeth by water.

Civill.

The Roue of *Civill* is 30 pound, 4 Roues make their C weight, which is 120 pounds. The 100 pounds of *Civill* maketh at *London* 102 pounds. Their other common meaſure is a Varre, whereof 100 maketh at *London* 74 Ells, and at *Rome* 40 Canes, &c.

Venice.

At *Venice* bee two ſorts of weight; the one called La *Groſſe*, the other La *Suttle*, with the groſſe is weighed all kinde of great wares; and with the ſmall all kinde of ſpicery, and ſuch like: 96 pounds of groſſe weight there, maketh at *London* 100 pound, and 100 pounds of ſpicery there without any tare or allowance, make at *London* 94, and with tare 65.

Their owne common meaſure are Braces, whereof 100 make at *London* $55\frac{4}{5}$ Ells, at *Antwerpe* $92\frac{1}{2}$, &c.

Florence Florence. Florence

At *Florence* the 100 li. weight maketh at *Aquila*, for Saffron 110, and 145 pounds of *Florence*, make at *Roan* but 100 pounds, the weight of *Florence* and that of *Luks* is all one.

Their

Their other measures are Braces, whereof 100 maketh at *Antwerpe Burges* measure, 81 $\frac{2}{3}$ Ells, 100 Braces there, make at *London* 49 Ells, &c.

Lucque.

¶ The *Lucque* Sattens are commonly sold at *Lyons* by weight, and 133 $\frac{1}{3}$ pounds maketh at *Lyons* 100 pound, so that 1 pound $\frac{1}{3}$ maketh at *Lyons* but one pound.

Their other measures are Braces, whereof 100 of them make at *London* 50 Ells, at *Antwerpe* 83 $\frac{1}{3}$ Ells, &c.

Aquila.

At *Aquila* their 100 pounds maketh at *London* 71 $\frac{3}{4}$, their 136 $\frac{2}{3}$ pounds of Saffron maketh at *Geneva* but 100, and 11 li. of *Geneua* maketh 15 li. at *Aquila.*

Valentia.

At *Valentia* be two sorts of weights, a great & a small. The C weight or great weight containeth foure Roves, the Roue 36 li so the C. great weight is 144 li. and the C. weight small containeth but 120 pounds, and is also parted into foure Roues, which is 30 pounds to a Roue. By the small is sold the scarlet graine, with all other kinde of spicery, and by the great is sold wooll, with all such like grosse wares. The 1 $\frac{1}{3}$ pounds of silke at *Valentia*, maketh at *Lyons* one pound *Geneua* weight. The charge of great Merchandize at *Valentia* containeth

432

432 pound, and in small wares 360 pounds.

The weight here and at *Barsellone* is all one.

There 100 pound weight maketh at *London* 78 pound, at *Antwerpe* 75.

Dansicke.

At *Dansicke* or *Spruce land* the rule is, that whosoever buyeth any Merchandize there, buyeth it by the ship-pound, which is 320. li. 20 Lispounds make a Ship-pound, and the Lispound containeth 16 pound, which Ship-pound of *Dansicke* maketh at *Antwerpe* 266 $\frac{2}{3}$ li. Their 100. li. weight, maketh at *London* 86. &c.

Their other common measures are Ells, whereof 100 maketh at *London* 72 $\frac{1}{4}$ and at *Antwerpe* 820. $\frac{1}{2}$ Ells.

Toulhouse.

At *Toulhouse* 6 Cabes of woad maketh a Charge, two Cesternes of Corne-measure, and all kind of graine maketh a Charge, the Cesterne weigheth 160 li weight of that place. Their 100 in weight, maketh at *London* but 91 $\frac{1}{4}$ pound.

Geanes.

At *Genua* or *Geanes*, 100 li of their weight maketh at *London* 71 $\frac{1}{4}$, and at *Antwerpe* 68 $\frac{1}{8}$ 100 li, weight at *Genua*, maketh at *Venice*, to wit, Suttle 106. li.

Their other common Measures are *Palmes*, whereof 100 make at *London* 20 $\frac{1}{3}$ Ells: and at *Antwerpe* 34 $\frac{3}{7}$.

The

The rest are suppliedin two Tables, which hereafter followeth : whereby the ingenious may gather his desire.]

The Table of the agreement of the weights of divers Countreyes, the one with the other, being reduced to an equality, as followeth.

112 pounds weight at *London*, make at		112 pounds weight at *London*, make at	
Antwerpe.	$107\frac{5}{7}$	*Venice grosse weight.*	$105\frac{3}{8}$
Franckeford.	099		
Collen and Ausburg.	$102\frac{1}{4}$	*Venice subtle weight.*	$166\frac{7}{7}$
Noremburg.	$100\frac{1}{2}$		
Roan.	098	*Aquila*	$157\frac{1}{7}$
Paris.	$102\frac{1}{4}$	*Vienna.*	$089\frac{3}{8}$
Lyons.	$118\frac{1}{2}$	*Preslaw*	$134\frac{5}{8}$
Diepe.	$100\frac{1}{4}$	*Leipsig.*	$101\frac{1}{4}$
Geneua.	$090\frac{1}{3}$	*Dansick*	$129\frac{1}{4}$
Towlouse	$122\frac{1}{4}$	*Lubeck.*	$097\frac{3}{8}$
Rochel.	$124\frac{1}{4}$	*Barcellon.*	$144\frac{1}{3}$
Marcellis	$124\frac{1}{4}$	*Lisbon.*	099
Ciuill,&c.	$109\frac{3}{4}$	*Geanes..*	$157\frac{1}{5}$

The

The other Table of agreement of Measures of divers Countries reduced unto an equality, by the aide whereof you may with the use of the Rule of three, convert either more or lesse of any one measure unto the other.

60 Ells, or 7 Yards at *London* make at

Antwerpe	100	*Ells.*
Norenburg	$104\frac{1}{2}$	*Ells.*
Franckeford	125	*Ells.*
Leipsig	125	*Ells.*
Preslaw	125	*Ells.*
Dansicke	183	*Ells.*
Vienne in Austria	87	*Aulnes.*
Lyons in France	$60\frac{3}{4}$	*Aulnes.*
Paris in France	57	*Aulnes.*
Roan in Normandy	52	*Aulnes.*
Lisbon	60	*Varres.*
Civil in Spaine	81	*Varres.*
Castile in Spaine	81	*Varres.*
Methera Iles	62	*Varres.*
Venice	108	*Braces.*
Lucques	120	*Braces.*
Florence	$122\frac{1}{2}$	*Braces.*
Millane	138	*Braces.*
Rome	90	*Canes.*
Geanes	$288\frac{6}{13}$	*Palms*

The

The eighteenth Chapter treateth of Sports and Pastimes, done by Number.

IF you would know the number that any man doth think or imagine in his minde, as though you could divine, bid them triple it, or put twice so much more to it as it is, which done, aske him whether it bee even or odde: if he say odde, bid him take one to it, to make it even, and for that one keepe one in your minde. Now after he hath taken one to it, to make it even, bid him give away halfe, and keep the other halfe for himselfe which when he hath done, bid him triple that halfe, and againe after hee hath tripled it, aske him whether it be even or odde: if he say odde: then bid him take one to make it even again, and for that last one, keepe two in your minde: now after he hath made his number even, bid him cast away the one halfe, and keep the other stil, from which halfe that hee keepeth cause him subtilly to put away or give you 9 out of his number, as oft as he can, and for each 9 that he giveth you, keepe 4 in your minde, and thereunto joyne the 3 which I bade you keepe, and you shall have

Example.

Imagine hee thought 7, the triple whereof is 21

16

21, *and because it is odde, hee is to take 1 to make it euen, which first 1 giuen, is for you to keepe in minde. Then the halfe of his 22 being cast away, hee reserueth still 11, which after you haue bid him triple, it maketh 33: then in giuing of him one againe to make it euen, vpon that last 1 reserue 2 in your minde, then his halfe of 34 maketh 17; from whence hee can giue you 9 but once. Therefore that yeelding to you 4, and the 3 that you keepe, make 7, your desire.*

Another kinde of Diuination, to tell your friend how many pence or single peeces, reckoning them one with another, he hath in his purse or should thinke in his minde.

Which to doe, first bid him double the peeces hee hath in his purse, or the number hee thinketh (if he participate his number or secrecie vnto some one friend that sitteth by him that can but multiply, and adde neuer so little: if their number bee great, then shall they worke as you bid them so much the surer.)

Now after hee hath doubled his number, bid him adde thereunto 5 more; which done, bid him multiply that his number by 5 also; which done, bid him tell you the iust summe of his last multiplication, which summe the giuer thinketh it nothing auailable, because it is so great aboue his pretended imagination: yet therby shall you presently with the helpe of Substraction tell his proposed number.

The Rule is this.

Imagine hee thought 17, Double 17, and it maketh 34, whereunto if you adde 5, it maketh 39: which multiplyed by 5, as here is practiſed, it yeeldeth 195, which 195 is the ſumme deliuered you in the worke: then for a generall Rule you ſhall euermore cut off the laſt figure toward your right hand, with a daſh of your pen, as here is performed, as a figure nothing auailable vnto your worke, and then rebate 2 from your firſt figure, after 5 is cut off, and the reſt ſhall euermore be your deſire, as by this example doth appeare.

17
2
34
5
39
5
19|5
2|
17

Another of a Ring.

If in any company you are diſpoſed to make them merry by manner of diuining, in deliuering a Ring vnto any one of them, which after you haue deliuered it vnto them, that you will abſent your ſelfe from them, and they to deuiſe after you are gone, which of them ſhall haue the keeping thereof, and that you at your returne will tell them what perſon hath it, vpon what hand, vpon what finger, and what ioynt: Which to doe, cauſe the perſons to ſit downe all in a row, and to keepe likewiſe an order of their fingers: now, after yee are gone out from them to ſome other place, ſay vnto one of the lookers on, that he double the numbers of him that hath the

Ring

Ring, and vnto the double bid him adde 5, and then cauſe him to multiply the Addition by 5, and vnto the product, bid him adde the number of the finger of the perſon that hath the Ring. And laſtly, to end the work, beyond that number towards his right hand, let him ſet downe a figure ſignifying vpon which of the ioynts hee hath the Ring, as if it bee vpon the ſecond ioynt, let him put downe 2. Then demand of him what number hee keepeth, from the which you ſhall abate 250, and you ſhall haue three figures remaining at the leaſt, The firſt toward your left hand; ſhall ſignifie the number of the perſon which hath the Ring, the ſecond or middle number ſhall declare the number of the finger, and the laſt figure towards your right hand ſhall betoken the number of the ioynt.

Example.

Imagine the ſeuenth perſon is determined to keepe the Ring vpon the fifth finger, and the third ioynt: firſt double 7, *it maketh* 14, *thereto adde* 5, *it maketh* 19, *which multiplyed by* 5, *yeeldeth* 95, *vnto which* 95, *adde the number of the finger, and it maketh* 100: *and beyond* 100 *toward the right hand, I ſet downe* 3 *the number of the ioynt, all maketh* 1003, *which is the number that is to bee deliuered you, from which abating* 205, *there reſteth* 753, *which prefigureth vnto you the ſeuenth perſon,*

the fifth figure, and the third ioynt.

But note, that when you haue made your subtraction, if there doe remaine 0, in the place of tens, that is to say, in the second place, you must then abate 1, from the figure which is in the place of the hundreds, that is to wit, from the figure which is next your left hand, and that shall bee worth 10 tenths, signifying the tenth finger, as it there should remaine 803, you must say, that the seuenth person vpon his tenth finger, and vpon his third ioynt, hath the Ring.

Another of three Dice.

If a man do cast 3 Dice, you may know the points of one of every of them. For if you cause him to double the points of one Die, and to the double to adde 5, and the same summe to multiply by 5, and vnto the product adde the points of one of the other Dice; and behinde the number towards the right hand, to put the figure which signifyeth the points of the last Die, and then to aske what number hee keepeth, from which abate 250, and there will remaine 3 figures, which doe note vnto you the points of euery Die.

Another of things hidden.

If three diuerse things are to bee hidden of three diuerse persons, and you to diuine, which of the three persons hath the three diuerse things, doe thus: imagine the three things to bee represented *A*, *B*, *C*. Then secondly keepe well

well in your minde which of the perſons you meane to be the firſt, ſecond, and third. Then take 24 counters or ſtones, and your three things, and give *A* to the party whom you imagine to be your firſt man, and therewithall give him one of your 24 counters in his hand, and *B*, unto your ſecond man, and therewithall 2 counters. And *C*, unto your third man, and therwithall 3 counters: and leave the reſt, which are 18, ſtill among them: which done ſeparate your ſelfe from them, and afterwards bid them change the things among them as they ſhal thinke good: which done, after they are agreed, bid him that hath ſuch a thing, as before you have repreſented by *A*, for every counter that he hath in his hand, to take up as many more. And for him that hath *B*. For every one in his hand to take up two. And for him that hath *C*. for every one in his hand to take vp 4, and the reſt of them to leave ſtill upon the board. Theſe 3 things and the three perſons being fully printed in your minde, come to the table, and you ſhall evermore finde one of theſe 6 numbers, 1, 2, 3, 4, 5, 6, or 7. If therefore one remaine ſtill upon the board, then have they made no exchange, but keepe them ſtill as they were delivered unto them. So that the firſt man hath *A*, the ſecond *B*, and the third man *C*. But if 2 remaine, then the firſt man hath *B*, your ſecond man *A*, and your third man *C*. The reſt of the worke and the order thereof are

here apparant by the Table following,

	1	A		1	B
1	2	B	5	2	C
	3	C		3	A
	1	B		1	C
2	2	A	6	2	A
	3	C		3	B
	1	A		1	C
3	2	C	7	2	B
	3	B		3	A

Another diuination of a number upon the caſting of two Dice.

Firſt let the Caſter caſt both the Dice, and marke well the number: then let him take up one of them, it maketh no matter which, and looke what number it hath in the bottome, and adde all together: then caſt the Dye againe, and keepe in his mind what altogether maketh: then let the Dice ſtand, and bring ſeven with you, and thereunto adde the reſt of the pits that you ſee upon the upper ſide of the Dice, and ſo many did the caſter caſt in all.

FINIS.

An Appendix concerning the Reſolution of the *Square* and *Cube* in numbers, to the finding of *their ſide, by* Ro. Hartwell.

A Figurate number is a number made by the multiplication of one number or more by another. A figurate number what.

The ſides of a figurate number, are the numbers by whoſe multiplication it is made. The ſides of a figurate number what.

A Figurate number is two-fold, a { *Plaine.* / *Solid.* }

And it is { *Of one multiplication,* / *Or conſequently of many,* } *as a* { *Plaine,* / *Solid.* }

And in each { *Both Equilater.* / *And Inequilater.* } Equilater & Inequilater

A figurate number made of one multiplication, by two ſides or numbers multiplied together, is called a plaine figurate number.

For every number made by the mutuall multiplication of two numbers, may be called a *Plaine*, becauſe it bringeth forth a right-angled parallelogramme, according to his unities diſpoſed in *length* and *breadth*, the *ſides* whereof are the two multiplying numbers. As the number 20, made by the mutuall multiplication of 4 and 5 is called a *Plaine*, and the A plaine figurate number.

the *sides* thereof are 4 and 5 as here.

```
*  *  *  *  *
*  *  *  *  *
*  *  *  *  *
*  *  *  *  *
```

Becauſe the vnities thereof diſpoſed in *length* and *breadth*, as the *ſides* doe expreſſe, doe bring forth an *inæqualiter Parallelogramme*, for that the numbers, or *ſides* are inequall.

By like reaſon 36 made by multiplication of 6 by 6, is called an *æqualiter plaine*, for the *ſides* thereof 6 and 6 are equall.

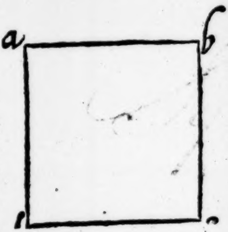
Moreouer one and the ſame *plaine number* may haue many *ſides*, as the *plaine number* 24. hath ſides 4 and ſixe: 3 and eight: 2 and 12. For it is produced from the mutuall multiplication of theſe numbers: whereupon for the inuention of the ſides, to wit, in *inequilater Plaines*, it is needfull to giue one of the *ſides*, by which the *plaine* it ſelfe diuided, the other *ſide* is made knowne. As the *plaine* 48 being diuided by the *ſide* 8, the *quotient* 6 is the remaining *ſide*. Notwithſtanding another reſolution and inquiſition doth happen in the *ſides* of the *æquilater plaines*.

An equilater Plaine or quadrat what.

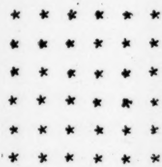
An æquilater plaine is a number made by two equall ſides, or by any number multiplyed by it ſelfe. It is vulgarly called a ſquare or quadrat; by the Arabians Zenſus, *it is commonly expreſſed by this note* ʒ, *by vs q.*

A quadrat or *ſquare* in *Geometrie* is called a right lined *plaine figure*, made by foure equall right lines, and ſo many right angles, and euery one of the lines is called the *ſide* of the *quadrat*, *as*

as this *Figure a b c d* whose *side* is *a b*, or *b c*, as also *c d*, and *a d*.



To the similitude heereof, that number is called a *Quadrate* which is made by the multiplication of two equall numbers, or of one in it selfe of which manner 36 is made, by 6 multiply in it selfe, or by the mutuall multiplication of 6 and 6. For if 36 vnites bee placed in plaine forme, it bringeth forth a perfect *Geometricall Quadrat* hauing in euery side fixe vnities as here.



The number whereof the Quadrat is produced by multiplication in it selfe, is called the side or root of the Quadrat. The side or root of a number what.

Concerning the extraction of the *quadrat or square root.*

THerefore to finde the *quadrat root*, or the *side* of any *quadrat number*, is to search a number, which brought or multiplyed in it selfe, maketh the number propounded: concerning the finding whereof, as it is requisite that the *sides* (being lesser then 10) of the *squares* vnder an hundred should be gathered by the Table of Multiplication: so the *sides* of the

the *greater ſquares* are to bee ſought out by *Art.* Firſt, the *ſquares* whoſe *ſides* are ſimple numbers, are here ſet downe as you ſee.

The roots.	1	2	3	4	5	6	7	8	9
The ſquares	1	4	9	16	25	36	49	64	81

The knowledge of a ſquare is by finding out his ſide expreſſed by a whole number.

Although the finding out of the *ſide* of a *ſquare* bee applyed to each number giuen, as to a *ſquare*, yet *ſquare numbers* onely haue a *ſide* to bee expreſſed by a certaine number of vnites, or by rationall numbers, the other are to bee expreſſed but onely in power. The *ſides* are commonly called *Rootes* by a *metaphoricall phraſe.*

The Roote or ſide of a ſquare is to be found by the Theorem following.

4 If the odde degrees of a *ſquare number* being marked from the right toward the left hand with points, you ſubduct from the number giuen, the particular *ſquare* of the laſt *period*, ſetting the *ſide* thereof alone by it ſelfe.

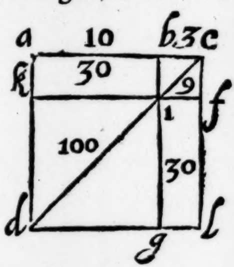
2 Then going on, if you diuide the remainder (if there be any) with the figure going before it, by the double of the *ſide* ſet alone by it ſelfe.

3 And multiply the *quotient*, found out (being placed by the *ſide*, which was firſt ſet alone by it ſelfe, and alſo before the doubled number on

on the right hand) by both the numbers (namely by the double number, and the Figure ſet by it ſelfe) being counted as one *diuiſor*, ſubducting the products from the giuen number, and then renue this laſt worke of diuiſion ſo many times as there are prickes remayning, the *ſide* of the *ſquare* ſhall bee found out.

This artificiall deuice is taken out of the 4 *P.* 2. *of* Euclide. *Where by demonſtration it is prooued, that if a right line bee cut into two* ſegments, *howſoeuer the* ſquare *of the whole line is equall to the* ſquares *of the* ſegments, *and to the two* right-angled figures *made of the* ſegments *as in the figure annexed, the two* diagonalls, *k* g, and *b f, are the* ſquares *of the* ſegments, *a b*, and *b c*. *Alſo the* complements *b k*, and *f g*, *are the* right-angled figures *made by multiplying the line a b*, by *b c*.

To extract the ſquare roote.



The ſelfe-ſame parts are to bee found in any *ſquare number*. As for *example*, let the *number* bee 169, whoſe *ſide* is 13. This *ſide* being diuided into two pieces, 10 and 3, multiplying each piece by it ſelfe once, namely, 10 by 10, and 3 by 3: then multiply one by another, as 10 by 3, and 3 by 10, ſo ſhall you haue foure *plaine numbers*, whereof 2 are *ſquare*, as here you ſee.

The firſt example.

There-

F Therefore as the *ſquare* 169 is made by adding together of theſe foure *plaine numbers*, ſo by ſubducting them ſeuerally it is reſolued.

10	3
10	3
100	
30	
30	
9	
169	

First therefore I marke each odde place with points, becauſe the *particular ſquares* are to bee found in the odde places. Then for ſo much as the vnity ſtanding vnder the firſt point next the left hand, and repreſenting the laſt *period*, is both a *ſquare* and the *ſide* of a *ſquare*: that *figure* therefore being ſet alone in the *quotient*, and being ſubducted from the vnitie ſtanding ouer the point, there remaineth nothing.

This vnitie ſet alone by it ſelfe in the *quotient*, ſhall ſignifie 10, when another *figure* is ſet by it, repreſenting the ſide of ſome other particular *ſquare*. Whereupon I ſay, that the greater *Diagonall* *k g*, is now ſubducted from the whole *ſquare*, and the ſide of it *k i*, or a *b*, (for they are equall one to another) and alſo the *ſide* of one of the *complements* is found out.

This is the firſt ſtep to this reſolution.

Moreouer, I double the *figure* found out, becauſe being doubled, it is the ſide of both the *complements* taken ioyntly together, namely, *k i*, and *g i*. Then ſetting 2 the doubled number vnder

vnder 6, I diuide 6 (which in this place is as much as 60, and repreſenteth both the *complements*) by 2, the *quotient* is 3, repreſenting the other *ſide* remaining of the *complement*, namely, *if*, or *b c*, which number I ſet in the *quotient*, and count it for the *ſegment remaining* of the right line giuen. Wherefore becauſe this number 3 is the ſide of the remaining *Diagonall*, that is to ſay, of the leſſer *ſquare b f*; therefore being ſet by the *diuiſor* on the right hand, and multiplyed by it ſelfe, and alſo by the *diuiſor*, it bringeth forth three *plain numbers*, namely, the *ſquare b f*, and the two *complements a i*, and *i l*, which being ſubducted from the numbers ſtanding ouer them, there remaineth nothing.

The example is thus.

169 (13	*Which is all one, as if you had put down the numbers found out in this manner.*	169	
~~1~~ 2 ~~3~~		~~1~~ ~~0~~ ~~0~~	*The greater diagonal.*
~~3~~		~~6~~ ~~0~~	*The comple: two-fold*
69		~~9~~	*The leſſer Diagonal,*
		169	

The ſubtilty of this invention is illuſtrated by many examples.

Let the *ſquare* given bee 1764. This number being marked with two points, telleth us, that the ſide thereof is to bee written with two Figures. — The ſecond example.

First

Firſt therefore beginning at the point on the left hand, I ſeeke the ſide of the laſt period, namely 17. But for ſo much as it is no *ſquare* number, I take 4 the *ſide* of the next leſſer ſquare, which I ſet alone by it ſelfe in the *quotient*, and then multiply it by it ſelfe, the *product* is 16, which being ſubducted from 17, there reſteth 1. Moreouer I double the *ſide* found out, the product is 8, I place this doubled number under 6, and by it diuide 16 ſtanding aboue it, the *quotient* is 2, which muſt be ſet by 4. This *quotient* 2 muſt be ſet before the *diuiſor* 8 on the right hand under the point, and then muſt it be multiplied both by it ſelfe, and into 8, the *product* is 164. which being ſubducted from the figures ſtanding ouer them, there remaineth nothing: whereby I gather that the number given is a iuſt *ſquare*.

The example ſtandeth thus.

~~1~~
~~1~~~~7~~~~6~~4 (42
———
1 6 8 ~~2~~
2
———
1 6 4
———
1 7 6 4 *The Collection.*

The ſame manner of working is to bee followed in greater ſquare numbers given, ſauing that the former part of the worke is to bee vſed

vſed but once, but the latter part is to bee followed ſo many times as there are points remaining, excepting the laſt.

As in 54756, I ſay, that the *ſide* of the *ſquare* next vnto 5 is 2: therefore 2 being ſet in the *quotient*, and multiplyed by it ſelfe, make 4, and taken from 5, the remainder is 1. Moreouer I double the *quotient*, the *product* is 4, which I ſet vnder the next *figure* toward the right hand, and thereby diuide 14. the *quotient* is 3. which three being ſet both in the *quotient*, and alſo before the *diuiſor* toward the right hand, I multiply both the numbers by it, the *product* is 129: this beeing ſubducted from 147 ſtanding aboue it, the remainder is 18. But becauſe there is yet one point remaining, with which I haue not medled; I therefore againe double all the whole *quotient*, for in this caſe I muſt take 23 for the ſide of one former *ſquare*, and generally in great numbers, when I light vpon more particular *ſquares* then two, I muſt eſteeme them but as two, and take the *ſides* which are firſt found out, but as the *ſides* of one onely *ſquare*. Therefore twice 23 is 46: by this I diuide 185, the number to bee ſet in the *quotient* is 4, which number alſo muſt bee ſet before the *diuiſor* on the right hand: then muſt 464 bee multiplyed by 4: the *product* is 1856, this *product* being ſubtracted from the numbers ſtanding ouer it, there remaineth nothing. The example ſtandeth thus.

The third Example

Note.

 The

~~1~~~~1~~8
~~5~~~~4~~~~7~~~~5~~~~6~~(234
~~4~~~~4~~~~3~~~~6~~~~4~~
129

4
1836
54756 *The Collection.*

See also the Example following.

10942864 (3308.

Therefore out of this inuention is this *consectarie.*

The number whose side cannot bee expressed by whole numbers, is not a square number.

4 Example of a surd number. Such are all *prime numbers*, and (the *squares* themselues excepted) all other *compound numbers.* For if in them you desire to finde out the *square side*, you shall labour in vaine, because they are not *squares*, for to the whole numbers arising in the *quotient*, there will bee some *fraction* adioyned, whereby it commeth to passe, that the number of the *side* is not to bee expressed by a true number, and it is commonly called a *surd number.*

Notwithstanding, if you adioyne to the *side* found out, the number remaining, taking his *denomination* from the double of the *side augmented*

mented by an unity, you ſhall finde the next *ſide* that may be like to the *ſide of a ſquare*.

As if from 40 you take the neareſt *ſquare*, to wit, 36, the remainer is 4. Here therefore the *ſide* ſought for of the *ſquare*, exceedeth not the *ſide* found out by an unity, but either by one, or more parts of ſome whole number: wherefore I double 6, the *ſide* found out, & adde an unity to it being doubled, the totall is 13, this number I ſet under 4 the remainder, and ſay that the *ſide* of 40 demanded as neere as may be, is $6\frac{4}{13}$: the *Denominator* of the *Fraction* being added to the greateſt *ſquare* in the number given, namely, unto 36, maketh the next greateſt *ſquare* aboue it, namely, 49, whoſe *ſide* is 7. But this *ſurd ſide*, to wit $6\frac{4}{13}$, multiplyed by it ſelfe, maketh $39\frac{133}{169}$, which are are not juſt equall to 40, the given number.

Iudge the like concerning the reſt which are not *ſquares*.

Thus much concerning plaine figurate numbers, but eſpecially ſuch as are ſquare numbers.

Concerning ſolid figurate Numbers.

A *Solid figurate Number is made of two multiplications by three numbers, or ſides, multiplyed together, admitting length, breadth, and thickneſſe.*

Therefore every *number* made by the

A solid figurate number.

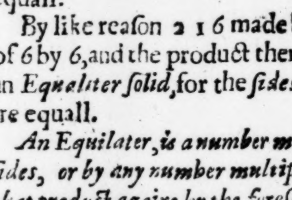
mutuall multiplication of three numbers, may be called a *solid*, because it bringeth forth *a right angled Parallelipipedon*, disposed according to his unities in *length, breadth*, and *thicknesse*, the *sides* whereof are the three multiplying *numbers*. As the *number* 30 made by the mutuall multiplication of 2, 3, and 5, is called an *inequilater solid number*, and the *sides* thereof are 2, 3, and 5, because the unities thereof disposed by a certaine distance one from another, in *length, bredth*, and *depth*: as the *sides* doe expresse, doe bring forth in resemblance an *inequilater Parallelipipedon*, for that the *numbers* or *sides* are inequall.

By like reason 2 1 6 made by multiplication of 6 by 6, and the product thereof by 6, is called an *Equalater solid*, for the *sides* thereof 6 6 and 6 are equall.

An Equilater Solid or Cube.

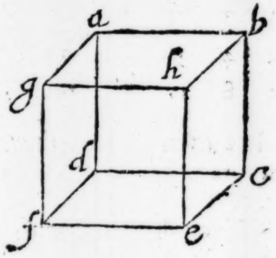
An Equilater, is a number made by three equall sides, or by any number multiplied by it selfe, and that product againe by the foresaid number. And it is called an Equilater and Equiangled Parallelipipedon or Cube, and is commonly represented by us thus C.

A Cube in Geometry is a right angled *Parallelipipedon* having six equall *surfaces*, and 8



solid

solid angles, & 12 *sides*, as this figure *a*, *b*, *c*, *d*, *e*, *f*, *g*, *h*, whose *side is a b*, or *a d*, also *b c*, or *c d*, either *c e*, or *e f*, likewise *c h*, or *b g*, also *g f*, or *d f*, or *d a*, and *g a*.



The number whereof the Cube is produced by multiplication in it selfe twice, is called the side or roote of the Cube, which being found out in whole numbers, the Cube is knowne. The side or root of a Cube.

Concerning the extraction of the *Cubicke roote*.

THerefore every *Cube* in numbers hath such a *side* as may be expressed in whole numbers, but in magnitudes it is not alwaies so, as indeed in *magnitudes* there are many things not to be expressed in *whole number*. Now for as much as the *side* of any *Cube* under 1000, is a simple figure, it is necessary, before we undertake to finde out the *side* of any great number, to know what *Cube* is made of each *simple figure*, and what is the side of any *Cube* lesser then 1000, as I have here set them downe.

Roots.	1	2	3	4	5	6	7	8	9
Squares.	1	4	9	16	25	36	49	64	81
Cubes.	1	8	27	64	125	226	343	512	729

But in ſearching out the *ſides* of greater *Cubes*, we are to proceed as the *theorem* following teacheth us.

If you diſtinguiſh with *points* as it were into periods, the given *Cube*, beginning at the firſt figure on the right hand, and omitting each two figures continually, and firſt of all ſubduct the *particular Cube*, of the laſt *period* from the given number, ſetting the *ſide* thereof in the *quotient*: and then ſet triple of the *quotient* under the *figure* next following the former *point* on the right hand, and the *ſquare* of the *quotient* being tripled beneath it one *degree* more toward the left hand: and afterward divide the number above written by the triple of the *ſquare*, ſetting the *quotient* by it ſelfe, and then multiply the *diviſor* by the *quotient* found out, and the tripled *ſquare* by the *ſquare* of the *quotient*, and the *quotient cubically*, ſubducting the *products* (ſo orderly added together, that each *figure* may anſwer the *numbers* whereof it was multiplied) from the *number* given, and renue this laſt manner of diviſion ſo many times as there are *points* remaining, the *ſide* of the *Cube* ſhall be found out.

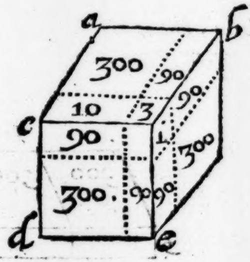
This

This artificiall deuice is drawne out of that *theorem* which *Ramus* made, imitating that of *Euclide* concerning *square numbers* in this manner.

If a right line be cut into two segments, the Cube of the whole line shall be equall to the Cubes of the segments, and to the two solid figures comprehended three times vnder the square of his segment, and the segment remaining.

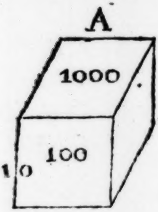
The extraction of the Cubick side or root

As the line *ci*, which is 13, is cut into two *segments*, 10 *and* 3 therfore the *Cube* of the whole line, namely 2197, is equall vnto the *Cubes* of the *Segmẽts*, namly vnto 1000, and 27 also to the *two-fold solids* or *Parallelipipedons* thrice taken, whereof three haue like *soliditie*, the *soliditie* of each of the three lesser is 90, being made of the *square of the segment* 3, that is to say of 9, multiplyed by the other *segment* 10. These three *Parallelipipedons* ioyntly taken together, make 270. But of the three greater *Parallelipipedons* each containeth 300, being made of 100, the *square* of the greater *segment* 10, multiplyed by the *lesser segment* 3,

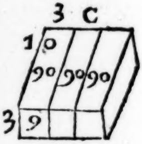


 and

and they being taken joyntly together, make 900.

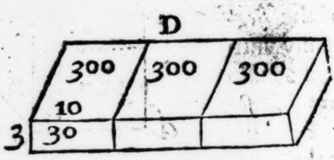


The Cube of the lesser segment 3.



The Cube of the greater segment 10.

The 3 *lesser Parallelipipedons.*



The three greater Parallelipipedons.

The *Cube* therefore hath eight *particular solids in number*, which are made of the parts of the number given, namely of 10 and 3. in this manner. First let there be foure *plaine numbers* made, each part being multiplied by it selfe; and one by another.

10	3
10	3
	9
	30
	30
	100

If againe I multiply the *Plaines* by the ſame parts, there will ariſe 8 *ſolids*, as you ſee here.

9	9
30	30
30	30
100	100
3	10
27	90
90	300
90	300
300	1000

All theſe being added together, are equall to the Cube *of the whole, to wit,* 2197.

The firſt example to extract the cubick Roote.

Therefore the ſame way that is kept in making the Cube, *is alſo to bee followed in reſoluing the* Cube.

As for example, I marke the *Cube* giuen with points in this manner, 2197.

Then I ſubduct the particular *Cube* of the number ſet vnder the laſt point: but for ſo much as that number is no *Cube*, I take the neereſt to it, namely, an vnity, which alſo I ſet in the *quotient*. This vnity in the number giuen, is 100, but in the *quotient* it is but 10, the vnite ſubducted from 2, the remainder is 1, which muſt bee written ouer the number giuen

giuen. So that the greater *Cube A*, is to be supposed to bee subducted from the number giuen.

This is the first step of this worke.

1
2197(1
1

After I triple the *quotient* found out (that is to say, I multiply it by 3) this triple representeth the three *sides* (iointly taken together) of the three lesser *solids* marked with *C*, I place the tripled number vnder 9. Againe, I multiply the *quotient* square wise, & triple the *product*, which maketh likewise 3. This *product* resembleth the three *square sides* (taken iointly together) of the three *greater solids*, marked with *D*, I place the *product* on a degree lower toward the left hand vnderneath 1. With it I diuide 11, which written aboue it, the quotient is 3. This *segment* or *quotient* 3, being multiplyed by 3 the *Diuisor*, maketh 9, which in respect of the place wherein it standeth, is 900, and representeth the three greater *solids*, marked with *D*, taken ioyntly together. Furthermore the same *quotient* being multiplyed squarewise, maketh 9, and multiplyed afterward by the tripled *number* standing vnder 9, it maketh 27, which in respect of the place wherein it standeth, is 270, & representeth the three *lesser solids* marked with *C*. Last of all, the same *quotient* multiplyed

plied *cubically*, breedeth the leſſer *Cube B*. Theſe three *products* therefore being added together, and the totall ſubducted from the *numbers* ſtanding over it, there remaineth nothing, which importeth the giuen number is a *Cube*.

The example is as you ſee.

1	2197 (13
2197 (13	1000 *The greater Cube.*
1 3	3
3	3
9 Or thus:	900 *The 3 greater Parallelipipedons*
27	270 *The leſſer Parallelipipe.*
27	27 *The leſſer Cube.*
2197	2197

The matter may be explained by many examples.

Let the ſide of the given *Cube* 16387064, be ſought out, contrive it therefore (as it were) into certaine periods with points. Then firſt of all, ſearch out the *ſide* of the *Cube* next to the left hand: But for as much as 16 is no *Cube*, take 2 the *ſide* of the next *Cube* under it, that is to ſay, of 8, and ſet it in the *quotient*, and ſubduct 8 the *Cube* thereof from 16, there remaineth 8. This firſt worke is not to be renued throughout the whole *number*, but the *rules* following muſt be repeated as often as there are *points* remaining.

The ſecond example of the Cubick roote.

The

The first step to finde out the roote, is in this manner.

```
   8
   . .  .  .
   16387064 (2
   8
```

Moreover, triple the *quotient* now found out, and the *product* is 6, which is to be placed under 8, namely, under the *figure* following the next *pricke* toward the right hand. Then multiply the *quotient* by this tripled *number* (or which is all to one purpose, square the *quotient*, and then triple the *product*) it maketh 12, set that number in a lower place one degree neerer the left hand, and make it the *divisor*: divide 83 by 12, observing this *rule* in choosing your *quotient*, that it be no greater, then that the numbers afterward produced by multiplication may not exceed the *numbers* standing over it. So that here you shall take 1 in 8, but 5 times. Afterward by this number 5, multiply the *divisor* 12, and by the *square* of 5, multiply the tripled *number* 6, and last of all multiply 5 *cubically*: so shall you produce three *numbers*, namely, 60, 150, 025, to bee described in such sort as you see. These numbers added together, and subducted from 8387, the remainer is 762.

The second step to finde out the roote, is in this manner.

```
  8762
  16387064 ( 25
  ------
       6
  12
  ------
  60
  150
    125
  ------
  7625
```

16387064 | 254

7400) 8387
6000

And because there is yet one point remaining, this last manner of Division must be wrought againe.

First therefore I triple the *quotient*, the *product* is 75. which must be so placed, that the *first figure* thereof, namely 5, may stand under 6, the *second* under the 0. Againe, multiply the *quotient* by this *tripled number* (or which is all one, *square the quotient*, and triple the *product*) it maketh 1875, which must be the *Divisor*, whose first *figure* namely 5, must be placed under 7, the *last figure* of the *tripled number*. Then see that 1 may be contained in 7, many times, but I can take it but 4 times, I set 4 in the *quotient* and multiply the *Divisor*, by 4, the *product* is 7500, afterward *I square* 4, it maketh 16 which I multiply by the *tripled number* 75, the *product* is 1200. Last of all I multiply 4 *cubically*, it maketh 64, these *products* added all together,

ther, make 762064, which number being subducted from the *Cube* given, there remaineth nothing, whereby I gather that the number given is exactly *cubicall*.

The third step to finde out the side is in this manner.

```
      7 6 2
1 6 3 8 7 0 6 4 (2 5 4
          7 5
      1 8 7 5
      7 5 0 0
       1 2 0 0
           6 4
  7 6 2 0 6 4
```

The third example of the cubick roote.

Behold also the example following.

6 1 4 1 2 5 0 0 0 (8 5 0

¶ Another manner of working.

Hitherto the Princely high-way to finde out the side of the Cube hath beene declared.

But there are moreover certaine other waies also bending thereto, and leaning to the same principles, whereof this is one.

The second forme.

1 Having found out in the Table of simple *Cubes*, the first figure representing the side of the *Cube* contained in the number standing under the first *point* on the left hand, set it in the *quotient*

quotient, and subduct the particular *Cube* of that figure as you did before: then *square* that *figure*, and triple that *square*, the *product* shall bee the *Diuisor*, the first *figure* whereof shall bee set vnder that *figure* which is on the right hand next of all to the *point* (now examined) beforegoing.

See how many times the *Diuisor* is contained in the number written ouer it, and multiply the *Diuisor* in the *quotient*, and subduct the *product* from the *diuidend*: yet here you must take heed, that you take not a greater *quotient*, then that the *products* made afterward thereby may bee subducted from the *number* giuen. 2

The subduction being done, triple the first *figure* which was set in the *quotient*, and adde to the triple the last *number* which was set in the *quotient* on the right hand of the *product*. 3

This totall multiplyed by the square of the *figure* last found out, and set downe the *product* so, that the first *figure* thereof toward the right hand may stand vnder the *point* next before going on the same hand, and finally subduct the same from the *number* giuen. 4

As in 804357. The particular *Cube*, namely, 729 being taken from the *number* standing vnder the last *period* vpon the left hand, there remaineth 75357, the side of that particular *Cube* being 9, I set in the *quotient*. Then I *square* that *side*, it maketh 81, & triple the *square*, the *product* 243 is my *diuisor*, which I set vnder the giuen *number*, so that 3 may stand vnder 3 with

The fourth example of the Cubick roote.

with this *diuisor*, diuide the *number* ſtanding ouer it, you ſhall finde 2 to bee conteined in 7 three times. Therefore I ſet 3 in the *quotient*, and multiply the *diuiſor* by it, the *product* is 729, which being ſubducted from 753, the remainder 24.

The Induction is thus:

2
~~754~~
~~8043~~57 (93
243
~~729~~

Moreouer I triple 9, the *product* is 27, by which on the right hand I ſet 3, the *quotient* laſt found out, the totall is 273.

This *number* I multiply by 9 the *ſquare* of the *quotient* laſt found out, the *product* ſhal be 2457, which being ſubducted from the ſuperiour *number*, there remaineth nothing.

The Induction is thus:

~~2~~4
~~804357~~ (93
273
9
2457

Another

Another manner.

THe selfe-same worke may bee dispatched another way a little differing from the former in this manner.

The *figure* in the *quotient* being found out by subducting the particuler *Cube*, and also the second *figure* in the quotient being found by diuision, let the totall *quotient* bee tripled and let the tripled *number* bee multiplyed by the former *figure* in the *quotient*. Then let the *product* bee multiplyed againe, by the latter *figure* found out, and let a *cipher* be set on the right hand of that *product*. Last of all, let the *Cube*, of the latter *figure* found out, bee added to this *product*, and let the totall summe be subducted from the number giuen. As in 373248. The third forme.

The first induction is in this manner.

$$\begin{array}{l} \;\;30 \\ \underline{\cancel{373}\,248\,(7} \\ 343 \end{array}$$

Moreouer I *square* the *side* found out, it maketh 49, and triple the *square*, the *product* is 147, which shall be the *diuisor*, by this I diuide 302, the number written ouer it, the *quotient* is 2. Now I triple the totall *quotient* 72, it maketh 216, and multiply this triple by 7, the former *figure* in the *quotient*, the *product* is 1512. I multiply this *product* also by 2, the latter *figure* of the *quotient*, and set a *cipher* on the right hand of it, so as it maketh 30240, vnto this number last of all I adde 8, the *Cube* The fifth Example.

 of

of the latter *figure* found out, the totall is 30248 which being subducted from the *figure* aboue it, there remaineth nothing.

The second Induction is thus.

$$
\begin{array}{r}
30 \\
\cancel{3}\cancel{7}\cancel{3}\dot{}248\,(72 \\
\hline
147 \\
\hline
30248
\end{array}
$$

¶ All the *points* of the *number* giuen being examined, if any thing remaine, it signifyeth the *number* giuen is no *Cube*: wherefore the true *side* of it cannot bee exactly giuen in *numbers*. Yet if it please you to sift out the neerest *side* that may bee, by the first kinde of reduction of *mixt numbers*, you shall reduce the *number* giuen vnto a *cubicall* fraction of a greater *denomination*, and afterward seeke out the *cubicall side* of that *fraction*.

To find the neerest Cubicke roote in a surd number.

For example sake, because 120 is no *Cube*, therefore let it bee reduced into sixty *cubicall* parts, after this manner. Multiply 60 *cubically* in it selfe, it maketh 216000, by this being taken for the *denominator* of the *fraction*, multiply 120 the *number* given, the *product* is 2592000 whose *cubicall side* is $\frac{295}{60}$ that is $4\frac{11}{12}$ the neerest to the true *side* that can be.

For the extraction of all sorts of roots, the table of Logarithmes *set forth by* Mr. Briggs *are most excellent, and ready.*

FINIS.

A Table of Board and Timber measure, more perfect then euer hath beene made; shewing also the Squares between 4 and 37 from quarter to quarter, Calculated by *Ro. Hartwelle*

Board measure.	Inches and quarters.	Squares.	Timber measure.	Board measure	Inches and quarters.	Squares.	Timber measure.
36.0.0	4	16	108.0.0	16.0.0	9	81	21.3.3
33.8.8	1	18	96 0.0	15.5.6	1	85	20.3.3
32.0.0	2	20	86.4.0	15.1.6	2	90	19.2.0
30.3.1	3	22	78.5.4	14.7.7	3	95	18.1.8
28.8.0	5	25	69.1.2	14.4.0	10	100	17.2.8
27.4.3	1	27	64.0.0	14.0.5	1	105	16 4.6
26.1.8	2	30	57.6.0	13.7.1	2	110	157.1
25.0.4	3	33	52.3.6	13.3.9	3	115	15.0.2
24.0.0	6	36	48.0.0	13 0.9	11	121	14.2.8
23.0.4	1	39	44.3.0	12.8.0	1	126	13.7.1
22.1.5	2	42	41.1.4	12 5.2	2	132	13 0.9
21.3.3	3	45	38.4.1	12.2.7	3	138	12.5.2
20.5.7	7	49	35.2.6	12.0.0	12	144	12.0.0
19.8.6	1	52	33.2.3	11.7.5	1	150	11.5.1
19.2.0	2	56	30.8.6	11.5.2	2	156	11.0.7
18.5.8	3	60	28.8.0	11.2.9	3	162	10.6.6
18.0.0	8	64	27.0.0	11.0.7	13	169	10.2.2
17.4.6	1	68	25.4.1	10.8.7	1	175	9.8.7
16.9.4	2	72	24.0.0	10.6.7	2	182	9.4.9
16.4.6	3	76	22.7.5	10.4.7	3	189	9.1.4

Board measure.	Inches and quarters.	Squares.	Timber measure	Board measure	Inches and quarters.	Squares.	Timber measure
10.2.8.	14	169	8.8.1	6:8:6	21	441	3:2:2
10.1.1	1	203	8.5.1	6:7.7	1	451	3:8:2
9.9.3	2	210	8.2.3	6:6:6	2	462	3:7.2
9.7.6	3	217	7.9.6	6:6:1	3	473	3.6.5
9.6.0	15	225	7.6.8	6:5.4	22	484	3:5:7
9.4.5	1	232	7.4.3	6:4:7	1	495	3:4:9
9.2.9	2	240	7.2.0	6:4:0	2	506	3:4:1
9.1.4	3	248	6.9.7	6:3:3	3	517	3:3:3
9.0.0	16	256	6.7.5	9:2:6	23	529	3:2:7
8.8.6	1	267	6.5.4	6:1:9	1	540	3:2:0
8.7.3	2	272	6.3.5	6:1:2	2	552	3.1:3
8.6.0	3	280	6.1.6	6:0:6	3	564	3:0;6
8.4.7	17	289	5.9.8	6:0:0	24	576	3:0:0
8.3.5	1	297	5.8.1	5:9:4	1	588	2:9:4
8.2.3	2	306	5.6.4	5:8:8	2	600	2.8:8
8.1.1	3	315	5.4.8	5:8:2	3	612	2:8:2
8.0.0	18	324	5.3.3	5:7:6	25	625	2:7:6
7.8.9	1	333	5.1:8	5:7:0	1	937	2:7:1
7.7.8	2	342	5.0.5	5:0:5	2	650	2:6:5
7.6.8	3	351	4:9:2	5:5:8	3	662	2:6:0
7.5.8	19	361	4:7:8	5:5.4	26	676	2:5:5
7.4.8	1	370	4:6:7	5:4:8	1	689	2;7:0
7.3.9	2	380	4:5:5	5:4:3	2	702	2:4:7
7.2.9	3	390	4:4:3	5:3:8	3	715	2:4:1
7.2.0	20	400	4:3:2	5:3:3	27	729	2:3:7
7.1.1	1	410	4:2:1	5:2:8	1	742	2.3:2
6 0.2	2	420	4:1:1	5:2:3	2	756	2:2:7
6.9.3	3	431	4:0:1	5:1.8	3	767	2:2:4

Board measure.	Inches and quarters.	Squares.	Timber measure.	Board measure	Inches and quarters.	Squares.	Timber measure.
5.1.4	28	784	2.2.0	4.3.6	33	1089	1.5.9
5.0.9	1	798	2.1.6	4.3.3	1	1104	1.5.6
5.0.5	2	812	2.1.2	4.3.0	2	1120	1.5.4
5.0.0	3	826	2.0.9	4.2.7	3	1136	1.5.2
4.9.6	29	841	2.0.5	4.2.3	34	1156	1.4.9
4.9.2	1	855	2.0.2	4.2.0	1	1174	1.4.7
4.8.8	2	871	1.9.8	4.1.7	2	1190	1.4.5
4.8.4	3	885	1.9.5	4.1.4	3	1210	1.4.3
4.8.0	30	900	1.9.2	4.1.1	35	1225	1.4.1
4.7.6	1	915	1.8.9	4.0.8	1	1237	1.4.0
4.7.2	2	930	1.8.6	4.0.5	2	1247	1.3.7
4.6.8	3	945	1.8.3	4.0.3	3	1280	1.3.5
4.6.4	31	961	1.7.9	4.0.0	36	1296	1.3.3
4.6.1	1	974	1.7.7	3.9.7	1	1313	1.3.1
4.5.7	2	987	1.7.5	3.9.4	2	1331	1.2.9
4.5.3	3	1000	1.7.2	3.9.1	3	1350	1.2.8
4.5.0	32	1024	1.6.9	3.8.9	37	1369	1.2.6
4.4.6	1	1040	1.6.6	3.8.7	1	1388	1.2.4
4.4.2	2	1056	1.6.3	3.8.4	2	1416	1.2.2
4.4.0	3	1072	1.6.1	3.8.2	3	1425	1.2.0

The vſe of this former Table.

IF vpon a *Scale or Ruler* you divide one *inch* into tenne *equall parts or primes*, and againe by *diagonals* and *parallell lines*, you ſubdivide each of them into tenne *equall parts or ſeconds*, with your *compaſſes*, you may take of more exact running meaſure for *board and timber*, then by any other meanes whatſoever, and ſo place the ſame, or this *Table* if you will, vpon any *ruler*.

Alſo by meanes of the *columnes of ſquares*, you may readily finde a *ſquare* equall to any *parallelipipedon* or piece of *timber* which is thicker then it is broad. As for example, ſuppoſe a piece of *timber* to bee tenne *inches thicke*, and 29 *inches broad*: if I multiply thoſe *ſides* one by another, they will produce 290. then ſeeking the *columne of ſquares* for 290. which I finde not; but I finde 280 the neareſt *number* to 290, to ſtand againſt 17: therefore I ſay 17 *inches fere*, will make a *ſquare* equall to ſuch an unlike *ſquared piece*: then looking in the *columne of Timber meaſure* againſt 17. you ſhall finde that 5 *inches*, 9 *prime*, or $\frac{9}{10}$, and 8 *ſeconds*, or $\frac{98}{100}$ of an *inch in length* of that piece will make a *foot of timber*.

Likewiſe for *board meaſure*, you may finde how

how much in *length* of *breadth* of *board* must be in one *foot*.

By the like meanes suppose *for example* that a *board*, appointed to bee measured, is 15 *inches* $\frac{3}{4}$ *broad*, if I desire to know how much in *length* thereof will make a *foot*, I seeke in the *columnes* that stand vnder *vnites* and *quarters*, for 15 $\frac{3}{4}$, and also against the same in the *columne* vnder the title of *board measure*, where I finde 9 *inches* 1 *prime* or *tenth* of an *inch*, and 4 *seconds* or *hundreths* of an *inch* will make a *foote at* that *breadth*: the like may be practised for another *breadth of board* whatsoeuer.

Jas Prescott

Certain tables shewing the Interest of any *summe of money* whatsoeuer vnto 40 years; how much *Annuities* respired or forborne commeth vnto. And for *buying* or *selling* of *Annuities* for the said time; and also the same in *reuersion* after any *number* of *yeares* vnto 30. what they may be worth in present ready money, by *R.C.* and now diligently corrected and amended by *Robert Hartwell*.

Definition of Interest.

PRincipall, *is the summe from which the* Interest *is reckoned*.

2 Interest *is the summe reckoned for the lending or forbearance of the* Principall *for any terms or time.*

3 Interest simple *is that which is counted from the* Principall onely.

4 Interest compound *is that which is counted for the* Principall, *together with the* Arrerage.

5 Interest profitable *is that which is added to the* Principall.

6 Interest Damageable *is that which is to be subtracted from the* Principall.

The vse per annum of	1 li.	is	2 s
	10 s		12 d.
	5 s.		6 d.
	2 s. 6 d.		d.
	12		1 d ⅕ of a penny.

A

A Table shewing what 1 l. *with interest, and interest vpon interest after* 10 *in the* 100. *comes to euery yeare vnder* 41 *yeares. As followeth.*

yeeres.	li	s	d	li	s	d	*yeeres.*
1	1	2	0	7	8	0	21
2	1	4	2	8	2	9	22
3	1	6	7	8	19	1	23
4	1	9	3	9	16	11	24
5	1	12	2	10	16	8	25
6	1	15	5	11	18	4	26
7	1	18	11	13	2	2	27
8	2	2	10	14	8	5	28
9	2	7	1	15	17	3	29
10	2	11	10	17	8	11	30
11	2	17	1	19	3	10	31
12	3	2	9	21	2	3	32
13	3	9	0	23	4	6	33
14	3	15	11	25	10	11	34
15	4	3	6	28	2	0	35
16	4	11	10	30	18	2	36
17	5	1	1	34	0	0	37
18	5	11	2	37	8	1	38
19	6	2	3	41	2	10	39
20	6	14	6	45	5	[illegible]	40

By the former Table, if you deſire to know what 1 li. commeth to with *intereſt*, and *intereſt* upon *intereſt* after 10 in the 100. for any *number* of yeares unto 40. Looke in the *rowe or margent* (over which is written *yeeres*) and againſt it on the right hand cloſe vnto it in the *rowe or Margent of pounds, ſhillings, and pence*, (which is titled thus, li.s.d. you ſhall finde your deſire.

Example.

I would know what 1. li. *with intereſt, and intereſt upon intereſt commeth to in* 7. *yeares*?

I looke in the rowe of *yeeres* for the *number* 7. And againſt it on the right hand I finde 1 li. 18 s. 11 d. Alſo what it commeth unto in 13. yeares. I ſeeke among the yeares for 13. and againſt it I finde 3 li.--9 s. Againe for 21. yeares. I looke for 21. among the yeares and I finde 7 li. 8 s. 0 d. But if you would know for a greater ſumme then 1 li. Then multiply your ſumme by that ſumme of 1 li. in the Table for any of thoſe yeeres, and you ſhall eaſily finde it. As thus, I would know what 10 li. commeth to for 7 yeares with intereſt, &c. I ſee that 1 li commeth to 1 l. 18 s. 11 d. in that time. Then ſay I that 10 li. muſt bee 10 times as much in that ſpace, which is 19 li. 9 s. 2 d. Alſo of 10 li. in 13. yeeres. I ſee that 1 li. in that time commeth unto 3 li. 9 s. Then muſt 10 li. be 10 times as much in that ſpace, which is 34 li. 10 s. Alſo what 10 li.

commeth

100 Yards of Cordroy to worth 17
yards 1 Calm brick
The Cambrick 11 yard

commeth to in 21 yeares. I finde firſt that 1 li. in that ſpace commeth to in 7 li. 8 ſ. Then I ſay 10 muſt bee 10 times as much, which is 74 li. Laſtly, I would know what 100 li. commeth to in 7 yeares, I ſee it muſt bee 100 times as much as 1 li. commeth to in that ſpace, which is 194 li. 11 s. 8 d. Hereby you ſee the common ſaying is not true, that 100 li. doth double it ſelfe in 7 yeares, for it wants thereof 5 li. 8. s. 4 d. But in 8 yeares 100 li. commeth to 210 li. 8 s. 4 d. Which you ſee is more then double it ſelfe by 10 li. 8 s. 4 d. And in this ſort may any that can but caſt with Counters, or indeed by memory finde the increaſe of any ſumme whatſoeuer for any of the number of yeares in the foreſaid Table, after they haue found what 1 li. commeth vnto for that time, as before is ſpecifyed.

If a Dozen of Ounces be worth half so many Crowns how many Drams can I have for 20 Pounds

30 — 12 — 22/400
16
192
400
76800
Ans. 2560 Drams

A Table shewing if 1 li. *annuity to endure for any number of years, vnder* 41, *be all respited or forborn, vntill the last payment grow due, and then all be receiued together: with interest, and interest vpon interest after* 10 *in the* 100 per annum, *what they wil amount vnto by any of the said number of yeares. As followeth.*

yeeres	li	s	d	li	s	d	yeeres.
1	1	0	0	64	0	0	21
2	2	2	0	71	8	0	22
3	3	6	2	79	10	10	23
4	4	12	10	88	9	11	24
5	6	2	1	98	6	11	25
6	7	14	3	109	3	7	26
7	9	9	8	121	1	11	27
8	11	8	8	134	4	2	28
9	13	11	7	148	12	7	29
10	15	18	8	164	9	10	30
11	18	10	7	181	18	10	31
12	21	7	8	201	2	9	32
13	24	10	5	221	5	0	33
14	27	19	5	245	9	6	34
15	31	15	9	271	0	5	35
16	35	18	11	299	2	6	36
17	40	10	10	330	0	9	37
18	45	11	11	364	0	10	38
19	51	3	2	401	8	11	39
20	57	[illegible]	6	442	11	10	40

By this Table you may know what any Annuity being respited or forborne for any number of yeares unto 41. with interest upon interest, after 10 in the 100 will come unto: first seeking in the Table what 1 li. will come unto: in that time, and that being found to multiply it by the summe you desire to know.

Example.

First, I would know what 1 li. *Annuity being forborne or respited for* 14 *yeares commeth unto.*

I looke in this last Table (which is for purpose, and I finde 27 li. 19 s. 5 d.

Againe, what 1 li. *Annuity* respited for 21: *yeares* commeth to, I looke in the said Table for 21 *yeares*, and I finde 64 li. Also the like for 1 li. for 30 *yeares* respited. I looke, and I finde it to bee 164 li. 9 s. 10 d. as by the said Table may appeare. Now for greater *Annuities*, as 30 li. *per annum*, respited or forborne, what it amounteth to in 16 *yeares*. I seeke first for 1 li. in this last *Table* before for 16 *yeares*, and against it I finde 35 li. 18 s. 11 d. Then say I, that 30 li. *per annum* being respited for that time, will come to 30 times as much, which is 1078 li. 7 s. 6 d. Also if there be an *Annuity* of 45 li. due and unpayed for 12 *yeares*, I looke in the said Table what 1 li. commeth to, 12 yeares being respited, and I finde it is 21 li. 7 s. 8 d. Then I conclude that

5 li.

5 li. must bee 45 times as much, which is 962 li. 5 s.

Lastly, *I haue an Annuity of* 50 li. *per annum, which hath beene behinde for 16 yeares, and must be answered vnto me with interest, and interest upon interest, all at one payment, what shall or ought I to receiue in all at the 16 yeares end?*

I seeke what 1 li. comes vnto in that time (as before taught) and I finde 35 li. 18 s. 11 d. Then must my 50 li. *per annum* forborne for that time, come to 50 times as much, which is 1797 li. 6 s. 10 d. And thus may you finde any other *summe* great or small, for any *number* of *yeares* contained in the foresaid *Table*, without the helpe of *Arithmeticke*, if you can but vse your *Counters*, or by memory count well.

Joan Strong
her Booke

A

A Table shewing if 1 l. *Annuity (to indure for any number of yeares vnto* 41) *bee to bee sold for present ready money, how much ought that ready money to bee, reckoning* 10. per 100. per annum *abating interest, and interest upon interest*, *As followeth.*

yeeres.	l	s	d	l	s	d	yeeres.
1	0	18	2	8	12	11	21
2	1	14	8	8	15	5	22
3	2	9	8	8	17	7	23
4	3	3	4	8	19	8	24
5	3	15	9	9	1	6	25
6	4	7	1	9	3		26
7	4	18	4	9	4	8	27
8	5	6	8	9	6	1	28
9	5	15	2	9	7	4	29
10	6	2	10	9	8	6	30
11	6	9	9	9	9	7	31
12	6	16	3	9	10	6	32
13	7	2	0	9	11	4	33
14	7	7	4	9	12	2	34
15	7	12	1	9	12	10	35
16	7	16	5	9	13	6	36
17	8	0	5	9	14	1	37
18	8	4	0	9	14	7	38
19	8	7	3	9	15	1	39
20	8	10	3	9	15	6	40

This Table before laſt ſpecifyed is very neceſſary and commodious for all *Gentlemen* or others, that ſhall have cauſe to buy or ſell *Annuities* or ſuch like, for by this they ſhall know what they doe, whether they demand, or take too little or too much, after the rate of 10. in the 100, by which proportion all theſe *Tables* are ruled,

As for example, I am to buy an Annuity of 16 li. per annum *for* 12 *yeares, and am demanded for it in ready money* 120 li. *I would know, if I giue this rate, whether I giue too much or too little, according to the proportion of* 10 *in the* 100 *per annum, &c.*

I looke in the *Table* laſt before what 1 li. is worth for 12 *yeeres*, and I finde againſt 12 this *ſumme* 6 li. 16 s. 3 d. Now I ſay that 16 li. *Annuity* for that time, and after that proportion commeth to 16 times as much, which is 100 li. So that I ſee the party demaunded of mee 11 li. too much after the rate of 10 in the 100 *per annum.* and therefore I muſt draw him to a lower price, or leaue it.

Againe, I am offered an Annuity of 20 li. *per annum of* 14 *yeares for* 130 li. *I would know if I giue it, whether I giue too much or too little, according to the proportion aforeſaid.*

I ſeeke firſt what 1 li. *Annuity* is worth for 14 *yeares*, and I finde in the ſaid laſt *Table* 7 li. 7 s. 4 d. Then ſay that the *Annuity* of 20 li. *per annum*, will come to 20 times as much, and will be worth 147. li. 6 s. 8 d. according to the

the proportion before mentioned : and is more then his demand by 17 li. 6 s. 8 d. So that I see, if I accept of it, I shall haue a good bargaine. And thus may you know readily by looking in your Table, and finding what 1 li. is worth for any time therein contained, how much any greater summe will come unto, if you multiply it by that summe of 1 li. as before is sufficiently shewed.

But suppose this I have 300 li. ready mony and would bestow the same for a valuable Annuity answerable therunto according to the proportion aforesaid. I would know what Annuity to endure 21 yeares this 300 li. will buy?

I looke in the former Table what 1 li. Annuity will cost for that time, and I finde 8 li. 12 s. 11 d. Then I say by the Rule of Proportion. If 8 li. 12 s. 11 d. will buy 1 li. Annuity for 21 yeares : what Annuity shall 300 li. buy or bee worth for that time. I reduced the *summes* to the least denomination (which is pence) and I finde 34 li. 10 s. 9 d. And after this manner (by the helpe of this *rule*) may you find all other summes for any time contained in the foresaide last *Table.*

A Table shewing what 1 *li. in reversion for any number of yeares under* 31 *is worth in ready money, the buyer staying untill the thing be fallen in hand.*

yeares	li.	s	d.	li.	s.	d.	yeares.
1	0	18	2	0	4	4	16
2	0	16	6	0	3	11	17
3	0	15	0	0	3	7	18
4	0	13	7	0	3	3	19
5	0	12	5	0	2	11	20
6	0	11	3	0	2	8	21
7	0	10	3	0	2	5	22
8	0	9	3	0	2	2	23
9	0	8	5	0	2	0	24
10	0	7	8	0	1	10	25
11	0	7	0	0	1	8	26
12	0	6	4	0	1	6	27
13	0	5	9	0	1	4	28
14	0	5	3	0	1	3	29
15	0	4	9	0	1	1	30

If [illegible] Groats [illegible] buy 10 yeares [illegible]
How many Groats [illegible]
for 20 yeares [illegible]

This last *Table* differeth, and is contrary to the other three before mentioned: For whereas the others increased more and more according to the *number of yeares* specified. This doth grow and diminish lesse and lesse, as the *number of yeares* increaseth. As for *example.*

There is a Tenement, the fee simple whereof after 7 yeares will be worth 40 li. *what am I to give for it in ready money, now staying untill it fall in hand.*

To know this I looke in this last Table for 7 *yeares*, and against it I finde 10 s. 3 d. So that a thing that after 7 *yeares* will bee worth 1 li. is worth now in ready mony but 10 s. 3 d. Then say I that the foresaid Tenement (which after 7 *yeares* will be worth 40 li.) is now worth 40 times 10 s. 3 d. which is 20 li. 10 s.

Againe, there is a Farme which after 9 yeares will be worth the Fee-simple 420 li. *what is it now worth in ready money staying untill it fall in hand.*

I looke in the said Table what 1 li. is worth in reversion after 9 *yeares*, and I finde 8 s 5 d. Then say I, that the *Farme* of 420 li. so long in reversion, will bee now worth in ready money, 420 times as much, which is 176 li. 15 s.

Lastly, there is a Lordship to be sold, the Fee-simple whereof after 14 *yeares will bee worth* 7500 li. *I would know what the same is now worth in ready money for the Reversion.*

I looke in this last *Table* for 14 yeares, and against it I finde 5 s. 3 d. so much 1 li. is worth in

in reversion after 14 yeares. Then say I, that 7500 li. is worth no more in reversion for that time then 7500 times 5 s. 3 d. which is 1968 li. 15 s. And after this manner may you finde out any other summe whatsoever. And though some men of their owne experience can ayme (as they thinke) neare enough the marke to serve their owne turnes: yet I dare undertake they shall never so exactly doe it nor justifie what they doe, as if they did it by Art.

FINIS.

New *Tables* of *Interest* at 8 *per centum, per annum*, exactly calculated for 30 yeares by *Robert Hartwell*, with necessary *questions* for the use of them.

The first Table expressing the increase of one pound principall, put out and forborne for any number of yeares under 31, at 8 per centum, per annum,

yeeres.	li	s	d	q	li.	s	d	q	yeeres.
1	1	1	7	0	3	8	6	0	16
2	1	3	3	3	3	14	0	0	17
3	1	5	2	1	3	19	11	0	18
4	1	7	2	2	4	6	3	3	19
5	1	9	4	2	4	13	2	2	20
6	1	11	8	3	5	0	8	0	21
7	1	14	3	1	5	8	8	3	22
8	1	17	0	0	5	17	5	0	23
9	1	19	11	3	6	6	9	3	24
10	2	3	2	0	6	16	11	2	25
11	2	6	7	2	7	7	11	0	26
12	2	10	4	1	7	19	9	0	27
13	2	14	4	2	8	12	6	2	28
14	2	18	8	3	9	6	4	0	29
15	3	3	5	1	10	1	3	0	30

The deſcription and uſe of the Tables of Intereſt at 8 per 100. per annum, being profitable.

The firſt of them.

THeſe Tables conſiſt of foure *Columnes*, in the firſt and fourth whereof is written ouer the head, yeares, and under the firſt number of yeares deſcending from 1 to 15. likewiſe in the fourth the number of yeares deſcending from 16 to 30. And againſt every yeare in the ſecond Columne toward the right hand, the pounds, ſhillings, pence and farthings, which one pound, or 20 s. principall will amount unto, being put forth and forborne for the number of yeares ſet againſt it; (but the pounds, ſhillings, pence, &c. in the third Columne belongeth to the yeares ſet in the laſt Columne.

9 JY 64

1 Exam-

1 Example.

Let it be required what one pound or 20 ſhillings, being put forth and forborne for 12 yeares ariſeth to at 8 per 100, per annum, *intereſt upon intereſt.*

Seeke in the firſt Columne under the title of yeares, for 12 the number of yeares propoſed in the queſtion, & right againſt it toward the right hand in the ſecond Colume, you ſhall finde 2 li. —— 10 —— 4 d —— 1 q, which is the principall and increaſe thereof due for the time required.

2 Example.

If 100 li. *be put forth for* 17 *yeares according to the ſame intereſt, I demand what it will amount to in that time?*

Looke in the Columne under the title of yeares for 17, and right againſt it towards the left hand in the Table is found 3 l. —— 14 ſ —— o d —— o q, which is the encreaſe of 1. li.

 by

by which you may thus gather the increaſe of 100 li. or any other *ſumme*; a hundreth times 3 li. is 300 li. then 100 times 14 *ſhillings* is 70 li. both which added together doe make 270 li ——08——0 d. which is the *increaſe* of 100 li. put forth and forborne 17 *yeares* the ſolution to the queſtion.

li	ſ	d	q
300	0	0	0
70	0	0	0
370	0	0	0

3 Example.

Suppoſe 60 li. bee put forth for 19 yeares according to that rate, what will it increaſe to in that time?

Seeke 19 vnder the *title* of *yeares*, and againſt it toward the left hand is found 4 li——6 s---3 d-3 q now ſay 60 times 4 li. is 240, and 60 times 6 *ſhillings*, is 360 *ſhillings*, or 18 li. and 60 times 3 d. is 180 d. or 15 *ſhillings*, and 60 times 3 *farthings* is 3 *ſhillings* 9 d. all which added together make 258 li. 18 s. 9 d. the increaſe thereof demanded.

li	ſ	d	q
240	0	0	0
18	0	0	0
0	15	0	0
	3	9	0
258	18	9	0

The

The second Table shewing what one pound Annuity or yearly rent is worth at the end of any number of yeares under 31, being forborne, at 8 per centum, per annum.

yeeres.	*li*	*s*	*d*	*q*	*li.*	*s*	*d*	*q*	*yeeres.*
1	1	0	0	0	30	6	5	3	16
2	2	1	7	0	33	15	0	0	17
3	3	4	1	0	37	9	0	0	18
4	4	10	1	1	41	8	11	0	19
5	5	17	3	3	45	15	2	3	20
6	7	6	8	0	50	8	5	2	21
7	8	18	5	1	55	9	1	2	22
8	10	12	8	3	60	17	10	1	23
9	12	9	9	0	66	15	3	2	24
10	14	9	8	3	73	2	1	1	25
11	16	12	10	3	79	19	1	0	26
12	18	19	6	2	87	7	0	0	27
13	21	9	10	3	95	6	1	2	28
14	24	4	3	2	113	19	3	3	29
15	27	3	0	2	11	5	7	3	30

The uſe of the ſecond Table (whoſe diſpoſition is altogether like the former) according to the title thereof, being profitable

1. Example.

There is a Leaſe worth 28 li. per annum. *to endure* 14 *yeares, I demand what it will riſe vnto at the end of thoſe yeares, being all forborne with the intereſt, and intereſt upon intereſt at the rate preſcribed in this Table.*

Look in the third Table for 14 yeares, againſt which toward the right hand, you ſhall finde 24 li——4 ſ——3 d——2 q. Now multiply 28 li. by 24 there ariſeth 672 li. then 28 li. by 4 ſ. yeeldeth 112 ſ. or 5 l-2 ſ. Againe 28 l. by 3 d. produceth 84 d or 7 ſ. finally, 28 by 2 farthings yeeldeth 56 farthings or 1 ſ---2 d.

li	ſ	d	q
672	0	0	0
5	12	0	0
	7	0	0
	1	2	0
678	0	2	0

All which added together make 678 li——0 ſ——2 d. to be received at the end of 14 yeares, the ſame rent or annuity being reſpited.

2 Exam-

2 Example.

If 60 li. yearely rent or annuity be forborne 20 yeares: I demand how much it will increase at the end of the said terme?

In the Table I finde that 1 pound in 20 years will arise to 45 li —— 15 s —— 2 d —— 3 q therefore 60 li. in the like terme will yeeld 60 times as much; which I will reckon thus: 60 times 45 li. is 2700 li. 60 times 15 s. is 900 s. or 45 l. 60 times 2 d. is 120 d. or 10 s. last of all, 60 times 3 q is 180 farthings, or 3 s. —— 2 d. all which together amount unto 2745 li —— 13 s —— 9 d the value thereof to bee received at the end of the terme.

li	s	d	q
2700	0	0	0
45	0	0	0
	10	0	0
	3	9	0
2745	13	9	0

3 Example.

The yearely rent of li --- 13 s --- 4 d. *being behind and unpaid the space of 7 years at the end of which terme the Tenant is compelled to pay the same with the interest thereof according to the above named rate. I demand what the payment ought to be?*

The

The increaſe of 1 li. *yearely rent* anſwering to 7 *ye res*, is 8 li -- 18 ſ -- 5 d. -- 1 q. which for 6 li. *rent* is to bee taken 6 times, which ariſeth to 53 li -- 10 s -- 7 d -- 1 q, now becauſe 13 s -- 4 d is two third parts of 1 li. therefore I take $\frac{2}{3}$ of 8 li -- 18 ſ -- 5 d -- 1 q, which is the increaſe of 1 li. forborne for 7 *yeares*, that is 5 li -- 18 s -- 11 d -- 2 q, which together make 59 li -- 9 s -- 6 d -- 3 q, the *ſumme* to bee receiued, as was required.

li	ſ	d	q
35	10	7	q
5	18	11	2
59	9	6	3

The

The third Table declaring what one pound due at the end of any number of yeares vnder 31 is worth ready mony at 8 per centum, per annum.

yeeres	li	ſ	d	q	li	ſ	d	q	yeares.
1	0	18	7	0	[illegible]		10	0	16
2	0	17	10	3	0	5	4	3	17
3	0	15	1	2	0	5	0	0	18
4	0	14	8	1	0	4	7	2	19
5	0	13	7	1	0	4	3	1	20
6	0	12	7	[illegible]	0	3	11	2	21
7	0	11	8		0	3	8	0	22
8	0	10	9	2	0	3	4	3	23
9	0	10	0	0	0	3	1	3	24
10	0	9	3	0	0	2	11	0	25
11	0	8	6	[illegible]	0	2	8	1	26
12	0	7	11	1	0	2	6	0	27
13	0	7	4	0	0	2	3	[illegible]	28
14	0	6	9	2	0	2	1	3	29
15	0	6	2		0	1	11	3	30

This third Table is diſpoſed as the firſt, the vſe according to the Title thereof, being damageable.

1 Example.

Suppoſe there is 750 li. *due to bee payed at the end of 9 yeares, the Creditour would ſell this debt for preſent money, what ought that money to bee at the rate deſcribed in the table.*

Seeke in this third Table for 9 yeares at the left ſide of the Table, and right againſt it toward the right hand, you ſhall finde 10 ſhillings, which multiplied or taken 750 times, yeeldeth 7500 ſhillings, which is 375 li. the value of that debt in preſent money.

2 Example.

There is a Leaſe worth 500 li. *after the end of the* 7 *yeares; what is it worth preſent money, according to the rate deſcribed in the table ſtaying till it fall.*

I ſeeke in the *Table* for the 7 *yeare*, and right againſt it I finde 11 ſ——8 d; now I multiply 5500 by 11, it yeeldeth 5500 *ſhillings*, or 275. li. then 500 times 8 d. maketh 4000 d, which is 16 li——13 ſ——4 d, which added together is 291 li.----13 ſ——4 d. the value of the Leaſe to be payed before it fall in hand.

li	ſ	d	o
275	0	0	0
16	13	4	0
221	13	4	0

The

The fourth Table *expressing what one pound yearely rent or annuity for any number of yeares not exceeding* 30 *is worth ready mony at* 8 per cent. per annum.

yeeres.	li.	ſ	d	q	li.	ſ	d	q	yeeres.
1	0	18	6	0	8	17	0	1	16
2	1	15	7	3	9	2	5	0	17
3	2	11	6	2	9	7	5	1	18
4	3	6	2	3	9	12	0	3	19
5	3	19	10	1	10	16	4	1	20
6	4	12	5	1	10	0	4	0	21
7	5	4	1	2	10	4	0	0	22
8	5	14	11	0	10	7	5	0	23
9	6	4	11	[illegible]	10	10	6	3	24
10	6	14	2	1	10	13	5	3	25
11	7	2	9	1	10	19	2	1	26
12	7	10	8	2	10	18	8	1	27
13	7	18	0	3	11	1	0	0	28
14	8	4	10	2	11	3	2	0	29
15	8	11	2	1	11	5	1	3	30

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THE GROVND OF ARTS;

TEACHING THE PERFECT worke and practise of Arithmeticke, both in whole Numbers and Fractions, after a more easie and exact forme then in former time hath beene set forth: Made by M. ROBERT RECORD D. *in Phisick.*

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Scientia non habet inimicum, nisi ignorantem.

Fide ——— sed ——— ——— vide.

LONDON:

Printed by THO. HARPER, for *Iohn Harison*, and are to be sold at his shop in Pater noster Row, at the signe of the Vnicorne 1636.

That which my friend hath well begun,
For very loue to Common weale,
Need not all whole to be new done,
But new increase I do reueale.

Something herein I once redrest,
And now againe for thy behoofe,
Of zeale I do, and at request,
Both mend and adde, fit for all proofe.

Of numbers vse, the endlesse might,
No wit nor language can expresse,
Apply and try both day and night,
And then this truth thou wilt confesse.

I. Dee.

The Books Verdict.

To please or displease sure I am,
But not of one sort to euery man:
To please the best sort would I faine,
The froward displease shall I certaine.
Yet wish I will, though not with hope,
All eares or mouthes to please or stop.

TO THE MOST mighty Prince *Edward* the ſixth, by the grace of God King of *England, France*, and *Ireland*, *&c.*

THe excellency of mans nature being ſuch, as it is by Gods diuine fauour (moſt mighty Prince) not only created in highneſſe of degree farre aboue all other corporall things, but by perfection, reaſon and ſearch of wit, much approaching toward the Image of God, as not onely the holy Scriptures do teſtifie, but alſo thoſe naturall Philoſophers, which exactly did conſider the nature of man, and namely the far reach and infinite compaſſe of the works of the minde, were inforced to confeſſe, that man ſcarſely was able to know himſelfe. And if he would duely ponder the nature of himſelfe, he would find it ſo ſtrange, that it might ſeeme vnto him a very miracle. And thereof ſprang that ſaying: *Magnum miraculum eſt homo, maximum miraculum ſapiens homo.* For vndoubtedly, as man is one of the greateſt miracles that euer God wrought, ſo a wiſe man is plainly the greateſt.

And therefore was it that ſome did account the head of a man the greateſt miracle in the world, becauſe not onely of the ſtrange workmanſhip that is in it, but much more of the efficacie of reaſon, wit, memory, imagination, and ſuch other powers, and works of the minde, which can more eaſily conceiue any thing in a manner, then vnderſtand it ſelfe. Amongſt all the creatures of God, it findeth none more difficult to be perceiued then theſe ſame powers of it ſelfe; whereby it doth conceiue and iudge: as it may be well coniectured by the diuerſity of opinions, that the wiſeſt Philoſophers did vtter touching the ſpirit of man, & the ſubſtance of it: whereof, I now intend to make no rehearſall; but whoſo liſteth to

number of the places) by 4, thereof commeth 24, which I adde to 5, that maketh 29, and that is the last number which I desire to know. And this you may straightway proue by continuall procéeding from 5, till the seuenth place, encreasing euery one by 4, as thus.

5 9 13 17 21 25 29.

Lo here, the last, being also the seuenth, is 29.

Scholar. I perceiue already one good property in this Rule, which in all works is to be desired: that is, it will ease one from great labour, if a Progression were propounded of a hundred or two hundred places, or moe: And also it is very easie to worke, and most necessary for the totall summe finding, in a very long Progression.

Master. It is true, and therefore now let me sée if you can answer me this question by this proposition.

A Merchant buyeth 50 pounds of Spices, & agreeth to pay for the first pound 4 pence, for the second 7 pence, for the third 10 pence, for the fourth 13 pence, &c.

The question is, how much he must pay for the last pound, and then how much the 50 pound commeth to?

Scholar. According to the proposition, I multiply 49 (which is lesse by one then the number of the places) by the excesse, which is

3,

3, to the product 147, I adde the first number which is 4 it maketh 151 pence, the price of the last pound. Now I adde 4, the price of the first pound, to 151 the price of the last pound, it maketh 155, which I multiply by halfe the number of the places, which is 25 the product 3875 pence is the totall summe or price of the 50 pounds of Spices, as appeareth.

49	Places 1 lesse.	151	last
3	excesse	4	first
147		155	
4	first	25	halfe places
151	the last	775	
		310	
		3875	totall summe

which amounteth to

li ſ d

16——2——11

Master. It is truly wrought.

Scholar. Then I intreat you to proceed to your second proposition.

Master. *The second Rule is this. From the last subtract the first, the remainder diuide by the common excesse, to the* Quotient *adde* 1, *and you haue the number of the places, which you would know: As in this Progression.* 2. Proposition.

6 11 16 21 26 31

If I know onely 6 and 31, and that they

encrease

encreaſe by 5, then according to the rule, from 31 I ſubtract 6, there remaineth 25 : which 25 I diuide by 5, (the common exceſſe) the Quotient commeth forth 5, to which I adde 1 that maketh 6: and ſo many are the places, as you ſee.

Scholar. This Rule is ſo eaſie, that I were much to blame, if I could not remember it.

3 Propoſition.

Maſter. *The third Propoſition may alwayes thus be ſolued. Multiply the* exceſſe *by a number leſſe by one, then the diſtance of the place is from the* firſt, *or the* laſt number giuen: *the of-come adde to the* firſt, *if the* diſtance *be reckoned from the* firſt, *and the* firſt *alſo knowne, or ſubtract from the* laſt, *if the diſtance be from the* laſt counted, *and the* laſt giuen *alſo, and that which commeth forth, either in that* Addition to the firſt, *or* ſubtraction from the laſt *is the number* ſought. As for example, *I propound you this* Progreſsion

8 15 22 29 36 43 50 57

And for the apt conſidering the manner of this queſtion, I will note ouer euery place his diſtance from the firſt, and under euery place his diſtance incluſiuely from the laſt, thus.

1	2	3	4	5	6	7	8
8	15	22	29	36	43	50	57
8	7	6	5	4	3	2	1

Now if the exceſſe whereby this Progreſ-
ſion

sion standeth, be knowne to be 7, and the first numbers giuen, being 8, I would know what number standeth vnder 4 that is to say, in the fourth place. I multiply 7 by 3 (which is lesse by 1 then the number of the place proponed) that yeeldeth 21, to which I adde 8, the first number so commeth 29 : which I say to belong to the fourth place, as ye see in the example it also doth: or if in the third place from the last, you would know what number in this example should stand, the last number being knowne to be 57, and the common excesse 7, then by 2 which is lesse by 1 than the place propounded, I multiply 7, that giueth 14, which appertaineth to the third place inclusiuely reckoned from the last, and so my example giueth you.

Scholar. I perceiue right good vse of this rule: for if I had forgotten what the first number were, and remember still but the last, the common excesse, & the number of the places, then might I come by the knowledge of my first number againe.

And me thinketh, that it differeth not much from the first proposition, sauing that which you make here a middle number, there was made the last: and also in this point it differeth, that in it the last was onely sought, and no consideration had in numbring the places from the last, as here I marke in your numbers noted vnder your Progression.

Master. And thinke you not, the middle

numbers of progression standing of a hundred or three hundred places or moe, may as much cumber a man to come to the knowledge of them by continuall encreasing from the first by the common excesse, or abating from the last continually the common excesse as the very small numbers in a shorter progression would doe?

Scholar. Yes sir, that I think right well, & therefore I am glad of this new framed proposition, and the manner of the working of it.

4 Proposition. Master. *The rule of the fourth is this. Adde the first and the last together, and by the of-come diuide the totall summe. Double the quotient, and that will be the number of the places.*

Scholar. Then if in a Progression, whose summe were 207, and the first number 12, & the last 57, if I adde 57 and 12 together, that maketh 69, and by it I diuide 207, the quotient will be 3, which I double: and so I haue 6, and so many must be the number of the places that this Progression standeth on.

Master. Whether it be so or no, how wil you trie?

Scholar. Halfe 6, which is 3, being multiplied by 69, must make 207, the totall summe; if 6 be the number of the places. For so the whole works of your rule in summing any Arithmeticall Progression did en-forme me. I will then multiply 69 by 3, thus,

	69
	3
It commeth forth iustly.	207

Master.

Master. I must much herein commend your promptnesse both in memory and in well applying your rule: although in manifest words it did containe no such matter.

Scholar. Sir I pray you heare me frame one example more.

Master. I am well pleased, so that ye bée short, for you make mée more longer here then willingly I would haue béen: but I cannot perceiue how I could haue omitted any thing as yet, without your great lack thereof.

Scholar. If I had receiued 85 pounds of certaine men, but of how many I haue forgotten, yet I remember that the first gaue me 7 pound, and the last 27 pound, and euery payment after other did rise by a like sum. And the man for whom I receiued this mony, conditioned with me, that of euery payment I should haue twelue pence for my labour: now vnlesse I can by Art finde the truth of this case, I am like to lose the most part of my reward. A question of money.

Master. I perceiue you can handsomely frame an example, which should concern your owne gaine: I pray you let me sée how you would do iustice in this point.

Scholar. I adde the first and the last together, that maketh 34: by which I diuide 85, thus:

	1	
	~~2~~7	17
	~~8~~ ~~5~~	(2 —
	~~3~~ ~~4~~	34

Why how now? Sir, here is a remnant of 17, in which 34 cannot bée

 had:

had: ſo that now I am in the bryers for doubling of my Quotient, and farewell then both my Iuſtice, and a good lumpe of my gaines.

Maſter. Ye are neuer the farther from the matter, though it fall into a Fraction. For you ſhall vnderſtand, that the Fraction which of any ſuch worke proceedeth, is euer halfe of one ſuch, as the vnites of the Quotient before are. And that you may trie, if you double that which ſo remaineth, for then it will be equall to your Diuiſor, as if ye double 17 (the remnant) it maketh 34, & your diuiſor alſo was 34 this noteth the remainder to be halfe of one.

Scholar. Now I am glad of this hard example. For with it I haue a generall rule for the Fraction that may hap in this worke. So that the Quotient being two and a halfe, I double that, it maketh 5, therefore ſhould my gains be 5 ſhillings. And to be ſure (by your leaue) I will trie it, for I will multiply halfe of 34: (which is the firſt and laſt number ioyned together) by 5, thus.

$$\begin{array}{r} 17 \\ 5 \\ \hline 85 \end{array}$$

It is moſt true (I ſee) that I ſhould leeſe nothing by the former working.

5 Propoſition.

Maſter. *The fifth proportion hath this rule appertaining vnto it: By the fourth rule finde the number of the places, that being done, from the laſt ſubtract the firſt, & the reſidue diuide by a number leſſe by 1, then the number of the places, and the quotient will ſhew the exceſſe which is ſought for.*

An

An example hereof ſhall be this: If ye had disburſed 685 pounds to a certaine number of men, you neither can tell how many they were, or how much the ones mony exceeded his next before, but you are ſure that the exceſſe was equall between euery two next: and alſo you remember that the firſt had 19, and the laſt 180 pounds, how would you finde the number of the name and the exceſſe, continually obſerued in the ſucceſsion of their payments. Example.

Scholar. Your rule doth plainly bid, first to finde the number of the places, which I will do according to the fourth rule: I adde 19, and 118 together thus.

118
19
137

By this 137, I diuide 685 thus.

Seeing there is no fraction but a whole number, being 5, I double that, and then muſt the number of the places be 10. Now from the laſt I ſubtract the firſt, as 19 from 118, thus: And ſo remaineth 99.

~~1~~ ~~3~~
~~6~~ ~~8~~ ~~5~~ (5
~~1~~ ~~3~~ ~~7~~

118
19
99

This 99 I diuide by a number leſſe by one then the number of the places, and ſeeing the places were 10, I diuide 99 by 9, thus:

The Quotient is 11, and ſo was the exceſſe, if I haue followed

~~9~~ ~~9~~
~~9~~ ~~9~~ (11

your rule right.

Master. You haue wrought euery part of this question both well in order, and truly in the practise of your rules.

Scholar. I will then set it downe also formably, so that the number of the places, the excesse and the totall summe may straight appeare, as your first example stood.

The common excesse. The progression.

11	11	11	11	11	11	11	11	11	11
19	30	41	52	63	74	85	96	107	118

That the places be 10, and that from the first to the last, the common excesse is 11, I perceiue most euidently: but whether the totall summe be 685. I haue not yet proued, which I will now doe: I adde 19 and 118 together, that maketh 137: I multiply that by halfe the number of the places, thus.

137
5
685

All things agrée most exactly, so that I am perfect enough in these rules, if I forget them not againe.

Master. Use maketh all things perfect.

6 Proposition. *Your sixth rule is this. By the number of the places diuide the totall summe, double the quotient, and that will bee the first and last ioyned in one summe. Then by a number lesse by 1, then the number of the places, multiply the excesse, that of-come subtract from the first doubled,*

doubled quotient, and the halfe of the reſidue is the firſt number. The laſt number you may diuerſly finde out, as by the firſt of our ſix rules, or by ſubtracting this firſt number from the ſumme which here contained both the firſt and laſt ioyntly, (or thirdly) by continuall adding the exceſſe.

Scholar. I pray you make this ſomewhat more plaine with an example.

Maſter. If euery moneth in the yeere (counting them now as 13) you gained clearly 40 ſhillings more then you did the moneth next going before, and at the yeeres end you finde the whole gaine 5720 ſhillings, but ye remember not how much either the gaine of the firſt moneth or the laſt was, by this rule it may be tried out.

Example of gaine.

Scholar. So that here ye ſeeme to apply the 13 moneths to thirteene places, the 40 ſhillings euery one more then the other next before it, to be the common exceſſe, and 5720 ſhillings to the totall ſumme.

Maſter. It is true: by 13 then I diuide 5720 in this manner.

~~1~~
~~15~~
~~5720~~ (440
~~1333~~
~~11~~

I double this quotient, ſo haue I 880 for the firſt and the laſt ſumme ioyned together; by 12 which is leſſe by one then the num-

ber of the places; I multiply 40 (the commen excesse) so commeth 480

This 480 I subtract from 480, so remaineth 400: halfe whereof is the first number which wee desired to know: that is 200.

40
12
80
4
480

And as for the last number, I can give you it three wayes. As by the first of my six rules I multiply, the excesse by a number lesse by 1 then the number of the places, as 40 by 12 that giveth 480, which I adde to the first, being 200, so shall the last be 680.

The same summe commeth forth, if ye subtract 200 from 880.

And thirdly, If I begin at 200, and so proceed, encreasing by 40, I shall at the thirteenth place have 680, as thus:

200	240	280	320	360	400	440
480	520	560	6 0	640	680	

Scholar. I thanke you most heartily for these sixe rules. Now if it be your pleasure I would heare and learne somewhat of Progression Geometricall.

Master. There are yet very many rules and propositions, which fall into this Arithmeticall Progression.

And for the use and practise of them, I will propone unto you certaine pleasant and necessary questions of Arithmeticall Progression, and to the performance of their workings,

such

ſuch neceſſary rules and documents, as are requiſite for the better vnderſtanding of them, or any ſuch like.

A certaine Mercer ſold 20 yards of Veluet to be paid in 12 weekes, by Arithmeticall proportion: that is to wit, to receiue the firſt weeke 6 ſhillings, the ſecond weeke 12 ſhillings, the third weeke 18 and ſo forth, encreaſing the number of weeks by 6 ſhillings, till the twelfth and laſt weeke were expired. The queſtion is how many pounds hée had for 20 yards of veluet.

A queſtion of Veluet.

6
12
18
24
30
36
42
48
54
60
66
72

To the performance of this queſtion, and ſuch other the like, I ſet forth the 12 payments in ſuch ſort, as for exãple, hére appeareth.

Then touching the adding together of theſe ſummes, without the aid of Addition, according to the rules I taught you in Progreſsion Arithmeticall, I note the number of the places, which are 12, then adding the laſt number of the Progreſsion, which is 72, and the firſt number together, make 78; and multiplying 78 by halfe the number of the places, which is 6, amounteth to 468 ſhillings, and in pounds maketh 23 pounds 8 ſhillings. And ſo much hath the Mercer for his 20 yards of Veluet, which is nigh about 23 ſhillings, 5 pence, a yard.

Scholar. I vnderſtand this worke very well: but is there any proofe for the iuſtifying hereof,

hereof,as you haue of other works?

Master. The works of it selfe (being so perfectly wrought) that in your proceeding and going forward from number to number, each number exceeding his fellow by an equall or like quantity, is all that is demanded for iustifying of the same : yet notwithstanding, because your request is reasonable, I wil propone an example for the proofe hereof.

The proof of the last question.

A certaine man is bound to pay for 20 *yards of veluet,the summe of* 23 *pound* 8 *shillings, and it is to be paid weekely, in* 12 *weekes or termes by Arithmeticall Progression.* The question is therefore to know with what number the same Progression is to be begun and continued in such equall proportion Arithmeticall, that in 12 weekes the same may iustly be accomplished.

For the resolution whereof, and of all such other like,reduce 23 pound 8 shillings,all into shillings,which maketh 468 shillings.

A generall rule.

Then adde 1 vnto 12, the number of the termes,it maketh 13,which 13 you shall multiply by halfe the number of the termes,which is 6.it maketh 78;then diuide 468 by 78,and you shall finde 6 in the quotient, which is the true number that shall begin and continue the said Progression. That is to say, the first weeke 6 shillings,the second 12 shillings,and the third week 6 shillings more, which is 18 shillings, and so euery weeke as they rise, 6 shillings

ſhillings more then the weeke before, as is manifeſt in the queſtion aforeſaid.

A Farme is to be ſold to be payed by the weekes in a yeare, the firſt weeke to pay 4 *ſhillings, the ſecond weeke* 8 *ſhillings, the third weeke* 12 *ſhillings and ſo forth, increaſing each number by* 4, *till the number of* 52 (*which are the number of weeks in a yeare be expired.*) The queſtion is, what the price of the Farme commeth to? A queſtion of a Farme

Scholar. I doubt not, but by that you haue already taught me, to end this queſtion very well; wherefore I ſet forth the Progreſſion with his exceſſe 52 times.

Maſter. Nay ſtay a while: And here for your further eaſe (to abridge you of great labour that appeareth to fall out in this queſtion, and ſo may do in any other the like) if a queſtion were proponed of 100 or 200 places or moe, and that this queſtion, nor any other the like can be ended, vnleſſe you know abſolutely what the laſt number of the Progreſſion at the 52 place is, or ought to bée) I will giue you a generall rule how to know the laſt number of any Progreſſion Arithmeticall, as well as if you had ordinarily procéeded by continuall Addition, till you had come to the laſt worke, which is this.

Multiply the exceſſe by a number leſſe by one then the number of the places, and thereto put the firſt number of the Progreſſion, and you ſhall haue your deſire. A generall rule.

Scholar.

Scholar. This rule is well worth the noting: for if I vnderſtand you aright, I conſider that my exceſſe is 4, which I multiply by 51, which is one leſſe then the number of the places, and it maketh 204, whereunto I adde the firſt number of the Progreſſion, which is 4, and then it is 208, which you ſay is, or ſhould be the laſt number of the Progreſſion.

Maſter. This is a moſt approoued truth, if there were neuer ſo many places.

Scholar. This rule is ſo eaſie, that I were much to blame, if I do not remember it. For by the benefit hereof, I haue ſuch an eaſe and light into this excellent Art, that my firſt entrance doth ſeeme to paſſe a great many mens further ſtudy, and longer continuance.

Maſter. Many moe conſiderations could I propound you in theſe *Arithmeticall Progreſſions*; but theſe are ſufficient for a taſte, to giue you occaſion to thinke that *Rules* of knowledge and *Arts*, are infinite capable of enlargement.

Scholar. Happy were I, if I did but well vnderſtand that which is already inuented and written. But theſe things, in my ſimple fantaſie, offer themſelues to be greatly beneficiall vnto the aide of Progreſsion. Therefore now I will go forward with your queſtion.

Now conſidering that the 52 and laſt place is 208, I adde thereunto the firſt

number

number of the Progression, which is 4, it maketh 212, which I multiply by halfe the number of the places, which is 26, and it amounteth to 5512 shillings. And so much is the totall summe or addition of this Progression: which maketh 275 pounds, 12 shillings, as appeareth here by my Tables.

Master. I like well your labour, and commend you for your diligence; I will here propone one example more, and therewithall for this time will end Progression Arithmeticall.

A certaine man bought 20 Ells of Holland, to be paid in 17 weekes or termes by Progression Arithmetical. And the first weeke to pay 1 shilling 8 pence, the second weeke 3 shillings, 4 pence, the third weeke 5 shillings, the fourth weeke 6 shillings 8 pence, and so forth, each weeke succeeding 20 pence more then the weeke before. The question is, what the summe of his 20 Elles commeth to? A question of Holland

Scholar. Because here is mention made both of shillings and pence, I feare there is some harder matter contained herein, then in the other before: therefore I pray you worke it your selfe, and I will diligently mark your labour.

Master. There is no more to be done in this, then in the other before; but because your request is so reasonable, be attentive unto me.

First by the generall Rules I seeke to finde out the last number of the 17 place, what

this

this Progreſſion ought to bée. Therefore here in my Tables multiplying the exceſſe 20 by 16, which is one leſſe then the number of the termes or places, and it commeth to be 320; and thereunto adding the firſt number of the Progreſſion, which is 20 pence, all is 340 pence, or 28 ſhillings 4 pence: for ſo much ought the laſt number of the payments to be.

Then finally, to know what the whole 17 places amount unto, I adde the firſt number of the Progreſſion and the laſt together, which make 360. Now becauſe 17 is an odde number, whoſe halfe cannot be taken, I take the halfe of 360, which is 180, and multiplying 180 by 17, commeth to 3060 pence, which maketh as you ſée by Diuiſion 12 pound, 15 ſhillings. And ſo much is the buyer to pay for his 20 Elles of Holland. Which 3060 pence if you diuide by 20, the number of Elles that was bought, you ſhall finde 12 ſhillings 9 pence, and ſo much payed he for an Elle one with another.

The proofe.

A queſtion of debt.

A certaine man doth owe 12 pound, 15 ſhil. to bee payed in 27 weekes or termes Arithmeticall Progreſſion. The queſtion is, to know with what number he ſhall begin and continue the Progreſſion in ſuch equall proportion, as the ſame may be truly payed

paid and ſatisfied in 17 *weeks.*

The anſwer.

First I reduce 12 pounds 15 ſhillings, all into pence, which as you ſee here in many Tables, make 3060 pence, that I let ſtand by a while.

Then I adde 1 to 17, the number of the places or terms, which maketh 18, which I ſhould multiply by halfe the number of the weeks or termes, which is 8 $\frac{1}{2}$ which 8 $\frac{1}{2}$ multiplied by 18 cannot well be done, vnleſſe you were acquainted with Fractions or broken numbers, therefore you ſhall let that paſſe and multiply 17. by the halfe of 18: which is 9, (for that is all one with the Multiplication of 8 $\frac{1}{2}$:and the Multiplication of 9 into 17 maketh as you ſee 153, with which number you ſhall diuide the 3060 pence beforeſaid, and the Quotient bringeth forth 20 pence, which is the firſt number or payment to begin the Progreſſion withall: and ſo each week ſucceeding to riſe 20 pence more then the weeke before, and thereby in 17 weekes ſhall 12 pound 15 ſhillings be paid: as before was ſufficiently declared. Thus much for Progreſſion Arithmeticall.

Scholar. Certainly Sir, I know not how to render you condigne thanks for theſe benefits ſhewed me, which me thinketh are ſo eaſie, delightfull, and pleaſant, that I count my ſelfe happy to be in your company.

Maſter.

Maſter. I am glad you delight ſo well herein, which is an Art of wonderfull dexteritie to all ſorts of men of what degrée or profeſsion ſoeuer they be. And now will I procéed to Progreſsion Geometricall wherein I will be more briefe, both becauſe I haue béene ſo long in this part of Arithmeticall Progreſsion, and alſo for that it would require the knowledge of Roots and ſurd numbers, (whereof ye haue learned nothing) if I ſhould frame the like propoſitions in them as I haue done in theſe. Therefore I will onely teach you two practiſes about it, and ſo end the conſiderations and works of theſe Progreſsions.

Progreſsion Geometricall.

Progreſsion Geometricall is when the numbers increaſe by a like proportion, that is, if the ſecond number containe the firſt, 2, 3, or 4 times, and ſo forth: then the third containeth the ſecond ſo many times alſo: and ſo the fourth the third, and the fift the fourth, wherefore I ſet theſe 3 examples

3	6	12	24	48
1	3	9	27	81
2	10	50		250

Here in the firſt example you ſee, that euery number containeth the other (that goeth next before him) two times: and in the ſecond example three times, and in the third example fiue times. Now if you will know how to finde eaſily the ſumme of any ſuch number, do thus: Conſider by what numbers, they be multiplied,

multiplied, whether by 2,3,4,5,or any other, and by the ſame number multiply the laſt ſumme in the Progreſſion.

To finde the totall ſumme in any Geometricall Progreſſion.

Scholar. I pray you worke it by this example, 2,8,32,128,512,2048, which I haue framed by proceeding from 2, and continuall multiply by 4.

Maſter. Then muſt I multiply the laſt ſumme (which is 2048) by 4 alſo, and it will be 8192. Now muſt I abate from this ſum the firſt number of the Progreſſion, which here is 2; then reſteth 8190: which ſumme I muſt diuide by 1 leſſe then was the number that I multiplyed by. Seeing then I multiplyed by 4, I muſt diuide by 3, ſo diuiding 8190 by 3, the Quotient will be 2730, which is the ſumme of all the Progreſſion. And now to proue whether you can do the ſame, I giue you theſe numbers to adde by this rule 3, 15, 75, 375, 1875, 9375, 46875.

Scholar. I cannot well tell by what number this Progreſſion doth increaſe.

Maſter. In any ſuch doubt do thus: Diuide the ſecond number by the firſt, and the quotient will ſhew you the number that engendreth the Progreſſion.

Scholar. Then is that number in this example 5, for ſo many times is 3 in 15.

Maſter. So is it. Now worke as I taught.

Scholar. The laſt number is 46875, which I multiply by 5, and it yeeldeth 234375, from

The backer Rule, or the second part of the Rule of Proportion compound.

Master.

IN the second part of this Rule of Proportion composed, the third number is like vnto the first. And the Rule is to be wrought thus: you shall now, contrary to the last rule, multiply the third number and the fourth together, and that Product shall be your Diuisor. Then multiply the fifth by the second, and the Product thereof by the first: and that is the number that shall be diuided. For example, I propound this question, for a proofe of my last question of interest.

The proofe of the last question.

A Merchant hath receiued 8 pound 12 shillings, *for interest of certaine money for* 5 *moneths tearme, which he receiued after the rate of* 8 pound *in the* 100, *for a yeere. The question is now, how much money was deliuered to raise this interest.*

Behold therefore the manner, how the question is set forth.

li	moneths.	li	moneths.	li	s.
100	12	8	5	8	12

Scholar. Sir I perceiue it very well: and according

according to the doctrine which you prescribed for the working thereof: if please you now it is set downe, I thinke I can follow the worke.

Master. Nay stay a while, and before you worke, marke well how I deliuer a reason for the perfect vnderstanding of this Rule, which is thus: *If* 8 pound *in* 12 *moneths do yeeld me* 100 pound, *to take* 8 pound 12 shillings *for* 5 *moneths, must needs yeeld a great deale more.* Note.

So vpon the knowledge that I haue in this Art, the first part of this rule is answerable to the rule of three forward: and this latter part accordeth to the rule of three backward.

Scholar. Sir, I yeeld you most heartie thankes for these your last instructions, they haue giuen mee great light into these two rules, whereby I may the better by deliberation conceiue how to vse them hereafter when occasion shall require.

Master. You say well, go to now if you will, and trie your cunning in the question: But this note take with you by the way, in as much as here is mention made of shillings: turne all your money as you worke into shillings for your more ease in working. Note.

Scholar. If it please you to behold me a little, I will quickly end it: for I haue but my first, my second, and my last number to be multiplied together for my diuidend: And my third

into my fourth for my Diuisor.

li *Moneths.* *Moneths.* li s.

100——12———8———5——8——12

20 20 20

2000 160 172

12 5

4000 800

2000

24000

172

48000 4128000(

168000 8 00

24000

——4128000

Which 4128000 I diuide by 800, and my Quotient is 5160 shillings, which in pounds yéeldeth 258, my desire.

Master. I will here for this time in whole numbers end this Rule, and I will instruct you in the Rules of Fellowship. You may at your conuenient leasure for your exercise worke the same by the Rule of Three at twice. And for your aid and incouragement therein, I set downe héere a proffer how to apply it.

	A			B
Moneths.	li.	ſ	*Pound.*	li.
5	8 —	12	8	100
12	412$\frac{4}{5}$ ſ		412$\frac{4}{5}$	258 li

The

The Rule of Fellowſhip.

The Rule of Fellowſhip without time.

BVt now will I ſhew you of the Rule of Fellowſhip or Company, which hath ſundry operations, according to the diuers number of the Company. This Rule is ſometime without difference of time, and ſometimes there is in it difference of time. Firſt I will ſpeake of that without difference of time, of which let this be an example.

A queſtion of cōpany.

Foure Merchants of one Company made a banke of money diuerſly: for the firſt layed in 30 pound, *the ſecond* 50 pound, *the third* 60 pound, *and the fourth* 100 pound, *which ſtocke they occupy ſo long, till it was increaſed to* 3000 pound. *Now I demand of you what ſhould each receiue at the parting of this money.*

Scholar. I perceiue that this Rule is like the other, but yet there is a difference which I perceiue not.

Maſter. Then will I ſhew it to you, Firſt by Addition, you ſhall bring all the particular ſummes of the Merchants into one ſumme, which ſhal be the firſt ſumme in your working by the Golden Rule, and the whole ſumme of the gaines by that ſtocke ſhall be the ſecond ſumme. Now for the third ſumme

you

ſhall ſet the portion of each man one after another, and then worke by the Golden Rule, and the fourth ſumme will ſhew you each mans gaines: as in example.

30
50
60
160
240

The parcels of theſe foure Merchants make in one ſumme 240 pounds: ſet that in the firſt place, the gaines in the ſecond, and the firſt mans portion of ſtocke in the third place, thus:

240 3000
30

Now multiply the ſecond by the third, and it will be 90000, which you ſhall diuide by 240, and there will appeare 375 pounds thus:

And that is the gaines for the firſt man.

240 3000
30 375

Now for the ſecond man, ſet the 50 pound that he brought, in the third place, and worke as before: and his part will be 625 pound: as this figure ſheweth.

Likewiſe for the third man, ſet his money which was 60 pounds and his part of gaines will bee 750 pounds, as here appeareth.

240 3000
50 625

And so for the fourth man; if you set his sum which is 100 pound his gains will bee 1250 pound, as the work will declare.

240	3000
60	750

Scholar. This I perceiue: but is there any way to examine whether I haue well done or no?

240	3000
100	1250

Note this common proofe.

Master. For the triall hereof, adde together all their foure portions, and if their addition make the whole summe of their gaines, then is the worke well done.

Scholar. That will I try by and by, the foure parcels are these which added together make 3000, which is the full summe of money that they gained, whereby I know the worke is well done.

375
625
750
1250
3000

Master. Well, now another example will I put to you, not of gaines, but of losse: for one reason serueth for both.

A question of losse.

If three Merchants in one ship, and of one fellowship, had bought Merchandise, so that the first had laid out 200 pound, *the second* 300 *pound, the third* 500 *pound, and it chanced by tempest that they did cast ouer board into the Sea Merchandise of the value of* 100 pound, *how much should each man beare in this losse?*

Scholar.

Scholar. If I ſhall do in this, as you did in the other queſtion, then muſt I ioyne their three portions together 200,300, 500, which maketh 1000. Then ſay I, if 1000 loſe 100 then ſhall 200 loſe 20, and 300 ſhall loſe 30, and 500 ſhall loſe 50, as by the three figures it doth appeare plaine.

1000 — 100
200 — 20

1000 — 100
300 — 30

1000 — 100
500 — 50

Maſter. Well ſith now you haue done theſe, I will propound a queſtion of more importance, which ſhall make you not only the abler to vnderſtand this Rule, but alſo it will greatly aid you in the next Rule of Fellowſhip with time, if ſuch néed be that your money be of diuers denominations.

For this may not be forgotten in all ſuch queſtions: if the number be of diuers kinds, you muſt by reduction bring it into one kind, that is to ſay, to the leaſt value that is named in the queſtion. And likewiſe ſhall you do, if the time be of diuers kinds, as ſome yeares, ſome moneths, weekes, and dayes, you ſhall make all moneths, weekes, or dayes, according as the leaſt name of time in the queſtion is; as for example.

Firſt in diuerſity of mony. Three companions bought 2000 *ſheep & paid for them* 241 pound 13—4 *pence, of which ſumme one paide* 101 A queſtion of ſheepe.

The increase of 1 li. *yearely rent* answering to 7 *ye res*, is 8 li -- 18 s -- 5 d. -- 1 q. which for 6 li. *rent* is to bee taken 6 times, which ariseth to 53 li--10 s--7 d--1q, now because 13 s--4 d is two third parts of 1 li. therefore I take ⅔ of 8 li--18 s--5 d- 1 q, which is the increase of 1 li. forborne for 7 *yeares*, that is 5 li--18 s--11 d --2 q, which together make 59 li--9 s--6 d--3q, the *summe* to bee receiued, as was required.

li	s	d	q
35	10	7	q
5	18	11	2
59	9	6	3

The

The third Table declaring what one pound due at the end of any number of yeares vnder 31 is worth ready mony at 8 per centum, per annum.

yeeres	li	ſ	d	q	li	ſ	d	q	yeares.
1	0	18	7		0		10	0	16
2	0	17	10	3	0	5	4	3	17
3	0	15	1	2	0	5	0	0	18
4	0	14	8	1	0	4	7	2	19
5	0	13	7	1	0	4	3	1	20
6	0	12	7	0	0	3	11	2	21
7	0	11	8		0	3	8	0	22
8	0	10	9	2	0	3	4	3	23
9	0	10	0	0	0	3	1	3	24
10	0	9	3	0	0	2	11	0	25
11	0	8	6	3	0	2	8	1	26
12	0	7	11	1	0	2	6	0	27
13	0	7	4	0	0	2	3	3	28
14	0	6	9	2	0	2	1	3	29
15	0		3		0	1	11	3	30

This third Table is disposed as the first, the use according to the Title thereof, being damageable.

1 Example.

Suppose there is 750 li. due to bee payed at the end of 9 yeares, the Creditour would sell this debt for present money, what ought that money to bee at the rate described in the table.

Seeke in this third Table for 9 yeares at the left side of the Table, and right against it toward the right hand, you shall finde 10 shillings, which multiplied or taken 750 times, yeeldeth 7500 shillings, which is 375 li. the value of that debt in present money.

2 Example.

There is a Lease worth 500 li. after the end of the 7 yeares; what is it worth present money, according to the rate described in the table staying till it fall.

I ſeeke in the *Table* for the 7 *yeare*, and right againſt it I finde 11 ſ——8 d; now I multiply 5500 by 11, it yeeldeth 5500 *ſhillings*, or 275. li. then 500 times 8 d. maketh 4000 d, which is 16 li——13 ſ——4 d, which added together is 291 li——13 ſ——4 d. the value of the Leaſe to be payed before it fall in hand.

li	ſ	d	o
275	0	0	0
16	13	4	0
221	13	4	0

The

The fourth Table *expressing what one pound yearely rent or annuity for any number of yeares not exceeding* 30 *is worth ready mony at* 8 per cent. per annum.

yeeres.	li.	s	d	q	li.	s	d	q	yeeres.
1	0	18	6	0	8	17	0	1	16
2	1	15	7	3	9	2	5	0	17
3	2	11	6	2	9	7	5	1	18
4	3	6	2	3	9	12	0	3	19
5	3	19	10	1	10	16	4	1	20
6	4	12	5	1	10	0	4	0	21
7	5	4	1	2	10	4	0	0	22
8	5	14	11	0	10	7	5	0	23
9	6	4	11		10	10	6	3	24
10	6	14	2	1	10	13	5	3	25
11	7	2	9	1	10	19	2	1	26
12	7	10	8	2	10	18	8	1	27
13	7	18	0	3	11	1	0	0	28
14	8	4	10	2	11	3	2	0	29
15	8	11	2	1	11	5	1	3	30

The fourth *Table* is disposed altogether as the former, and the vse thereof in like sort being *dammageable.*

1 Example.

There is an annuity or rent of 20 s. per annum *to endure 25 years, it is required what it is worth ready money?*

Looke in the *Table* for the 25 *yeares*, and right against it you shall finde 10 li—13 s—5 d—3 q which is the solution.

2 Example.

What is the lease of certaine land valued at 140 li. per annum, *to beginne presently and endure* 18 *yeares worth ready money?*

Search in the *Table* for 18 *yeares*, the *terme* named in the *question*, and right against it toward the left hand you shall finde 9 li——7 s. 5 d——1 q. which expresseth that *one pound rent* to bee bought for that *terme* is worth so much; therefore that summe 140 times is

the value required. Now 140 times 9 li. is 1260. and 140 times 7 s. is 980 s. or 49 li; likewise 140 times 5 d. is 700 d. or 2 li. —18 s—4 d. and 140 *farthings* is 2 s—11 d. all which added together make 1312 li.—1 s. 3 d. for the value of the said *lease*, paying no *rent*.

li	s	d
1260	0	0
49	0	0
2	18	4
	2	11
1312	1	3

3 Example.

A lease taken for 21 yeares at 13 li—6 s 8 d. per annum, *which after 5 yeares expired, the Tenant is desirous to give a fine, and bring the rent downe to* 8 *li.* per annum, *for the rest of the terme, the demand is what fyne is to bee payed?*

Subtract 5 *yeares* from 21, the remaine 16, is the time vnexpired; likewise from the present *rent* abate 8 li. the rest wil be 5 li—6 s—8 d now the drift of the *question* is, what 5 li—6 s —8 d. *yearly rent* or *annuity* to indure 16 yeares is worth present money?

The value of 1 li. *rent* or *annuity* answering to 16 *yeares* is, 8 li—17 s—0 d—1 q. Now

5

li.	s	d	q
40	0	0	0
4	5	0	0
2	19	1	1
47	4	1	1

times 8 li. is 40 li. and 5 times 17 s. is 4 li. —5 s. and 5 times one farthing, is 1 d. —1 q. and because s.—8 d. is 1/3 of 1 li. take 1/3 of 8 li—17 s. —0 d—1 q. which 2 li—19 s.—0 d. all which added together, make 4 li—4 s—1 d— q. which is the *fyne* that ought to bee paid to bring the *rent* to 8 li. *per annum*.

The fifth Table declaring what yearely rent or annuity one pound ready money will purchase for any number of yeares under 31, *at* 8 per centum per annum.

yeeres.	*li*	*s*	*d*	*q*	*li*	*s*	*d*	*q*	*yeeres.*
1		1	7	0		2	9	3	16
2		14	0	0		2	8	3	17
3		[illegible]	8	2		2	8	0	18
4		7	6	2		2	7	0	19
5		6	3	0		2	6	2	20
6		5	4	3		2	5	3	21
7		4	9	2		2	5	1	22
8		4	4	0		2	4	3	23
9		4	0	0		2	4	1	24
10		3	8	2		2	4	0	25
11		[illegible]	6	0		2	[illegible]	3	26
12		[illegible]	3	3		2	3	1	27
13		3	1	3		2	3	0	28
14		3	0	1		2	2	3	29
15		2	11	0			2	2	30

In the fifth *Table* the *numbers* and *Columnes* are all disposed as the former *Tables*, and needeth no further explanation but onely *Examples*.

1 Example.

The Table declareth at first sight what yearely rent or Annuity one pound ready money will purchase for any terme in the Table expressed.

But if the *ready money* bee aboue one pound, then if any *value* or *rent* set downe in this *Table*, bee multiplyed by the *number* belonging to the *yeares in question*, the *product* will shew what *yearly rent* or *Annuity* that *ready money* will purchase for the *time proposed*.

2 Example.

A certaine man hath 750li. *to purchase an Annuity to endure* 27 *yeares, so as it may yeeld him the like profit, as if it were put out according to the rate in the Table expressed, it is required what that annuity ought to bee?*

Because

Because the *annuity* is to endure 27 *yeares*, seeke out the *value* or *rent* set against 27 *yeares* in this *fift Table*, which is 2 s—3 d—1 q now this being the *Annuity* 20 s. *ready money* will *purchase* for that *terme* it must bee multiplyed by 750 li. as followeth because 2 s. is the tenth part of 20 s. therefore take the tenth part of 750 li. which is 75 li. which set first downe, then 750 times 3 d. is 9 li—7 s—6 d. which set vnder the former, last of all 750 *farthings* is 15 s—7 d—ob. All which added together, produce 85 li.—3 s—1 d—ob. the *yearely annuity* required.

li	s	d	q
75	0	0	0
9	7	6	0
	15	7	2
85	3	1	2

Deo soli laus, omnis honor & gloria tribuatur. Amen.

FINIS.